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RESEARCH ARTICLE

Analysis of Prey, Predator and Top Predator Model Involving Various Functional Responses

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ABSTRACT

This research proposes a mathematical model to investigate the dynamical behavior of the system of three species, namely prey, predator and top predator. The feeding behavior of each predator serves as a functional response. The interaction between the species is carried out by a functional response. Crowley Martin functional response is incorporated between prey and predator while Holling type III functional response occurs between predator and top predator. The existence of positivity and boundedness of the system have been examined. The equilibrium points of the system are determined. The system has been linearized by applying the Jacobian matrix. The main perspective used to discuss the system's dynamics is that of permanence and stability. Further stability analysis of the system is carried out at around each equilibrium point. To comprehend the dynamics of the model system, the asymptotic stability of several equilibrium solutions, both local and global, is investigated. Routh Hurwitz criteria are used to analyze local stability at every equilibrium point. Using an appropriate Lyapunov function, the global asymptotic stability of the positive interior equilibrium solution is established. From a biological perspective, a system is considered to be permanent if all of its populations continue to exist in the future. The existence of permanence conditions of the system have been determined. To support the analytical results, several numerical simulations are carried out using the MATLAB software. Finally based on the results of the analytical and numerical simulations, the impact of the functional response between the prey, predator and top predator was discussed.

Keywords: Food chain model, Functional response, Global stability, Jacobian matrix, Logistic growth, Stability analysis

Introduction

In recent years, ecological modelling research has become more interesting to both mathematicians and biologists because of its dynamism. The richness of the dynamics is yielded by the interaction of the species in the ecology. Moreover, the interaction of the species is fascinating to investigate in the ecosystem. In 1798, Malthus formulated a single-species model. The modification of the single-species model was developed by Verhulst in 1838. Based on the single-species model many models were formulated. A two-species model was developed by Lotka and Volterra such as Prey and Predator in 1926.[1–3] Numerous researchers have established different kinds of functional responses like Crowley Martin functional response, Beddington functional response, and Holling type I, II, III, and IV to study the interaction between prey and predator employing harvesting, refuge, and these responses. The relationship between a predator's rate of prey consumption per unit of time and the quantity or density of its prey is known as the functional response.[4] Many researchers studied the dynamics of prey predator model in the presence of various functional response, Allee effect, reaction and diffusion.[5,6] Later on, three species and many species models were developed. In 1961, Kerner[7] expanded

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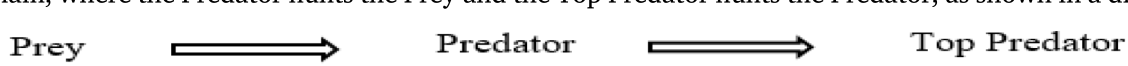
the Lotka-Volterra Model to include a three-species feeding chain. Chauvet et al.[8] investigated a linear food chain for a three-species lotka-Volterra model. Hasting and Powell[9] developed the three-level food chain model which is linear and demonstrates the chaotic dynamics in the ecosystem. The three-species model's oscillatory behavior has been investigated.[10] Klebanoff created an ecological model class that exhibits the chaotic behavior of a three-species model with bifurcation.[11] A tri-trophic food chain model with a hybrid functional response was investigated for its chaotic behavior.[12,13] The dynamics of the fractional order prey-predator model were studied by Prabir Panja together with harvesting.[14] Zabidin Salleh et al.[15] incorporated Holling type III functional responses in the tri-trophic food chain model. A cyclic three-species model's dynamic behavior was mathematically explained by Krishna das et al.[16] The dynamic behavior of the three-species food chain model was examined by numerous authors with various functional responses.[17–19] Ashok Mondal used the Crowley Martin functional response, which displays the Hopf bifurcation and persistence, to analyse the dynamical behavior of the food chain model.[20] Permanence and persistence refer to each species' ability to persist over the long term in a given population and it was first introduced by Goodman.[21] Many authors have looked into the longevity of the three-species model.[22,23] Arif et al.[24] investigated the non-autonomous prey-predator model's reaction to the fluctuation rescue effect. Ali et al.[25] studied the dynamics of food chain model involving Holling type IV and Holling type II functional response with leslie gower model. Naji[26] studied the chaotic dynamics of the prey-predator relationship. The chaotic behavior of food chain model with Holling type IV functional response was studied by Ali et al.[27,28] The stability analysis of three species model with prey T axis has been investigated.[29,30] The behavior of three species model with the effect of noise has been examined.[31,32] Researchers have looked at the dynamic behavior of a three-species model including intraspecific rivalry between predators.[33,34] The three-species model's durability and stability were extensively researched by several authors.[35–37] Many studies have incorporated the dynamic interactions of a three-species model with diverse functional responses, Allee effects, interspecific competition, refuges, and various types of delays. The models previously used were based on either a single functional response or the same type of functional response. Now, they include a mixed functional response in the model. The model introduces a novel approach by combining a mixed functional response, including Holling type III and Crowley Martin functional responses, in a three-species model. Crowley-Martin's functional response is a suitable choice among many functional responses. The Crowley-Martin functional response is utilized in situations where there is no predation occurring in a large population of both prey and predators. The ecosystem dynamics are more accurately represented by the Holling type III functional response when top predators are more effective at higher predator numbers and less effective at lower predator densities. The predator's persistence is maintained by utilizing the Holling type III functional response.

The main objective of the research is to investigate the dynamical behavior of the three-species food chain model in the presence of various functional responses such as Crowley Martin functional response and Holling type III functional response. The local and global stability of the system are analyzed. The stability of the system depends on the presence of equilibrium points. The behavior of the model is examined using the Jacobian matrix. The system's overall stability and longevity are also evaluated using the Routh Hurwitz Criteria and Lyapunov function.

Mathematical framework

Before the description of the mathematical model, some of the aspects are introduced. Three species models, comprising prey, predator, and top predator, have been considered. The three species are organized in a linear food chain, where the Predator hunts the Prey and the Top Predator hunts the Predator, as shown in a diagram.



The feeding on the three species involves mixed functional responses such as Crowley Martin Functional Response between Predator and Prey while Holling type –III functional response between Top predator and Predator. In the absence of a Predator, the Prey population grows logistically with the intrinsic growth rate (r) and the carrying capacity (K). The Population densities of Prey, Predator and Top Predator over time are represented as $N_1(t), N_2(t)$ and $N_3(\mathrm{t})$. The Crowley-Martin response function is affected by predator density, catch rate, handling time, and the level of disturbance among predators. The Crowley-Martin response functional response suggests that reciprocal interferences among predators still have a significant impact on eating rate when the prey population is huge. The Holling type-III functional response is defined by a sigmoidal relationship, where a substantial portion of predator devoured by the top predator increases in a density-dependent manner within specific predator population ranges. This physiological response enables the predator to persist.

By considering the above aspects, the mathematical model can be formulated which is given below:

$$\begin{aligned}\frac{dN_1}{dT} &= rN_1\left(1-\frac{N_1}{K}\right)-\frac{\alpha N_1N_2}{(1+AN_1)(1+BN_2)}\\ \frac{dN_2}{dT} &= \frac{\alpha_1 N_1N_2}{(1+AN_1)(1+BN_2)}-D_1N_2-\frac{\beta N_2^2N_3}{M^2+N_2^2}\\ \frac{dN_3}{dT} &= \frac{\beta_1 N_2^2N_3}{M^2+N_2^2}-D_2N_3,\end{aligned}\tag{1}$$

where $N_3(0), N_2(0), N_1(0) > 0$. The parameters of the model $\alpha, \alpha_1, \beta, \beta_1, D_1, D_2$ and M are assumed to be positive. Here α, β are the predation rates of Predator and Top predator while $\alpha_1$being the rate of transition from Prey to Predator and β_1 the rate of transition from Predator to Top Predator. M is the half-saturation constant. D_1, D_2 is the mortality rate of predator and Top Predator.

The following method (non-dimensionalization) is used to reduce the number of parameters in the system of Eq. (1). The dimensionless parameters are $n_1 = \frac{N_1}{K}, n_2 = \frac{N_2}{K}, n_3 = \frac{N_3}{K}$ and t = rT.

After non-dimensionalization, the above system is of the form

$$\begin{aligned}\frac{dn_1}{dt} &= n_1(1-n_1)-\frac{b_1n_1n_2}{(1+b_2n_1)(1+b_3n_2)}\\ \frac{dn_2}{dt} &= \frac{b_4n_1n_2}{(1+b_2n_1)(1+b_3n_2)}-d_1n_2-\frac{b_5n_2^2n_3}{1+b_6n_2^2}\\ \frac{dn_3}{dt} &= \frac{b_7n_2^2n_3}{1+b_6n_2^2}-d_2n_3,\end{aligned}\tag{2}$$

with $n_3(0) = n_{30} > 0, n_2(0) = n_{20} > 0, n_1(0) = n_{10} > 0,$ where $b_1 = \frac{\alpha K}{r}, b_2 = AK, b_3 = BK, b_4 = \frac{\alpha_1 K}{r}, d_1 = \frac{D_1K}{r}, b_5 = \frac{\beta K^2}{rM^2}, b_6 = \frac{K^2}{M^2}, b_7 = \frac{\beta_1 K^2}{rM^2}, d_2 = \frac{D_2K}{r}$.

Positivity and boundedness

The Existence of positivity in the system with its initial condition guarantees the model. The following illustrates the system's positivity and boundedness:

Positivity

The solution (n_1, n_2, n_3) for the system of Eq. (2) with its initial condition $n_1(0) \geq 0, n_2(0) \geq 0, n_3(0) \geq 0$ remains positive in R_+^3.

Proof: The system of Eq. (2) can be written in the following form with its initial condition as

$$\begin{aligned}\frac{dn_1}{n_1} &= \left[(1-n_1)-\frac{b_1n_2}{(1+b_2n_1)(1+b_3n_2)}\right]dt\\ \frac{dn_2}{n_2} &= \left[\frac{b_4n_1}{(1+b_2n_1)(1+b_3n_2)}-d_1-\frac{b_5n_2n_3}{1+b_6n_2^2}\right]dt\\ \frac{dn_3}{n_3} &= \left[\frac{b_7n_2^2}{1+b_6n_2^2}-d_2\right]dt.\end{aligned}\tag{3}$$

Integration of the above system of Eq. (3) now results in

$$n_1(t) = n_1(0)exp\left[\int_0^t \left\{(1-n_1(s)) - \frac{b_1 n_2(s)}{(1+b_2 n_1(s))\,(1+b_3 n_2(s))}\right\} ds\right] \to n_1(t) \geq 0$$
$$n_2(t) = n_2(0)exp\left[\int_0^t \left\{\frac{b_4 n_1(s)}{(1+b_2 n_1(s))\,(1+b_3 n_2(s))} - d_1 - \frac{b_5 n_2(s) n_3(s)}{1+b_6 n_2^2(s)}\right\} ds\right] \to n_2(t) \geq 0$$
$$n_3(t) = n_3(0)exp\left[\int_0^t \left\{\frac{b_7 n_2^2(s)}{1+b_6 n_2^2(s)} - d_2\right\} ds\right] \to n_3(t) \geq 0.$$

Thus all of the system of the Eq. (2) solution remains positive in R_+^3.

Boundedness

The system's of Eq. (2) possible solutions are uniformly bounded in R_+^3.

Proof: Since $\frac{dn_1}{dt} \leq n_1(1-n_1)$.

$$\text{Let B } = n_1 + \frac{b_1}{b_4} n_2 + \frac{b_1 b_5}{b_4 b_7} n_3$$
$$\frac{dB}{dt} = n_1\,(1-n_1) - \frac{b_1 d_1 n_2}{b_4} - \frac{b_1 b_5 d_2 n_3}{b_4 b_7}$$
$$\frac{dB}{dt} \leq n_1 - \frac{b_1 d_1 n_2}{b_4} - \frac{b_1 b_5 d_2 n_3}{b_4 b_7}$$
$$\frac{dB}{dt} \leq 2n_1 - KB, \text{ where } K = \min\{1, d_1, d_2\}.$$

Hence $\frac{dB}{dt} + KB \leq 2n_1 \leq 2$ [Using the known result $\text{Lim}_{t\to\infty} \sup n_1(t) \leq 1$] at the largest value of t. Solving the above differential inequality which results in

$$0 \leq B(n_1, n_2, n_3) \leq \frac{2}{K} + \frac{B(n_1(0), n_2(0), n_3(0))}{e^{Kt}} \Rightarrow 0 \leq B \leq \frac{2}{K} \text{ as t } \to \infty.$$

The system's of Eq. (2) solution lies in the region: $M = \{(n_1, n_2, n_3) : 0 \leq B \leq \frac{2}{K} + \delta, \quad for\ any\ \delta > 0\}$. Hence the theorem.

Equilibrium points

The stability of the system depends on the equilibrium points existing. The equilibrium points can be determined for the above system of Eq. (2).

1. $E_0(0, 0, 0)$ is the equilibrium point that exists trivially.
2. $E_1(1, 0, 0)$ is an axial equilibrium point that exists axially
3. $E_2(\hat{n}_1, \hat{n}_2 0)$ is the equilibrium point that exists

$$a_1 \hat{n}_1^3 + a_2 \hat{n}_1^2 + a_3 \hat{n}_1 + b_1 d_1 = 0 \text{ and}$$
$$\widehat{n}_2 = \frac{n_1\,(b_4 - b_2 d_1) - d_1}{d_1 b_3 (1 + b_2 n_1)},$$

where $a_1 = -b_2 b_3 b_4$, $a_2 = -[b_3 b_4 (1-b_2)]$, $a_3 = -[b_4\,(b_1 - b_3) - b_2 n_1 d_1]$. This equilibrium point exists only if it satisfies the condition that $b_4 > b_2 d_1$

4. The inner equilibrium point $E_3(n_1^*, n_2^*, n_3^*)$ of the system of Eq. (2) is given by

$$n_1^* = \frac{(b_2-1)}{2b_2} + \sqrt{\left(\frac{b_2-1}{2b_2}\right)^2 + \frac{b_4}{b_2}}, n_2^* = \left(\frac{d_2}{b_7 - b_6 d_2}\right)^{1/2}, n_3^* = \frac{1 + b_6\ (n_2^*)^2}{b_5 n_2^*}\left[\frac{b_4 n_1^*}{(1+b_2 n_1^*)\,(1+b_3 n_2^*)} - d_1\right]$$

where

$$b_4 = 1 - \left(\frac{b_1 {d_2}^{1/2}}{(b_7 - b_6 d_2)^{1/2} + b_3 {d_2}^{1/2}}\right)$$

This equilibrium point exists only if it follows the below conditions
i) $b_7 > b_6 d_2$ ii) $b_4 n_1^* > d_1 \left(1 + b_2 n_1^*\right)\left(1 + b_3 n_2^*\right)$ iii) $b_3 > b_1$.

Stability analysis

The Stability Analysis for the System of Eq. (2) is determined using the Jacobian matrix along with the existing equilibrium points. The Jacobian Matrix is of the form

$$\mathrm{J}\left(n_1, n_2, n_3\right) = \begin{bmatrix} \frac{\partial f}{\partial n_1} & \frac{\partial f}{\partial n_2} & \frac{\partial f}{\partial n_3} \\ \frac{\partial g}{\partial n_1} & \frac{\partial g}{\partial n_2} & \frac{\partial g}{\partial n_3} \\ \frac{\partial h}{\partial n_1} & \frac{\partial h}{\partial n_2} & \frac{\partial h}{\partial n_3} \end{bmatrix},$$

where $\mathrm{f} = n_1(1 - n_1) - \frac{b_1 n_1 n_2}{(1+b_2 n_1)(1+b_3 n_2)}$, $g = \frac{b_4 n_1 n_2}{(1+b_2 n_1)(1+b_3 n_2)} - d_1 n_2 - \frac{b_5 n_2^2 n_3}{1+b_6 n_2^2}$, $\mathrm{h} = \frac{b_7 n_2^2 n_3}{1+b_6 n_2^2} - d_2 n_3$.

Therefore, the Jacobian matrix for the system of Eq. (2) is

$$\mathrm{J}\left(n_1, n_2, n_3\right) = \begin{bmatrix} J_{11} & J_{12} & 0 \\ J_{21} & J_{22} & J_{23} \\ 0 & J_{32} & J_{33} \end{bmatrix}, \tag{4}$$

where $J_{11} = 1 - 2n_1 - \frac{b_1 n_2}{(1+b_2 n_1)^2 (1+b_3 n_2)}$, $J_{12} = -\frac{b_1 n_1}{(1+b_2 n_1)(1+b_3 n_2)^2}$, $J_{21} = \frac{b_4 n_2}{(1+b_2 n_1)^2 (1+b_3 n_2)}$, $J_{22} = \frac{b_4 n_1}{(1+b_2 n_1)(1+b_3 n_2)^2} - d_1 - \frac{2b_5 n_2 n_3}{(1+b_6 n_2^2)^2}$, $J_{23} = \frac{-b_5 n_2^2}{1+b_6 n_2^2}$, $J_{32} = \frac{2b_7 n_2 n_3}{(1+b_6 n_2^2)^2}$, $J_{33} = \frac{b_7 n_2^2}{1+b_6 n_2^2} - d_2$.

All potential equilibrium points are employed in Eq. (4) to determine the stability of the model. The procedures for determining the stability are as follows.

At the point $E_0(0, 0, 0)$ in Eq. (4) then the matrix is given by $\mathrm{J}(E_0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -d_1 & 0 \\ 0 & 0 & -d_2 \end{bmatrix}$. Thus the characteristic equation of the above matrix is given as $\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & -d_1-\lambda & 0 \\ 0 & 0 & -d_2-\lambda \end{vmatrix} = 0$. The Eigenvalues are $\lambda_1 = 1 > 0, \lambda_2 = -d_1 < 0, \lambda_3 = -d_2 < 0$. The system is unstable because the eigenvalues are real distinct and the point is saddle since one of its eigenvalue is an absolute value.

At the Point $E_1(1, 0, 0)$ in Eq. (4) then the matrix is given by $J\left(E_1\right) = \begin{bmatrix} -1 & \frac{-b_1}{1+b_2} & 0 \\ 0 & \frac{b_4}{1+b_2} - d_1 & 0 \\ 0 & 0 & -d_2 \end{bmatrix}$. Thus the characteristic equation of the above matrix is given as $\begin{vmatrix} -1-\lambda & \frac{-b_1}{1+b_2} & 0 \\ 0 & \frac{b_4}{1+b_2} - d_1 - \lambda & 0 \\ 0 & 0 & -d_2-\lambda \end{vmatrix} = 0$. The corresponding eigenvalues are $\lambda_1 = -1 < 0, \lambda_2 = \frac{b_4}{1+b_2} - d_1 < 0, \lambda_3 = -d_2 < 0$. The system is locally asymptotically stable only if $\frac{b_4}{1+b_2} < d_1$.

At the point $E_2(\hat{n}_1, \hat{n}_2, 0)$ in Eq. (4) then the Matrix

$$\mathrm{J}\left(E_2\right) = \begin{bmatrix} 1 - 2\widehat{n}_1 - \frac{b_1 \widehat{n}_2}{\left(1 + b_2 \widehat{n}_1\right)^2 \left(1 + b_3 \widehat{n}_2\right)} & -\frac{b_1 \widehat{n}_1}{\left(1 + b_2 \widehat{n}_1\right)\left(1 + b_3 \widehat{n}_2\right)^2} & 0 \\ \frac{b_4 \widehat{n}_2}{\left(1 + b_2 \widehat{n}_1\right)^2 \left(1 + b_3 \widehat{n}_2\right)} & \frac{b_4 \widehat{n}_1}{\left(1 + b_2 \widehat{n}_1\right)\left(1 + b_3 \widehat{n}_2\right)^2} - d_1 & \frac{-b_5 \widehat{n}_2^2}{1 + b_6 \widehat{n}_2^2} \\ 0 & 0 & \frac{b_7 \widehat{n}_2^2}{1 + b_6 \widehat{n}_2^2} - d_2 \end{bmatrix}$$

Thus the characteristic equation of the above matrix is given as follows

$$\begin{vmatrix} 1-2\widehat{n}_1-\dfrac{b_1\widehat{n}_2}{(1+b_2\widehat{n}_1)^2(1+b_3\widehat{n}_2)}-\lambda & -\dfrac{b_1\widehat{n}_1}{(1+b_2\widehat{n}_1)(1+b_3\widehat{n}_2)^2} & 0 \\ \dfrac{b_4\widehat{n}_2}{(1+b_2\widehat{n}_1)^2(1+b_3\widehat{n}_2)} & \dfrac{b_4\widehat{n}_1}{(1+b_2\widehat{n}_1)(1+b_3\widehat{n}_2)^2}-d_1-\lambda & \dfrac{-b_5\widehat{n}_2^2}{1+b_6\widehat{n}_2^2} \\ 0 & 0 & \dfrac{b_7\widehat{n}_2^2}{1+b_6\widehat{n}_2^2}-d_2-\lambda \end{vmatrix}=0$$

The corresponding eigenvalues are λ_1, λ_2 and λ_3. One of the root is $\lambda_3=\frac{b_7\hat{n}_2^2}{1+b_6\hat{n}_2^2}-d_2$ then the remaining roots can be found from the following characteristic equation $\lambda^2+B_1\lambda+B_2=0$ where

$$B_1=d_1-\frac{b_4\hat{n}_1}{(1+b_2\hat{n}_1)(1+b_3\hat{n}_2)^2}+\frac{b_1\hat{n}_2}{(1+b_2\hat{n}_1)^2(1+b_3\hat{n}_2)}+2\hat{n}_1-1$$

$$B_2=\frac{b_4\hat{n}_1}{(1+b_2\hat{n}_1)(1+b_3\hat{n}_2)^2}-d_1+2\hat{n}_1d_1+\frac{b_1d_1\hat{n}_2}{(1+b_2\hat{n}_1)^2(1+b_3\hat{n}_2)}-\frac{2b_4\hat{n}_1^2}{(1+b_2\hat{n}_1)(1+b_3\hat{n}_2)^2}.$$

The Eigenvalues of the above characteristic equation have negative real roots if and only if $B_1>0,\ B_2>0$ by using Routh Hurwitz criterion. Thus the system is locally asymptotically stable at E_2 only if $B_1>0,\ B_2>0$ and $b_7\hat{n}_2^2<d_2(1+b_6\hat{n}_2^2)$.

At this point, $E_3(n_1^*,n_2^*,n_3^*)$ in Eq. (4), the Jacobian matrix is given as

$$J(E_3)=\begin{bmatrix} 1-2n_1^*-\dfrac{b_1n_2^*}{(1+b_2n_1^*)^2(1+b_3n_2^*)} & -\dfrac{b_1n_1^*}{(1+b_2n_1^*)(1+b_3n_2^*)^2} & 0 \\ \dfrac{b_4n_2^*}{(1+b_2n_1^*)^2(1+b_3n_2^*)} & \dfrac{b_4n_1^*}{(1+b_2n_1^*)(1+b_3n_2^*)^2}-d_1-\dfrac{2b_5n_2^*n_3^*}{(1+b_6{n_2^*}^2)^2} & \dfrac{-b_5{n_2^*}^2}{1+b_6{n_2^*}^2} \\ 0 & \dfrac{2b_7n_2^*n_3^*}{(1+b_6{n_2^*}^2)^2} & \dfrac{b_7{n_2^*}^2}{1+b_6{n_2^*}^2}-d_2 \end{bmatrix}$$

Thus, the characteristic equation of the above matrix is given as follows

$$\begin{vmatrix} 1-2n_1^*-\dfrac{b_1n_2^*}{(1+b_2n_1^*)^2(1+b_3n_2^*)}-\lambda & -\dfrac{b_1n_1^*}{(1+b_2n_1^*)(1+b_3n_2^*)^2} & 0 \\ \dfrac{b_4n_2^*}{(1+b_2n_1^*)^2(1+b_3n_2^*)} & \dfrac{b_4n_1^*}{(1+b_2n_1^*)(1+b_3n_2^*)^2}-d_1-\dfrac{2b_5n_2^*n_3^*}{(1+b_6{n_2^*}^2)^2}-\lambda & \dfrac{-b_5{n_2^*}^2}{1+b_6{n_2^*}^2} \\ 0 & \dfrac{2b_7n_2^*n_3^*}{(1+b_6{n_2^*}^2)^2} & \dfrac{b_7{n_2^*}^2}{1+b_6{n_2^*}^2}-d_2-\lambda \end{vmatrix}=0$$

The Corresponding Characteristic Equation is as follows $\lambda^3+L_1\lambda^2+L_2\lambda+L_3=0$ where

$$L_1=d_1+\frac{2b_5n_2^*n_3^*}{(1+b_6{n_2^*}^2)^2}-\frac{b_4n_1^*}{(1+b_2n_1^*)(1+b_3n_2^*)^2}+d_2-\frac{b_7{n_2^*}^2}{1+b_6{n_2^*}^2}+2n_1^*+\frac{b_1n_2^*}{(1+b_2n_1^*)^2(1+b_3n_2^*)}-1,$$

$$\begin{aligned} L_2=\Bigg[&\frac{b_4b_7n_1^*{n_2^*}^2}{(1+b_2n_1^*)(1+b_3n_2^*)^2(1+b_6{n_2^*}^2)}-d_1d_2+\frac{2b_5n_2^*n_3^*d_2}{(1+b_6{n_2^*}^2)^2}-\frac{b_4n_1^*d_2}{(1+b_2n_1^*)(1+b_3n_2^*)^2}-\frac{b_7{n_2^*}^2d_1}{1+b_6{n_2^*}^2}\\ &+\left(\frac{b_1n_2^*}{(1+b_2n_1^*)^2(1+b_3n_2^*)}+2n_1^*-1\right)\left(d_1+d_2+\frac{2b_5n_2^*n_3^*}{(1+b_6{n_2^*}^2)^2}-\frac{b_7{n_2^*}^2}{1+b_6{n_2^*}^2}\right)+\frac{b_4n_1^*}{(1+b_2n_1^*)(1+b_3n_2^*)^2}\\ &-\frac{2b_4{n_1^*}^2}{(1+b_2n_1^*)(1+b_3n_2^*)^2}\Bigg],\end{aligned}$$

$$L_3 = \left[\left(\frac{b_1 n_2^*}{(1+b_2 n_1^*)^2 (1+b_3 n_2^*)} + 2n_1^* - 1\right)\left(\frac{2b_5 n_2^* n_3^* d_2}{(1+b_6 n_2^{*2})^2} - \frac{b_7 n_2^{*2} d_1}{1+b_6 n_2^{*2}} - d_1 d_2\right) + \frac{2b_4 b_7 n_1^* n_2^{*2}}{(1+b_2 n_1^*)(1+b_3 n_2^*)^2 (1+b_6 n_2^{*2})} - \frac{2b_4 d_2 n_1^{*2}}{(1+b_2 n_1^*)(1+b_3 n_2^*)^2} + \frac{b_4 n_1^* d_2}{(1+b_2 n_1^*)(1+b_3 n_2^*)^2} - \frac{b_4 b_7 n_1^* n_2^{*2}}{(1+b_2 n_1^*)(1+b_3 n_2^*)^2 (1+b_6 n_2^{*2})}\right].$$

The Eigenvalues of the above characteristic equation have negative real roots if and only if $L_1 > 0,\ L_3 > 0$ and $L_1 L_2 - L_3 > 0$ by using Routh Hurwitz criterion. Thus the system is locally asymptotically stable at E_3.

Global stability analysis

Here the global stability is done for the positive inner equilibrium point and not for the boundary points. The global stability exists for coexistence equilibrium point. The global stability around the interior equilibrium point $E_3(n_1^*, n_2^*, n_3^*)$ *for* the system of Eq. (2) is determined by constructing a suitable Lyapunov function. The function is given as $V(n_3, n_2, n_1) = V_3(n_3, n_2, n_1) + V_2(n_3, n_2, n_1) + V_1(n_3, n_2, n_1)$ where $V_3 = n_3 - n_3^* - n_3^* \ln\frac{n_3}{n_3^*}$, $V_2 = n_2 - n_2^* - n_2^* \ln\frac{n_2}{n_2^*}$, $V_1 = n_1 - n_1^* - n_1^* \ln\frac{n_1}{n_1^*}$.

This implies Lyapunov function (V) is continuous in R_+^3. Let us take the derivative of the V along with the time given as

$$\dot V = \frac{dV}{dt} = \frac{n_3 - n_3^*}{n_3}\frac{dn_3}{dt} + \frac{n_2 - n_2^*}{n_2}\frac{dn_2}{dt} + \frac{n_1 - n_1^*}{n_1}\frac{dn_1}{dt}$$

$$\frac{1}{n_1}\frac{dn_1}{dt} = (1 - n_1) - \frac{b_1 n_2}{(1+b_2 n_1)(1+b_3 n_2)}$$

$$\frac{1}{n_2}\frac{dn_2}{dt} = \frac{b_4 n_1}{(1+b_2 n_1)(1+b_3 n_2)} - d_1 - \frac{b_5 n_2 n_3}{1+b_6 n_2^2}$$

$$\frac{1}{n_3}\frac{dn_3}{dt} = \frac{b_7 n_2^2}{1+b_6 n_2^2} - d_2$$

$$\dot V(n_3, n_2, n_1) = (n_3 - n_3^*)\left[\frac{b_7 n_2^2}{1+b_6 n_2^2} - d_2\right] + (n_2 - n_2^*)\left[\frac{b_4 n_1}{(1+b_2 n_1)(1+b_3 n_2)} - d_1 - \frac{b_5 n_2 n_3}{1+b_6 n_2^2}\right] + (n_1 - n_1^*)\left[(1-n_1) - \frac{b_1 n_2}{(1+b_2 n_1)(1+b_3 n_2)}\right] = (n_3 - n_3^*)\left[\frac{b_7 (n_2 - n_2^*)^2}{1+b_6 (n_2 - n_2^*)^2}\right] + (n_2 - n_2^*)\left[\frac{b_4 (n_1 - n_1^*)}{(1+b_2 (n_1 - n_1^*))(1+b_3 (n_2 - n_2^*))} - \frac{b_5 (n_2 - n_2^*)(n_3 - n_3^*)}{1+b_6 (n_2 - n_2^*)^2}\right] + (n_1 - n_1^*)\left[-(n_1 - n_1^*) - \frac{b_1 (n_2 - n_2^*)}{(1+b_2 (n_1 - n_1^*))(1+b_3 (n_2 - n_2^*))}\right]$$

$$\frac{dV}{dt} = -(n_1 - n_1^*)^2 - \left[\frac{b_1 (n_2 - n_2^*)(n_1 - n_1^*)}{(1+b_2 (n_1 - n_1^*))(1+b_3 (n_2 - n_2^*))} - \frac{b_4 (n_1 - n_1^*)(n_2 - n_2^*)}{(1+b_2 (n_1 - n_1^*))(1+b_3 (n_2 - n_2^*))}\right] - (n_2 - n_2^*)^2\left[\frac{b_5 (n_3 - n_3^*)}{1+b_6 (n_2 - n_2^*)^2} - \frac{b_7 (n_3 - n_3^*)}{1+b_6 (n_2 - n_2^*)^2}\right]$$

$$\frac{dV}{dt} \le -(n_1 - n_1^*)^2 - (n_2 - n_2^*)^2\left[\frac{b_5 (n_3 - n_3^*)}{1+b_6 (n_2 - n_2^*)^2} - \frac{b_7 (n_3 - n_3^*)}{1+b_6 (n_2 - n_2^*)^2}\right]$$

$$\dot V(n_3, n_2, n_1) \le -(n_1 - n_1^*)^2 - (n_2 - n_2^*)^2\left[\frac{b_5 (n_3 - n_3^*)}{1+b_6 (n_2 - n_2^*)^2} - \frac{b_7 (n_3 - n_3^*)}{1+b_6 (n_2 - n_2^*)^2}\right]$$

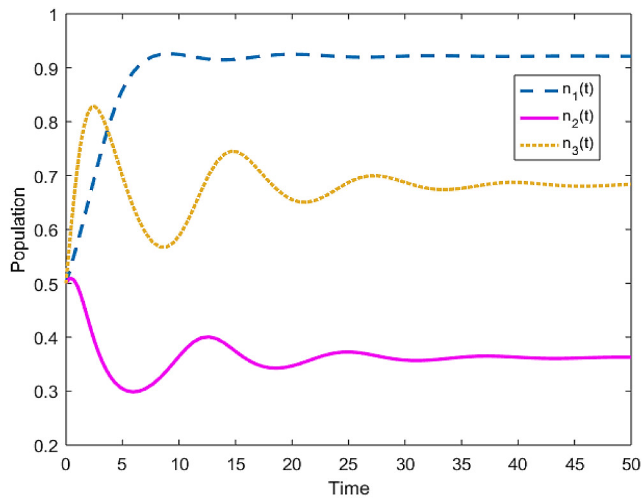


Fig. 1. Variations of n_1, n_2, and n_3 along time t converge to an equilibrium state.

Therefore, $\dot{V}$ is a negative definite function. This shows that the inner equilibrium point $E_3(n_1^*, n_2^*, n_3^*)$ of the system is globally asymptotically stable. Thus, $E_3(n_1^*, n_2^*, n_3^*)$ is globally asymptotically stable, according to the Lyapunov theorem.

Permanence

The Average Lyapunov function is used to demonstrate the system's of Eq. (2) permanence (Gard and Hallam).[22]

Theorem: The system of Eq. (2) is said to be permanent when it satisfies the following conditions as

a) $\frac{b_4}{1+b_2} > d_1$

b) $\frac{b_7 n_2^2}{1+b_6 n_2^2} > d_2$.

Proof: The typical Lyapunov function is represented by the following form $\sigma(n_1, n_2, n_3) = n_1^{p_1} n_2^{p_2} n_3^{p_3}$ where p_1, p_2 and p_3 are positive in the interior R_+^3. Then

$$\psi(n_1, n_2, n_3) = \frac{\dot{\sigma}}{\sigma} = p_1\left((1-n_1) - \frac{b_1 n_2}{(1+b_2 n_1)(1+b_3 n_2)}\right) + p_2\left(\frac{b_4 n_1}{(1+b_2 n_1)(1+b_3 n_2)} - d_1 - \frac{b_5 n_2 n_3}{1+b_6 n_2^2}\right) + p_3\left(\frac{b_7 n_2^2}{1+b_6 n_2^2} - d_2\right).$$

The Permanence of the system is validated when $\psi(n_1, n_2, n_3) > 0$. The values $\psi(n_1, n_2, n_3)$ at the equilibrium point E_0, E_1, E_2, are as follows

$$E_0: \; p_1 - p_2 d_1 - p_3 d_2$$

$$E_1: p_2\left(\frac{b_4}{1+b_2} - d_1\right) - p_3 d_2$$

$$E_2: p_3\left(\frac{b_7 \widehat{n}_2^2}{1+b_6 \widehat{n}_2^2} - d_2\right).$$

For certain $p_n > 0$ (n = 1, 2, 3), this $\psi(0,0,0) > 0$ is satisfied. Additionally, ψ is positive at E_1 *and* E_2 whenever the inequalities a) and b) hold. As a result, the system of Eq. (2) is permanent.

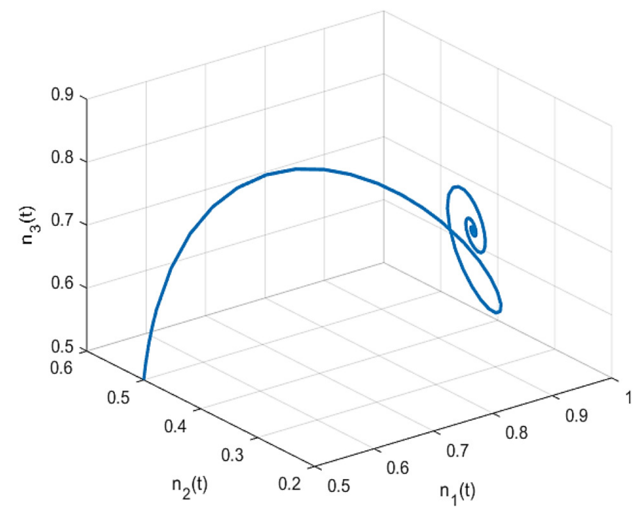


Fig. 2. The phase diagram of the system around point E (0.9212, 0.3629, and 0.6837).

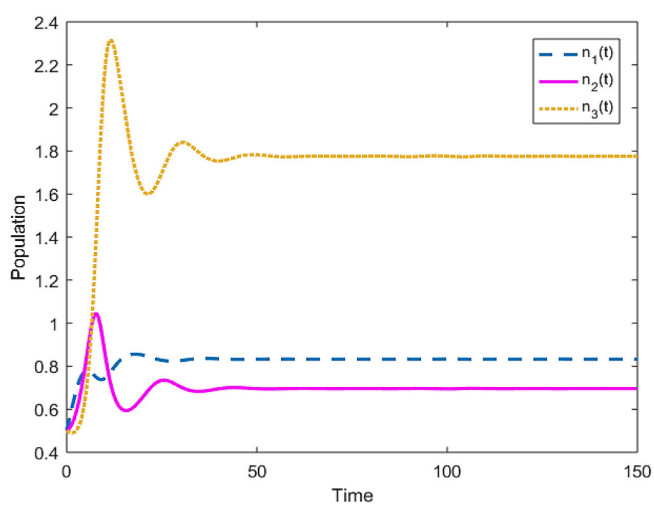


Fig. 3. The stable behavior of the system with time t.

Note

The conditions

$$E_1 = \frac{b_4}{1 + b_2} - d_1 > 0$$

$$E_2 = \frac{b_7 n_2^2}{1 + b_6 n_2^2} - d_2 > 0$$

assured that the boundary equilibrium points E_1 and E_2 are Unstable. At these conditions the system becomes unstable. Since in the Stability Analysis the system becomes stable at the equilibrium point E_1 at $\frac{b_4}{1+b_2} - d_1 < 0$ and E_2 at $\frac{b_7 \hat{n}_2^2}{1+b_6 \hat{n}_2^2} - d_2 < 0$.

Results and discussion

Analytical findings are justified by the numerical simulation. Here the dynamical behavior of the three species model has been studied analytically with mixed functional response such as Crowley martin functional response

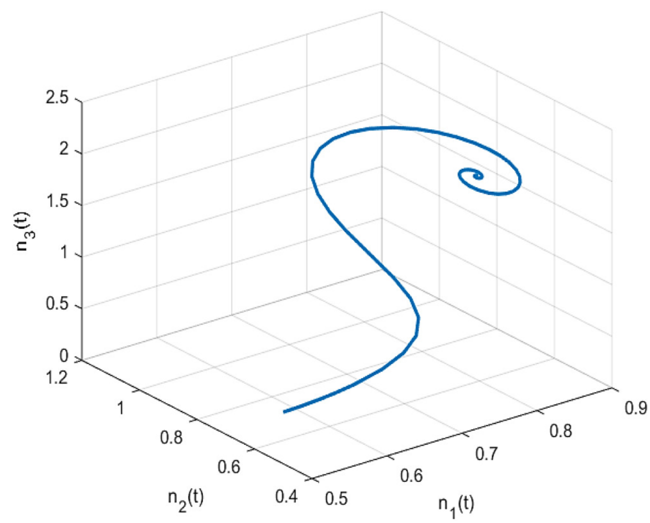


Fig. 4. The system is asymptotically stable at E (0.8327, 0.6967, and 1.777).

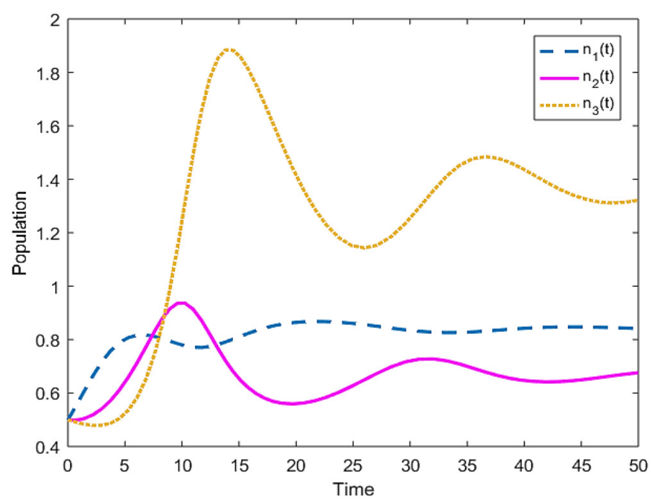


Fig. 5. Oscillatory behavior of the given system concerning time t.

and Holling type III functional response. The numerical simulations are done for the stability of the system. The Figs. 1 to 6 shows the stable, oscillatory behavior and phase portrait of the model. A numerical simulation has been done with the following set of parameters to show the dynamic behavior of the system of Eq. (2). The system's of Eq. (2) phase portraits are obtained, together with the associated time series graph. In the ecology the parameters value cannot be predicted exactly it varies. Therefore, the values are taken randomly for the simulation. The phase potraits are obtained by considering different initial points only. Let us consider the value of the parameter as $b_1 = 0.2$, $b_2 = 0.3$, $b_3 = 0.17$, $b_4 = 0.4$, $d_1 = 0.38$, $b_5 = 1.9$, $b_6 = 1.5$, $b_7 = 0.09$ and $d_2 = 0.4$ for the Fig. 1 which depicts that the system variations of n_1, n_2, and n_3 along time t converge to an equilibrium state. For the same set of values, Fig. 2 is obtained which shows the system's phase portrait, which is locally asymptotically stable at point E(0.9212, 0.3629, and 0.6837). The other set of values is $b_7 = 0.3$,$b_6 = 0.2$, $b_5 = 0.2$, $b_4 = 0.45$, $b_3 = 0.05$, $b_2 = 0.1$, $b_1 = 0.2$, $d_1 = 0.25$ and $d_2 = 0.2$ for the Fig. 3 which depicts the stable behavior of the system concerning time t. Fig. 4 shows the system's phase portrait for the values of $b_7 = 0.3$, $b_6 = 0.2$, $b_5 = 0.2$, $b_4 = 0.45$, $b_3 = 0.05$, $b_2 = 0.1$, $b_1 = 0.2$, $d_1 = 0.25$, and $d_2 = 0.2$ and the equilibrium point is asymptotically stable at E(0.8327, 0.6967, and 1.777). Fig. 5 depicts the oscillations of the populations concerning time t with the following values of parameters as $b_1 = 0.2$, $b_2 = 0.3$, $b_3 = 0.17$, $b_4 = 0.4$, $d_1 = 0.25$, $b_5 = 0.2$, $b_6 = 0.2$, $b_7 = 0.3$ and $d_2 = 0.2$. Fig. 6 depicts the phase diagram of the system with the same values of parameters which is asymptotically stable at E (0.8415, 0.676, and 1.322).

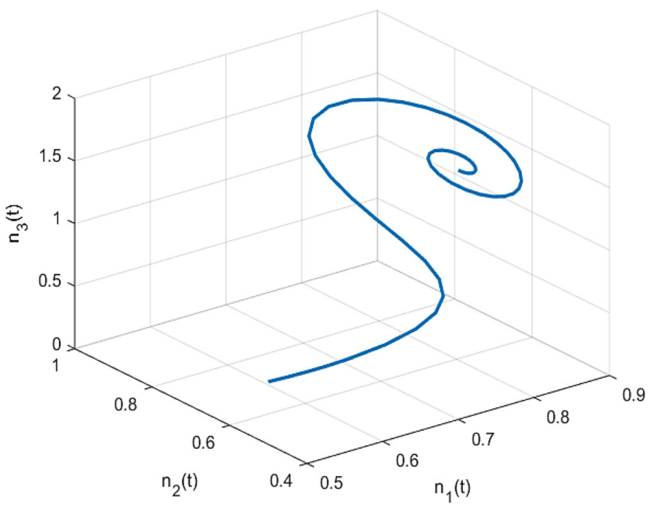


Fig. 6. The system is asymptotically stable at the point E (0.8415, 0.676, and 1.322).

Conclusion

The mathematical framework of three species models in the ecosystem with the densities of Prey (N_1), Predator (N_2) and Top Predator (N_3) has been studied in this present paper. The interaction between these species with a mixed functional response is examined. The system's positivity and boundedness are studied. The system's feasible equilibrium points are all determined. The implicit premise in deterministic scenarios is the models which are created have been justified by their stability around the interior equilibrium. The local stability is examined using Routh-Hurwitz criteria and global stability by the Lyapunov function around the interior equilibrium point $E_3(n_1^*, n_2^*, n_3^*)$. Further, the conditions for permanence are analyzed. Computational simulations are done using MATLAB software. The paper can be further extended by adding any kind of delay in the model or other functional responses such as debeddington functional response, non-monotone functional response, and Holling type IV functional response.

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Authors' declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Vellore Institute of Technology, India.

Authors' contribution statement

This work was carried out in collaboration between the authors B.D., K.K (Corresponding author) designed the study. B.D provided the research conceptualization and methodology. K.K has done the investigation. B.D performed the analysis and simulations. K.K performed the software validation. B.D made an original draft preparation of the manuscript. K.K supervised the findings of this work and reviewed. K.K edited the manuscript and made some changes in the manuscript. Both authors read the manuscript carefully and approved the final version of their manuscript.

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تحليل نموذج الفريسة والمفترس والمفترس الأعلى الذي يتضمن استجابات وظيفية مختلفة

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قسم الرياضيات، كلية العلوم المتقدمة، معهد فيلور للتكنولوجيا، فيلور، الهند.

الخلاصة

يقترح هذا البحث نموذجا رياضيا لدراسة السلوك الديناميكي لنظام ثلاثة أنواع، وهي الفريسة والمفترس والمفترس العلوي. يعد سلوك التغذية لكل حيوان مفترس بمثابة استجابة وظيفية. يتم التفاعل بين الأنواع عن طريق الاستجابة الوظيفية. يتم دمج الاستجابة الوظيفية لكراولي مارتن بين الفريسة والمفترس بينما تحدث الاستجابة الوظيفية من النوع الثالث هولينج بين المفترس والمفترس الأعلى. تم فحص وجود إيجابية وحدود النظام. يتم تحديد نقاط التوازن للنظام. لقد تم جعل النظام خطيا من خلال تطبيق المصفوفة اليعقوبية. المنظور الرئيسي المستخدم لمناقشة ديناميكيات النظام هو الديمومة والاستقرار. يتم إجراء مزيد من تحليل استقرار النظام عند كل نقطة توازن. لفهم ديناميكيات النظام النموذجي، تم دراسة الاستقرار المقارب للعديد من حلول التوازن، المحلية والعالمية. تُستخدم معايير روث هورويتز لتحليل الاستقرار المحلي عند كل نقطة توازن. باستخدام وظيفة Lyapunov المناسبة، تم إنشاء الاستقرار المقارب العالمي لحل التوازن الداخلي الإيجابي. من منظور بيولوجي، يعتبر النظام دائمًا إذا استمرت جميع سكانه في الوجود في المستقبل. تم تحديد وجود شروط دوام النظام. لدعم النتائج التحليلية، تم إجراء العديد من عمليات المحاكاة العددية باستخدام برنامج MATLAB. وأخيراً وبالاعتماد على نتائج المحاكاة التحليلية والعددية تمت مناقشة تأثير الاستجابة الوظيفية بين الفريسة والمفترس والمفترس العلوي.

الكلمات المفتاحية: ك نموذج السلسلة الغذائية، الاستجابة الوظيفية، الاستقرار العالمي، المصفوفة اليعقوبية، النمو اللوجستي، تحليل الاستقرار.