

Evaluation of the Minimum Transportation Cost of Asymmetric/Symmetric Triangular Fuzzy Numbers with α -Cut by the Row-Column Minima Method

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Abstract

In this article, the main idea is to obtain the minimum transportation total fuzzy cost of the triangular transportation problem using the row-column minima (*RCM*) method. Here, the capacity of supply, the destination of demand, and transportation cost are all fully triangular fuzzy numbers with asymmetric or symmetric but not with the negative triangular fuzzy number (TFN). Vagueness plays an active role in many fields, such as science, engineering, medicine, management, etc. In this idea, the TFN problem is decomposed into two interval integer transportation problems (IITP) using the α -cut method, by putting $\alpha = 0.5$ and $\alpha = 0$ to get the upper bound interval transportation problem and the lower bound interval transportation problem. These two interval problems are decomposed again into two problems: the right-bound transportation problem (RBTP) and the left-bound transportation problem (LBTP). First, compute an initial basic feasible solution for RBTP, then also obtain the optimum solution by the existing method; there is no need to solve LBTP directly because the solution of RBTP is the initial solution of LBTP. Apply the *RCM* method to LBTP, getting interval solutions for both interval transportation problems. Then the combined and computed the minimum fuzzy triangular transportation cost, in which an asymmetric or symmetric triangular fuzzy transportation problem (TFTP) is not changed into classical TP without using ranking methods, and the same result was obtained using the existing method. Some numerical examples are illustrated, and it is very suitable to clarify the idea of this concept. This idea is an easy way to understand the uncertainty that happens in a real-life situation.

Keywords: Asymmetric/symmetric triangular fuzzy transportation problem, Interval integer transportation problem, Row-column minima method, Transportation problem, α -cut method.

Introduction

The concept of a transportation problem (TP) is to ship a single product from one place to another place. In a TP, many methods are available to find an initial basic feasible solution (IBFS), such as the Least Cost Method as well as the North-West Corner Rule, and an optimal solution by the famous

MODI method. The TP concept was familiar to Hitchcock. TP was solved with the simplex method by Dantzig and Thapa, and then Cooper developed the modified simplex method. There are many places where uncertainty may occur due to some computational error, high cost of materials, no

accuracy in measurements, weather conditions, etc. Zadeh developed the fuzzy set concept in 1965 to deal with ambiguity in real-life settings. In 1970, solving the TP with uncertain conditions was introduced by Bellman Zadeh. In real-life situations, fuzzy is used in many places. Many methods were active to obtain an optimum solution using a α -cut mode. In an environment of fuzzy, calculate the optimum solution for asymmetric/symmetric TFN. In this article, the asymmetric/symmetric fuzzy triangular TP is decomposed into two problems, such as the Upper bound interval transportation problem (UBITP) and the Lower bound interval transportation problem (LBITP) by the α -cut method, by putting $\alpha = 0.5$ and $\alpha = 0$. These two interval problems are decomposed again into two problems the RBTP and the LBTP. First, compute an IBFS for RBTP then also obtain the optimum solution by the MODI method. Then to solve LBTP by using the RCM method, in which an optimal solution of RBTP is IBFS of LBTP and solved. In the same manner, LBITP is followed to get the minimum interval integer transportation cost of IITP. Also, combine the intervals of UBITP and LBITP to get TFTP and obtain the minimum triangular fuzzy transportation cost.

In the literature review, TFTP was transformed into a classical TP using many ranking methods and was solved by the Least cost method, Vogel's approximation method, and existing methods. But in this article, TFTP is converted into IITP by the α -cut method, then initially solved TP by MODI, and then the RCM method, where the parameters all are TFN and positive numbers, that is, availability, demand, and transportation cost. Silmi et al.¹ investigated the uncapacitated TP with supply and requirement intervals defined as a way to attain the closest optimum estimation of a heuristic idea. Habiba and Quddoos² examined an alternate optimum solution for the interval TP, in which the interval was changed into a bi-objective and then solved through fuzzy programming. Bisht and Srivastava³ determined the interval TP with data-based methods, which is converted into fuzzy TP through the trisection fuzzy idea, then applied in-center to convert the classical number. Dalman and Sivri⁴ presented and solved non-linear TP with multi-objective, where the factors are all interval and

unknown requirements. Das⁵ was presented and solved by linear programming with fractional type under conditions, and all the factors are TFN. Facade et al.⁶ suggested obtaining the least transportation cost of TFN, where TFN was transformed into classical TP by the Centroid Rank Method and all the remaining components were TFN. Then this solution was compared with the solution of the Robust Ranking Method. Hodel and Hasan⁷ derived the optimality constraints for the derivative of control theory with a fractional problem with multipliers of Lagrange. Ali Ebrahimnejad⁸ developed a method to find the standard transportation simplex algorithm, where TFN is transformed into three classical TPs, and the optimal solution is obtained when the parameters all are non-negative. Singh and Gupta⁹ suggested solving the fuzzy TP and finding the fuzzy optimum value. Here the degeneracy conditions have not occurred and compared the result with the existing solutions. Kaur et al.¹⁰ suggested a modified fuzzy programming procedure for computing the optimum solution of a solo objective TP, where the factors all are TFNs without transforming to a classical value. Ezzati et al.¹¹ recommended solving the fully fuzzy linear programming problem using the Lexicography process and it's transformed into a multi-objective linear programming problem, where the parameters all are TFN under two case studies. Gomathi and Jayalakshmi¹² developed a one's orientation approach for obtaining the optimum solution as well as an optimal schedule for a symmetrical assignment problem, in which all the factors are symmetric TFN or trapezoidal fuzzy numbers. That solution has very little iteration in getting the optimal solution. Kumar¹³ developed a modal to compute the optimum solution of intuitionistic, crisp, and fuzzy TP and assignment problems with the help of type-2 fuzzy and type-2 intuitionistic fuzzy sets. Alhindawee et al.¹⁴ suggested finding the optimum development to solid excess management through the channel of hierarchical approach. Hunwisai and Kumam¹⁵ developed a Modified Distribution Method (MODI) to obtain IBFS using the Allocation Table Method and an optimum solution by the MODI process used on the α -cut and Robust Ranking Method, where the origins, as well as demands, are real values and the fuzzy cost of transportation is TFN or trapezoidal number. Baykasoglu and Subulan¹⁶ suggested

solving fully fuzzy TP with the constrained arithmetic operation, where all the parameters are TFN and the transportation quantity is also TFN. Malik and Gupta¹⁷ suggested computing the optimal solution for the balanced TP and also obtaining the solution by goal programming approach with all parameters are intuitionistic fuzzy numbers. Dhanasekar et al.¹⁸ suggested an algorithm by the Fuzzy Hungarian MODI technique to obtain the optimum solution by transforming fuzzy triangular and trapezoidal numbers into classical TP numbers by Yagar's ranking method, where all the parameters are positive or negative TFN. Akil Basha et al.¹⁹ developed a Mid-Width Method for obtaining the optimal solution of full IITP, which involves the parameters that are real interval numbers. Balasubramanian and Subramanian²⁰ created a fuzzy and crisp optimal TFN solution in which TFTP is transformed into classical TP using indices ranking and supply, demand, and transportation costs are TFN. Pandian et al.²¹ suggested a level-bound process to obtain the optimum solution of fuzzy IITP and transform it into classical TP, where the parameters involved all are fuzzy interval triangular numbers. Prabha and Vimala²² suggested an Allocation Table Method to solve all the TP problems with the magnitude ranking method and also used for all kinds of fuzzy TP, where all the factors are trapezoidal numbers. Ravikumar et al.²³ recommended solving different types of fuzzy TP with a ranking function where all the factors are TFN or classical value. Muthuperumal et al.²⁴ developed a method for obtaining the IBFS for unbalanced TFTP, where the unbalanced TFTP was transformed into a modified triangular fuzzy unbalanced TP by increasing the availabilities or demands. Srinivasan et al.²⁵ suggested computing the minimum TP cost for TFTP and assigning the rank by ranking function and also ranking for a different type of TFN, where the availability, as well as demand, are transformed into the classical value using beta distribution for ranking. Indira and Jayalakshmi²⁶ proposed a RCM method to compute IBFS as well as an optimum solution for the fully interval integer transportation problem, in which all the factors are intervals. Faizi et al.²⁷ developed a COMET technique that is based on multi-criteria decision-making to find the solution for asymmetric and symmetric triangular fuzzy interval-valued normalized. Vidhya et al.²⁸

recommended obtaining the IBFS and optimality without transforming fuzzy TP into classical TP by the MODI method, in which the parameters involved are TFN, trapezoidal fuzzy numbers, and real numbers. Vidhya et al.²⁹ computed the IBFS and optimal solution for TFN, where all the factors are TFN with mixed constraints and without changing fuzzy numbers into crisp or classical numbers. Ammar et al.³⁰ suggested getting the rough optimal value for the triangular fuzzy interval integer TP with level and solving the interval part by the slice-sum as well as the branch-and-bound process, where the factors all involved rough TFNs. Saman and Farikhin³¹ suggested solving the fuzzy TP by using algorithms such as NNWC, NLC, and NVA, which involve ranking methods. Also, a total integral value is involved, where the factors are involved in non-normal TFN or trapezoidal fuzzy numbers. Deshmukh³² proposed a technique based on the locations of the centroid, rectangle, and trapezium in the α -cut approach. Gupta et al.³³ Suggested TP with multi-objective capacitated with mixed conditions in a fuzzy environment and parameters are linear and fractional, in which applied the fuzzy set theory and alpha cut in trapezoidal fuzzy numbers into classical value then obtain the compromise solution. Gupta et al.³⁴ developed TP with multi-objective capacitated, with linear and fractional objectives and multi-choice and trapezoidal fuzzy numbers as parameters. Here, multi-choice dealt with binary variables and ranking for trapezoidal numbers, following which the problem was solved and a compromise solution was obtained. Gupta et al.³⁵ developed a technique for the TP with multi-objective capacitated in an uncertain environment where the parameters are multi-choice and the probability distribution is turned to crisp by binary variable and stochastic programming to determine the best compromise solution by fuzzy goal programming. Gupta and Kour³⁶ investigated the TP in fractional form with discount cost in transporting time and obtained the optimum solution using the Karush Kuhn Tucker approach.

In this concept, all the parameters are TFN and positive only, solving TFN using the α -cut via, then applying the RCM method. The given TFTP is separated into two problems UBITP and LBITP using the α -cut method. Then UBITP is decomposed

into two TPs RBTP and LBTP, and to compute the optimal solution for UBTP, for which RBTP by any existing method and LBTP by the RCM method was implemented. In the same manner, proceed to solve LBTP, then combine the solutions of UBTP and LBTP to get TFTP and obtain the minimum fuzzy triangular transportation cost of symmetric/asymmetric TFN, where the TFN is not converted into classical TP by not using any ranking method. It is very easy for decision-makers to transport a single commodity from one place to another.

Motivation:

According to the literature, different techniques for solving FTPs could impact the outcome of the fuzzy optimal solution. As a consequence, this work develops a novel fuzzy transportation technique, namely the RCM method. In an algorithm, the fuzzy optimum interval solution is obtained. Many authors use ranking approaches to get an optimum solution for the TFN, but this algorithm decomposes the intervals and solves the upper interval using the existing method and the lower interval using the RCM method.

The following are the main contributions to this article:

- The TFN model with asymmetric/symmetric is developed.
- The given TFN is converted to interval form to obtain the UBTP and RBTP solution using the suggested algorithm.
- Using the RCM method to obtain the asymmetric or symmetric transportation cost.
- The comparison was made between the existing and recommended methods.
- The numerical case provides an understanding of the suggested algorithm.

Materials and Methods

PRELIMINARIES:

A. Grade of Function (or) Membership function: ¹⁸

A membership function maps an element of a domain, space, or universe of discourse to

the unit interval $[0, 1]$, that is, $\mu_A(x) = X \rightarrow [0, 1]$ this $\mu_A(x)$ is called the grade of function or membership function.

B. Fuzzy Set: ¹⁸

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval $[0, 1]$. That is, $A = \{(x, \mu_A(x); x \in X)\}$ where $\mu_A(x)$ the membership function is represented by real numbers ranging from $[0, 1]$.

C. Convex Function: ²

If a fuzzy set $\tilde{A} : R \rightarrow [0, 1]$ is convex then it satisfies the following condition $\tilde{A}(\lambda x_1 + (1 - \lambda)x_2) \geq \min\{\tilde{A}(x_1), \tilde{A}(x_2)\}, \forall x_1, x_2 \in R, \lambda \in [0, 1]$.

Note: A fuzzy set is a convex function all α level set is a convex function.

D. Fuzzy Number: ³

A fuzzy number \tilde{A} is

- Real number subset
- The function of continuous membership
- Convex, i.e. $\mu_{\tilde{A}}(\lambda s + (1 - \lambda)t) \geq \min\{\mu_{\tilde{A}}(s), \mu_{\tilde{A}}(t)\}, \forall s, t \in R, \lambda \in [0, 1]$
- Normal, in the sense that there exists at least one $x \in R$ such that $\mu_{\tilde{A}}(x) = 1$.

E. Triangular Fuzzy Number: ⁶

A fuzzy number $\tilde{B} = (b_1, b_2, b_3)$ in R is called a triangular fuzzy number if its membership function $\mu_{\tilde{B}}$ has the following resulting appearance.

$$\mu_{\tilde{B}}(x) = \begin{cases} \frac{x-b_1}{b_2-b_1}, & b_1 \leq x \leq b_2 \\ 1, & x = b_2 \\ \frac{b_3-x}{b_3-b_2}, & b_2 \leq x \leq b_3 \\ 0, & O.W \end{cases} \quad 1$$

Where b_2 is Principal(\tilde{B}), b_1 is the width of left & b_3 is the width of right.

E1: The fuzzy number of triangular $\tilde{B} = (b_1, b_2, b_3)$ is called a *positive* if $b_i \geq 0, i = 1, 2, 3$.

E2: The fuzzy number of triangular $\tilde{B} = (b_1, b_2, b_3)$ is called a **negative** if $b_i \leq 0, i = 1, 2, 3$.

E3: The fuzzy number of triangular $\tilde{B} = (b_1, b_2, b_3)$ is called a **symmetric** $b_2 - b_1 = b_3 - b_2$ otherwise, it is called asymmetric.

The diagram image of a triangular fuzzy number with an α -cut is exposed in Fig. 1.

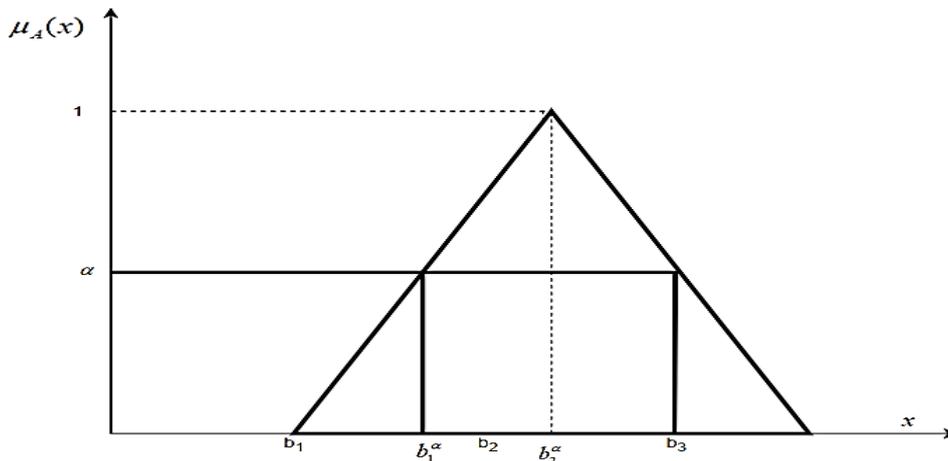


Figure 1. Triangular Fuzzy Numbers (b_1, b_2, b_3)

F. α - cut for triangular fuzzy number: ³²

The α -cut triangular fuzzy number $\tilde{B} = (b_1, b_2, b_3)$ is defined as $\bar{B}_\alpha = (b_1^\alpha, b_3^\alpha) = \{(b_2 - b_1)\alpha + b_1, b_3 - (b_3 - b_2)\alpha\}$ Where $\alpha \in [0, 1]$ and

$$b_1^\alpha = (b_2 - b_1)\alpha + b_1, \quad b_3^\alpha = b_3 - (b_3 - b_2)\alpha$$

from $\frac{b_1^\alpha - b_1}{b_2 - b_1} = \alpha = \frac{b_3 - b_3^\alpha}{b_3 - b_2}$

G. Arithmetic Operation of Triangular Fuzzy Number: ³²

The triangular fuzzy number is quantified as addition, subtraction, multiplication, and division for any two triangular fuzzy numbers $\tilde{L} = (l_1, l_2, l_3)$ and $\tilde{M} = (m_1, m_2, m_3)$ follows:

a. Add: $\tilde{L} + \tilde{M} = (l_1 + m_1, l_2 + m_2, l_3 + m_3)$

b. Sub: $\tilde{L} - \tilde{M} = (l_1 - m_3, l_2 - m_2, l_3 - m_1)$

c. Product: $\tilde{L} \times \tilde{M} = \left\{ \begin{matrix} \min(l_1 m_1, l_1 m_3, l_3 m_1, l_3 m_3), l_2 m_2 \\ \max(l_1 m_1, l_1 m_3, l_3 m_1, l_3 m_3) \end{matrix} \right\}$

d. Division: $\frac{\tilde{L}}{\tilde{M}} = \left(\frac{l_1}{m_3}, \frac{l_2}{m_2}, \frac{l_3}{m_1} \right)$

The Mathematical Model of Fuzzy Transportation Problem: ¹⁹

The mathematical form of a fuzzy transportation problem is defined as follows:

$$\text{Minimize } \tilde{Z} = \sum_{k=1}^m \sum_{l=1}^n \tilde{y}_{kl} \tilde{\beta}_{kl} \quad 2$$

Subject to the constraints

$$\sum_{l=1}^n \tilde{\beta}_{kl} \leq \tilde{s}_k \quad l = 1, 2, 3, \dots, n \quad 3$$

$$\sum_{k=1}^m \tilde{\beta}_{kl} \geq \tilde{t}_l \quad k = 1, 2, 3, \dots, m \quad 4$$

$$\tilde{\beta}_{kl} \geq 0 \quad k = 1, 2, 3 \dots, m, l = 1, 2, 3 \dots, n \quad 5$$

With the balance condition, $\sum_{k=1}^m \tilde{s}_k = \sum_{l=1}^n \tilde{t}_l$
 Where \tilde{y}_{kl} = cost of fuzzy transportation of a unit from k^{th} origin to l^{th} destination

\tilde{s}_k = Fuzzy availability in k^{th} origin

\tilde{t}_l = Fuzzy requirements in l^{th} destination

$\tilde{\beta}_{kl}$ = non-negative integer, which is a transported fuzzy transportation cost from k^{th} origin to l^{th} destination

Note:

The necessary and sufficient condition for the fuzzy LPP or fuzzy TP is to have a solution if and only if the problem is a balanced one.

Theorem 1: ²⁰

A fuzzy number A is called a positive (non-negative) iff $\mu_{\tilde{A}}(x) = 0, \forall x < 0$.

Theorem 2: ¹⁶

The necessary and sufficient conditions for a fuzzy LPP (or) fuzzy TPP to have a solution should be balanced.

Proof:

Suppose that there is an unbalanced problem. Then the constraint on the problem should be

$$\sum_{k=1}^m \tilde{s}_k \geq \sum_{l=1}^n \tilde{t}_l \quad 6$$

If the problem is unbalanced, the sources or destination must be added. Here, the source of availability and finding a feasible solution should not be satisfied by Eq. 3, 4, and 5.

That is, the problem has a solution if condition (2) is satisfied.

Obviously, $\sum_{k=1}^m \tilde{s}_k \leq \sum_{l=1}^n \tilde{t}_l$ 7

From (6) and (7),

$$\sum_{k=1}^m \tilde{s}_k = \sum_{l=1}^n \tilde{t}_l$$

Therefore, the problem is balanced. Then the problem has a solution, but the only possibility is that the given problem is balanced.

Proposed Methodology

Stage 1: Consider the given TFTP (\tilde{Z})matrix. It is separated into two interval transportation problems Upper Bound Interval Transportation Problem (UBITP) is denoted by Z_U and Lower Bound Interval Transportation Problem (LBITP) is denoted by Z_L using α - cut method, by setting $\alpha = 0.5$ and $\alpha = 0$.

Stage 2: Z_U is decomposed into two transportation problems, the right-bound transportation problem

(RBTP) and the left-bound transportation problem (LBTP), which are denoted by Z_{R1} and Z_{L1} respectively,

Stage 3: Solve Z_{R1} by the well-known method in two steps and it's indicated by η_{ij} :

- (i) An IBFS
- (ii) Optimum solution

Stage 4: Consider Z_{L1} as well as obtain the optimum solution for Z_{L1} using the RCM Method as follows:

Stage 4(a): Build Z_{L1} from the above Z_U .

Stage 4(b): Mark (*) as the optimal solution for Z_{R1} in Z_{L1} . At this point, allow the maximum amount in the fewest number of consecutive allocations that is possible or for the selected cell's column (*).

Stage 4(c): Repeat the process until the rim criteria are met. At that stage, to obtain an optimal solution Z_{L1} , which is indicated ϑ_{ij} , by the condition $\vartheta_{ij} \leq \eta_{ij}$.

Stage 5: Now combine Z_{R1} and Z_{L1} to get Z_U , which is denoted by $Z_U = (Z_{R1}, Z_{L1})$.

Stage 6: Replicate the same procedure Z_L as Z_U , to get the solution for LBITP (Z_L), which is denoted by $Z_L = (Z_{R2}, Z_{L2})$, where Z_{R2} is RBTP and Z_{L2} is LBTP.

Stage 7: Combine the intervals of Z_U and Z_L to get TFTP(\tilde{Z}), which is denoted by $\tilde{Z} = (Z_a, Z_b, Z_c)$.

Stage 8: Determine the lowest transportation cost for the given TFTP as $\tilde{Z} = \sum_{k=1}^m \sum_{l=1}^n \tilde{y}_{kl} \tilde{\beta}_{kl}$.

The architecture model for the recommended algorithm is presented in Fig. 2.

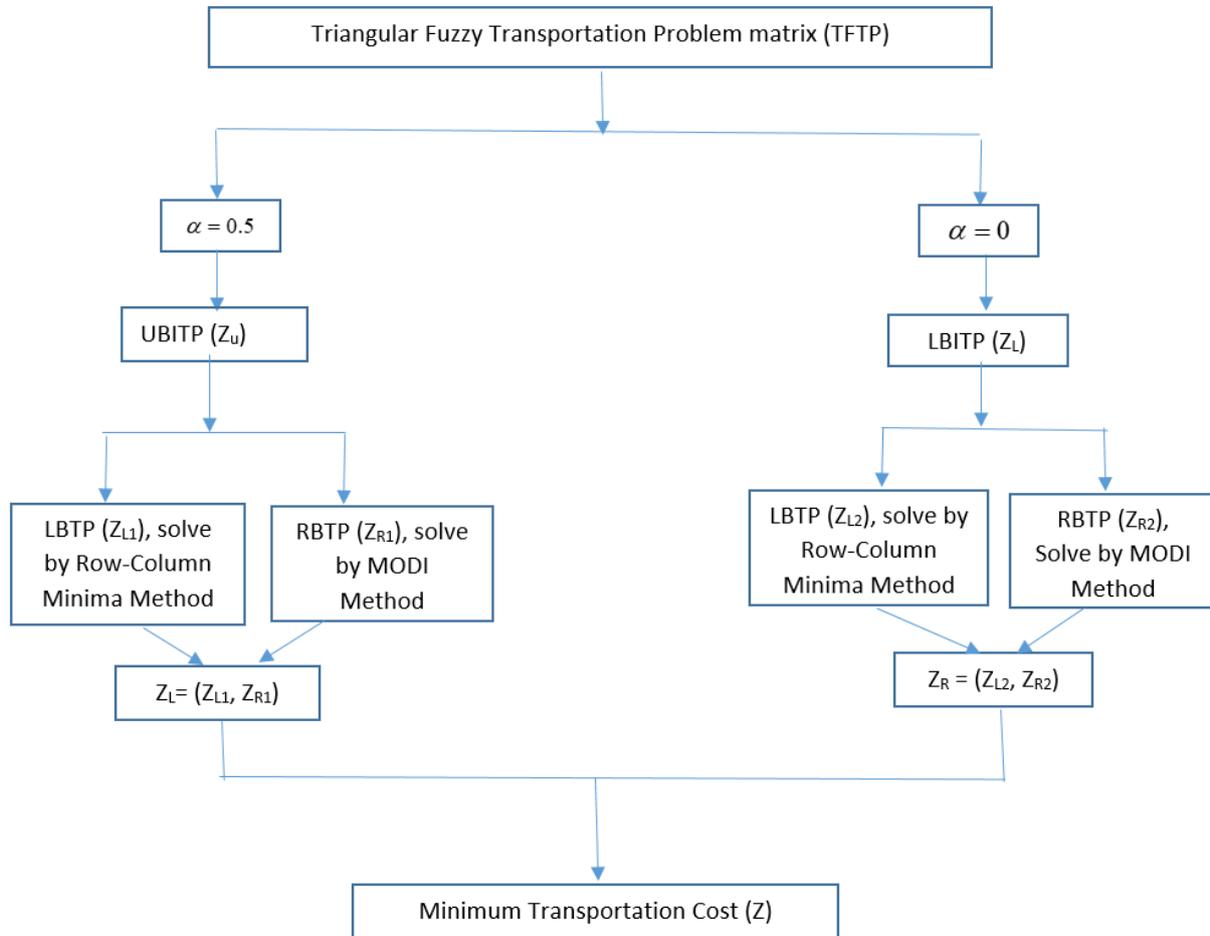


Figure 2. The recommended method flow chart is as the above

Numerical Examples:

Example 1: ^{11, 30}

A screw manufacturing concern supervisor is regarded as the best tactic to transport industrial centers η_1, η_2 and η_3 to $\gamma_1, \gamma_2, \gamma_3$ and γ_4 depots.

The unreliable weekly production and loads, along with transportation costs, are given below and the company needs to find whatever possible way to reduce costs because transportation is a major expenditure. Table 1 represents the triangular fuzzy transportation cost given below.

Table 1. Triangular fuzzy transportation Cost

	γ_1	γ_2	γ_3	γ_4	Supply \tilde{s}_k
η_1	(8, 10, 10.8)	(20.4, 22, 24)	(8, 10.2, 10.6)	(20.2, 21, 22)	(7.2, 8, 8.8)
η_2	(14, 15, 16)	(18, 20, 22)	(10, 12, 13)	(26, 28, 28.8)	(12, 14, 16)
η_3	(18.4, 20, 21)	(20.6, 22, 23)	(7.8, 9, 11)	(14, 15, 16)	(10.2, 12, 13.8)
Demand \tilde{t}_i	(6.2, 7, 7.8)	(8.9, 10, 11.1)	(6.5, 8, 9.5)	(7.8, 9, 10.2)	(29.4, 34, 38.6)

Solution:

The mathematical form of a triangular fuzzy transportation problem is

$$\text{Minimize } \tilde{Z} = \sum_{k=1}^m \sum_{l=1}^n \tilde{y}_{kl} \tilde{\beta}_{kl}$$

Subject to the constraints

$$\sum_{l=1}^n \tilde{\beta}_{kl} \leq \tilde{s}_k \quad l = 1, 2, 3, \dots, n$$

$$\sum_{k=1}^m \tilde{\beta}_{kl} \geq \tilde{t}_l \quad k = 1, 2, 3, \dots, m$$

$$\tilde{\beta}_{kl} \geq 0 \quad k = 1, 2, 3 \dots m, l = 1, 2, 3 \dots n$$

Since $\sum_{k=1}^m \tilde{s}_k = \sum_{l=1}^n \tilde{t}_l = (29.4, 34, 38.6)$ the preferred problem is balanced fuzzy TP.

Apply the α - cut method $(a, b, c) = [(b - a)\alpha + a, c - (c - b)\alpha]$ in the given asymmetric TFTP to get a below interval alpha form:

$$\left[\begin{array}{ccccc} 2\alpha + 8, 10.8 - 0.8\alpha & 1.6\alpha + 20.4, 24 - 2\alpha & 2.2\alpha + 8, 10.6 - 0.4\alpha & 0.8\alpha + 20.2, 22 - \alpha & 0.8\alpha + 7.2, 8.8 - 0.8\alpha \\ \alpha + 14, 16 - \alpha & 2\alpha + 18, 22 - 2\alpha & 2\alpha + 10, 13 - \alpha & 2\alpha + 26, 28.8 - 0.8\alpha & 2\alpha + 12, 16 - 2\alpha \\ 1.6\alpha + 18, 4, 21 - \alpha & 1.4\alpha + 20.6, 23 - \alpha & 1.2\alpha + 7.8, 11 - 2\alpha & \alpha + 14, 16 - \alpha & 1.8\alpha + 10.2, 13.8 - 1.8\alpha \\ 0.8\alpha + 6.2, 7.8 - 0.8\alpha & 1.1\alpha + 8.9, 11.1 - 1.1\alpha & 1.5\alpha + 6.5, 9.5 - 1.5\alpha & 1.2\alpha + 7.8, 10.2 - 1.2\alpha & \end{array} \right]$$

The above asymmetric interval alpha form is separated into two interval transportation problems such as UBITP and LBITP, which are denoted by Z_U and Z_L respectively. By substituting $\alpha = 0.5$ as UBITP and $\alpha = 0$ as LBITP in the above interval alpha form, as shown in Table 2 and 3. Now Z_U is decomposed into two transportation problems, RBTP (Z_{R1}) and LBTP (Z_{L1}). To compute the optimal solution Z_{R1} by an existing method, since all $d_{ij} \geq 0$, the optimal solution is represented by Table 4, and the minimum transportation cost is $Z_{R1}=546.33$. Now consider Z_{L1} , obtaining the optimal solution Z_{L1} by the RCM method which is shown in Table 5 with a minimum transportation cost of $Z_{L1}=431.58$. Therefore the minimum interval integer transportation cost of UBITP is $Z_U = (431.58,$

$546.33)$. Now Z_L is decomposed into two transportation problems, RBTP (Z_{R2}) and LBTP(Z_{L2}), and the optimum solution Z_{R2} by the existing method, since all $d_{ij} \geq 0$, the optimal solution is shown in Table 6, the minimum transportation cost is $Z_{R2}=605.54$. Now consider Z_{L2} , obtaining the optimal solution Z_{L2} by the RCM method, which is shown in Table 7 and the minimum transportation cost is $Z_{L2}=376.72$. Combinatorily, the minimum interval integer transportation cost of LBITP is $Z_L = (376.72, 605.54)$. Finally, Table 8 represents the minimum transportation cost of the given TFTP as follows: Combine the intervals Z_L and Z_U as the triangular fuzzy number and find the minimum asymmetric fuzzy transportation cost .

Table 2. Put $\alpha = 0.5$ in the above interval alpha form to get a UBITP (Z_U)

	γ_1	γ_2	γ_3	γ_4	Supply
η_1	[9, 10.4]	[21.2, 23]	[9.1, 10.4]	[20.6, 21.5]	[7.6, 8.4]
η_2	[14.5, 6.5]	[19, 21]	[11, 12.5]	[27, 28.4]	[13, 15]
η_3	[19.2, 20.5]	[21.3, 22.5]	[8.4, 10]	[14.5, 5.5]	[11.1, 2.9]
Demand	[6.6, 7.4]	[9.45, 0.55]	[7.25, .75]	[8.4, 9.6]	[31.7, 6.3]

Table 3. Put $\alpha = 0$ in the above interval alpha form to get LBITP (Z_L)

	γ_1	γ_2	γ_3	γ_4	Supply
η_1	[8, 10.8]	[20.4, 24]	[8, 10.6]	[20.2, 22]	[7.2, 8.8]
η_2	[14, 16]	[18, 22]	[10, 13]	[26, 28.8]	[12, 16]
η_3	[18.4, 21]	[20.6, 23]	[7.8, 11]	[14, 16]	[10.2, 13.8]
Demand	[6.2, 7.8]	[8.9, 11.1]	[6.5, 9.5]	[7.8, 10.2]	[29.4, 38.6]

Table 4. To compute the optimal solution (Z_{R1}) by an existing method

	γ_1	γ_2	γ_3	γ_4	Supply
η_1	10.4	23	10.4	21.5	8.4
η_2	16.5	21	12.5	28.4	15
η_3	20.5	22.5	10	15.5	12.9
Demand	7.4	10.55	8.75	9.6	36.3

Table 5. Obtaining the optimal solution Z_{L1} by the RCM method.

	γ_1	γ_2	γ_3	γ_4	Supply
η_1	9	21.2	9.1	20.6	7.6
η_2	14.5	19	11	27	13
η_3	19.2	21.3	8.4	14.5	11.1
Demand	6.6	9.45	7.25	8.4	31.7

Table 6. Compute the optimal solution Z_{R2} by the existing method

	γ_1	γ_2	γ_3	γ_4	Supply
η_1	10.8	24	10.6	22	8.8
η_2	16	22	13	28.8	16
η_3	21	23	11	16	13.8
Demand	7.8	11.1	9.5	10.2	38.6

Table 7. Obtaining the optimal solution Z_{L2} by the RCM method.

	γ_1	γ_2	γ_3	γ_4	Supply
η_1	8	20.4	8	20.2	7.2
η_2	14	18	10	26	12
η_3	18.4	20.6	7.8	14	10.2
Demand	6.2	8.9	6.5	7.8	29.4

Table 8. Minimum asymmetric triangular fuzzy transportation cost

	γ_1	γ_2	γ_3	γ_4	Supply
η_1	(6.2, 7, 7.8) (8, 10, 10.8)	(20.4, 22, 24)	(1, 1, 1) (8, 10.2, 10.6)	(20.2, 21, 22)	(7.2, 8, 8.8)
η_2	(14, 15, 16)	(8.9, 10, 11.1) (18, 20, 22)	(3.1, 4, 4.9) (10, 12, 13)	(26, 28, 28.8)	(12, 14, 16)
η_3	(18.4, 20, 21)	(20.6, 22, 23)	(2.4, 3, 3.6) (7.8, 9, 11)	(7.8, 9, 10.2) (14, 15, 16)	(10.2, 12, 13.8)
Demand	(6.2, 7, 7.8)	(8.9, 10, 11.1)	(6.5, 8, 9.5)	(7.8, 9, 10.2)	(29.4, 34, 38.6)

Using the recommended approach to obtain the minimum asymmetric triangular fuzzy transportation cost is $\tilde{Z} = (376.72, 490.2, 605.54)$.

Example 2:

A manufacturer of bold concern administrator is in the vision of the highest tactic to transport bolts from

these three trade centers η_1, η_2, η_3 to workshops $\gamma_1, \gamma_2, \gamma_3$. The unreliable daily creation and the difficulties in lengthwise transportation costs are given below (Table 9) and the concern essentials to find whatever possible way to reduce costs because transportation is a major expenditure.

Table 9. Triangular fuzzy transportation Costs

	γ_1	γ_2	γ_3	Supply \tilde{s}_k
η_1	(1, 2, 3)	(10, 11, 12)	(4, 7, 10)	(1, 6, 11)
η_2	(0, 1, 2)	(1, 6, 11)	(0, 1, 2)	(2, 3, 4)
η_3	(1, 5, 9)	(5, 15, 25)	(3, 9, 15)	(3, 4, 5)
Demand \tilde{t}_l	(3, 7, 11)	(1, 3, 5)	(2, 3, 4)	(6, 13, 20)

Solution:

The mathematical form of a fuzzy transportation problem is

$$\text{Minimize } \tilde{Z} = \sum_{k=1}^m \sum_{l=1}^n \tilde{y}_{kl} \tilde{\beta}_{kl}$$

Subject to the constraints

$$\sum_{l=1}^n \tilde{\beta}_{kl} \leq \tilde{s}_k \quad l = 1, 2, 3, \dots, n$$

$$\sum_{k=1}^m \tilde{\beta}_{kl} \geq \tilde{t}_l \quad k = 1, 2, 3, \dots, m$$

$$\tilde{\beta}_{kl} \geq 0 \quad k = 1, 2, 3 \dots m, l = 1, 2, 3 \dots n$$

Since $\sum_{k=1}^m \tilde{s}_k = \sum_{l=1}^n \tilde{t}_l = (16, 13, 20)$ the preferred problem is balanced fuzzy TP. Apply α -cut method $(a, b, c) = [(b - a)\alpha + a, c - (c - b)\alpha]$ in the given symmetric TFTP to get the below interval alpha form:

$$\left[\begin{array}{cc|cc} \alpha + 1, 3 - \alpha & \alpha + 10, 12 - \alpha & 3\alpha + 4, 10 - 3\alpha & 5\alpha + 1, 11 - 5\alpha \\ \alpha, 2 - \alpha & 5\alpha + 1, 11 - 5\alpha & \alpha, 2 - \alpha & \alpha + 2, 4 - \alpha \\ 4\alpha + 1, 9 - 4\alpha & 10\alpha + 5, 25 - 10\alpha & 6\alpha + 3, 15 - 6\alpha & \alpha + 3, 5 - \alpha \\ 4\alpha + 3, 11 - 4\alpha & 2\alpha + 1, 5 - 2\alpha & \alpha + 2, 4 - \alpha & \end{array} \right]$$

The above symmetric interval alpha form is separated into two interval transportation problems such as UBITP and LBITP, which are denoted by Z_U and Z_L respectively. By substituting $\alpha = 0.5$ as UBITP and $\alpha = 0$ as LBITP in the above interval alpha form, as shown in Table 10 and 11. Now Z_U is decomposed into two transportation problems, RBTP (Z_{R1}) and LBTP (Z_{L1}). To compute the optimal solution Z_{R1} by an existing method, since all $d_{ij} \geq 0$, the optimal solution is represented by Table 12, and the minimum transportation cost is $Z_{R1} = 94+d$. Now consider Z_{L1} , obtaining the optimal solution Z_{L1} by the RCM method which is shown in Table 13 with a minimum transportation cost of $Z_{L1} = 35+d$. Therefore the minimum interval integer transportation cost of UBITP is $Z_U = (94, 35)$. Now Z_L is decomposed into two transportation problems,

RBTP (Z_{R2}) and LBTP(Z_{L2}), and the optimum solution Z_{R2} by existing method, since all $d_{ij} \geq 0$, the optimal solution is shown in Table 14, the minimum transportation cost is $Z_{R2} = 131+d$. Now consider Z_{L2} , obtaining the optimal solution Z_{L2} by the RCM method, which is shown in Table 15 and the minimum transportation cost is $Z_{L2} = 13+d$. Combinatorily, the minimum interval integer transportation cost of LBITP is $Z_L = (131, 13)$. Finally, Table 16 represents the minimum transportation cost of the given TFTP as follows: Combine the intervals Z_L and Z_U as the triangular fuzzy number and find the minimum asymmetric fuzzy transportation cost.

Table 10. Put $\alpha = 0.5$ in the above interval alpha form to get UBITP (Z_U)

	γ_1	γ_2	γ_3	Supply
η_1	[1.5, 2.5]	[10.5, 11.5]	[5.5, 8.5]	[3.5, 8.5]
η_2	[0.5, 1.5]	[3.5, 8.5]	[0.5, 1.5]	[2.5, 3.5]
η_3	[3, 7]	[10, 20]	[6, 12]	[3.5, 4.5]
Demand	[5, 9]	[2, 4]	[2.5, 3.5]	[9.5, 16.5]

Table 11. Put $\alpha = 0$ in the above interval alpha form to get LBITP (Z_L)

	γ_1	γ_2	γ_3	Supply
η_1	[1, 3]	[10, 12]	[4, 10]	[1, 11]
η_2	[0, 2]	[1, 11]	[0, 2]	[2, 4]
η_3	[1, 9]	[5, 25]	[3, 15]	[3, 5]
Demand	[3, 11]	[1, 5]	[2, 4]	[6, 20]

Table 12. To find the optimal solution Z_{R1} to the existing method

	γ_1	γ_2	γ_3	Supply
η_1	4.5 2.5	4 11.3	8.5	8.5
η_2	d 1.5	8.5	3.5 1.5	3.5
η_3	4.5 7	20	12	4.5
Demand	9	4	3.5	16.5

Table 13. Find the optimal solution Z_{L1} by the RCM method.

	γ_1	γ_2	γ_3	Supply
η_1	1.5 1.5	2 10.5	5.5	3.5
η_2	d 0.5	3.5	2.5 0.5	2.5
η_3	3.5 3	10	6	3.5
Demand	5	2	2.5	9.5

Table 14. Compute the optimal solution Z_{R2} by the existing method.

	γ_1	γ_2	γ_3	Supply
η_1	6 3	5 12	10	11
η_2	d 2	11	4 2	4
η_3	5 9	25	15	5
Demand	11	5	4	20

Table 15. Obtaining the optimal solution Z_{L2} by the RCM method

	γ_1	γ_2	γ_3	Supply
η_1	0 1	1 10	4	1
η_2	d 0	1	2 0	2
η_3	3 1	5	3	3
Demand	3	1	2	

Table 16. Minimum symmetric triangular fuzzy transportation cost

	γ_1	γ_2	γ_3	Supply
η_1	(0, 3, 6) (1, 2, 3)	(1, 3, 5) (10, 11, 12)	(4, 7, 10) (2, 3, 4)	(1, 6, 11)
η_2	(0, 1, 2) (3, 4, 5)	(1, 6, 11)	(0, 1, 2)	(2, 3, 4)
η_3	(1, 5, 9)	(5, 15, 25)	(3, 9, 15)	(3, 4, 5)
Demand	(3, 7, 11)	(1, 3, 5)	(2, 3, 4)	(6, 13, 20)

Using the recommended approach to obtain the minimum symmetric triangular fuzzy transportation cost is $\tilde{Z} = (13, 62, 131)$.

Results and Discussion

In this proposed idea, to obtain IBFS and an optimal solution for TFTP, it is decomposed into two interval integer transportation problems (UBITP and LBITP) and then this interval TP's is decomposed into two TP's (RBTP and LBTP), solve RBTP by well-known method and LBTP by the RCM method for both UBITP and LBITP, combining to get minimal triangular transportation cost in fuzzy, where all factors are asymmetric/symmetric TFN. TFN is not transformed into classical TP by using any ranking methods. The numerical examples are explained easily to the decision-maker. In the below table, the minimal asymmetric transportation cost and optimal solution for Example 1 obtained by Sam'an et al.³¹ method and proposed methods are the same value, but the Ezzati et al.¹¹ method does not yield the same optimal solutions. Akilbasha et al.¹⁹ explained in example 2 how to find the optimal solution for fully symmetric TFN and the minimal transport cost in fuzzy is [13, 62, 131]. By the offered method, getting the same optimal solution is shown in Table 17.

Table 17. Comparison table of the offered as well as existing methods

Methods	Optimal Solution $\tilde{\beta}_{kl}$	Minimum Cost
Proposed	$\tilde{\beta}_{11} = (6.2, 7, 7.8)$ $\tilde{\beta}_{13} = (1, 1, 1)$ $\tilde{\beta}_{22} = (8.9, 10, 11.1)$ $\tilde{\beta}_{23} = (3.1, 4, 4.9)$ $\tilde{\beta}_{33} = (2.4, 3, 3.6)$ $\tilde{\beta}_{34} = (7.8, 9, 10.2)$	(376.72, 490.2, 605.54)
Sam'an et al.	$\tilde{\beta}_{11} = (6.2, 7, 7.8)$ $\tilde{\beta}_{13} = (1, 1, 1)$ $\tilde{\beta}_{22} = (8.9, 10, 11.1)$ $\tilde{\beta}_{23} = (3.1, 4, 4.9)$ $\tilde{\beta}_{33} = (2.4, 3, 3.6)$ $\tilde{\beta}_{34} = (7.8, 9, 10.2)$	(376.72, 490.2, 605.54)
Ezztai et al.	$\tilde{\beta}_{11} = (6.2, 7, 7.8)$ $\tilde{\beta}_{13} = (1, 1, 1)$ $\tilde{\beta}_{14} = (0, 0, 0.8)$ $\tilde{\beta}_{22} = (8.9, 10, 11.1)$ $\tilde{\beta}_{23} = (3.1, 4, 4.9)$ $\tilde{\beta}_{31} = (0, 0, 0.8)$ $\tilde{\beta}_{32} = (0, 0, 1.1)$ $\tilde{\beta}_{33} = (2.4, 3, 3.6)$ $\tilde{\beta}_{34} = (7.8, 9, 10.2)$	(376.72, 490.2, 643.2)

Conclusion

In this concept, IBFS and the optimal solution were computed for asymmetric and symmetric TFN, with all the parameters being positive for asymmetric and symmetric TFN. The proposed approach first applied the α -cut method to

asymmetric and symmetric TFN, where asymmetric and symmetric TFN are decomposed into UBITP for $\alpha=0.5$ and LBITP for $\alpha=0$. For the UBITP, it is transformed into two TP's, which are RBTP and LBTP, then solved the RBTP by the existing method,

and there is no need to solve the LBTP directly because the solution of RBTP is the initial solution, which is denoted by (*) for LBTP, and adopted the RCM method. In the same manner, the LBTP decomposed these interval TP's into two TP's problems, RBTP and LBTP, then applied the existing method for RBTP and adapted the RCM method for LBTP to get the minimum interval transportation cost. Also, by combining these two interval solutions, to get an asymmetric or symmetric TFN solution. The minimal transportation cost was calculated using this asymmetric/symmetric TFN,

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Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.

and the same result was obtained using the existing method. The minimum asymmetric or symmetric TFN cost was obtained without using any transformation or ranking methods. However, in most of the papers, the transformation and ranking methods were followed to get crisp values and to find the minimum asymmetric or symmetric TFN cost. In this paper, convert TFN into interval form and solve it using the proposed method. For better understanding, numerical examples with fewer iterations were provided for the minimum cost of asymmetric and symmetric TFN.

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- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Vellore Institute of Technology, Vellore, India.

Authors' Contribution Statement

P.I. is the creator of the recommended conception. P. I. carried out the computations in addition to the theoretical development. M. J. examined the analytical methods, and M. J. suggested P. I. investigate. M.J. supervised the work's outcomes, while P. I. was the manuscript's

primary author. M.J. developed the software, which independently confirmed the numerical results of the experiment that the P.I. had suggested. Both the P.I. and M.J. provided helpful explanations and facilitated to shape of the writing, analysis, and research.

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تقييم الحد الأدنى لتكلفة النقل للأرقام الغامضة المثلثية غير المتماثلة/المتماثلة مع قطع ألفا بطريقة الصف والعمود

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الخلاصة

في هذه المقالة، الفكرة الرئيسية هي الحصول على الحد الأدنى من التكلفة الإجمالية الغامضة للنقل لمشكلة النقل الثلاثي باستخدام طريقة الحد الأدنى للصف والعمود (RCM) هنا، فإن سعة العرض ووجهة الطلب وتكلفة النقل كلها أرقام غامضة مثلثية بالكامل مع غير متماثلة أو متماثلة ولكن ليس مع الرقم الغامض الثلاثي السلبي (TFN) يلعب الغموض دورًا نشطًا في العديد من المجالات، مثل العلوم والهندسة والطب والإدارة وما إلى ذلك. في هذه الفكرة، تتحلل مشكلة TFN إلى مسألتين لنقل الأعداد الصحيحة الفاصلة (IITP) باستخدام طريقة α -cut، وذلك بوضع $\alpha=0$ و $\alpha=0.5$ للحصول على مشكلة النقل الفاصل الزمني العلوي ومشكلة النقل الفاصل الزمني الأدنى. يتم تقسيم هاتين المسألتين الفاصلتين مرة أخرى إلى مشكلتين: مشكلة النقل إلى اليمين RBTP ومشكلة النقل إلى اليسار (LBTP). أولاً، قم بحساب الحل الأساسي الأولي الممكن لـ RBTP، ثم احصل أيضًا على الحل الأمثل بالطريقة الحالية؛ ليست هناك حاجة لحل LBTP مباشرة لأن حل RBTP هو الحل الأولي لـ LBTP. قم بتطبيق طريقة RCM على LBTP، للحصول على حلول الفاصل الزمني لمشكلتي النقل الفاصل. ثم تم دمج وحساب الحد الأدنى لتكلفة النقل الثلاثي الغامض، حيث لا يتم تغيير مشكلة النقل الغامض الثلاثي غير المتماثل أو المتماثل TFNP إلى TP الكلاسيكية دون استخدام طرق التصنيف، وتم الحصول على نفس النتيجة باستخدام الطريقة الحالية. وقد تم توضيح بعض الأمثلة العددية، وهي مناسبة جدًا لتوضيح فكرة هذا المفهوم. هذه الفكرة هي طريقة سهلة لفهم حالة عدم اليقين التي تحدث في مواقف الحياة الواقعية.

الكلمات المفتاحية: مشكلة النقل الغامض الثلاثي غير المتماثل/المتماثل، مشكلة نقل الأعداد الصحيحة الفاصلة، طريقة الحد الأدنى لعمود الصف، مشكلة النقل، طريقة قطع α .