Pre-Test Single and Double Stage Shrunken Estimators for the Mean of Normal Distribution with Known Variance

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Abstract:

This paper is concerned with pre-test single and double stage shrunken estimators for the mean (μ) of normal distribution when a prior estimate (μ0) of the actule value (μ) is available, using specifying shrinkage weight factors ψ(·) as well as pre-test region (R).

Expressions for the Bias [B(·)], mean squared error [MSE(·)], Efficiency [EFF(·)] and Expected sample size [E(n/μ)] of proposed estimators are derived. Numerical results and conclusions are drawn about selection different constants included in these expressions. Comparisons between suggested estimators, with respect to classical estimators in the sense of Bias and Relative Efficiency, are given. Furthermore, comparisons with the earlier existing works are drawn.

Key words: Shrinkage estimator, single- stage shrinkage estimator, double- stage shrinkage estimator, mean square error, expected sample size, efficiency.

Introduction:

Assume that $x_1, x_2, \ldots, x_n$ be a random sample of size (n) from a normal population with known variance (σ^2) and un-known mean (μ). In conventional notation, we write $x \sim N(\mu,\sigma^2)$.

In this paper we suggest the problem of estimating the mean (μ) when some prior information (μ_0) regarding the mean (μ) is available. More specifically we assume that the prior information regarding due the following reasons, Thompson [1]:

1. we believe that (μ_0) is close to the true value of μ, or

2. we fear that, (μ_0) may be near the true value of (μ), i.e.; something bad happens if (μ) approximately equal to (μ_0) and we do not known about it.

In such a situation it is natural to start with the MLE $(\hat{\mu})$ of μ and modify it by moving it closure to (μ_0) using shrinkage weight factor [ψ(·)], so that the resulting estimator though perhaps biased, has a smaller mean squard error [MSE] than that of $(\hat{\mu})$ in some interval around (μ_0).

Pre-test single stage shrinkage estimator of (μ) is a testimator $(\tilde{\mu})$ of level of significance (α) for test the hypothesis $H_0: \mu = \mu_0$ against the hypothesis $H_A: \mu \neq \mu_0$ using test statistic $T(\hat{\mu}/\mu_0) = \frac{\hat{\mu}-\mu_0}{\sigma/\sqrt{n}}$.

If H_0 accepted we feel comfortable in using the prior information (μ_0) with $(\hat{\mu})$ in estimating (μ) using shrinkage weight factor $\psi_1(\hat{\mu})$; $0 \le \psi_1(\hat{\mu}) \le 1$.

i.e.; $\tilde{\mu} = \psi_1(\hat{\mu})\hat{\mu} + [1-\psi_1(\hat{\mu})]\mu_0$ …(1)

however, if H_0 is rejected, we suggest modify shrinkage estimator using

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another shrinkage weight factor $\psi_2(\hat{\mu})$; $0 \le \psi_2(\hat{\mu}) \le 1$.

i.e.; $\tilde{\mu} = \psi_2(\hat{\mu})\hat{\mu} + [1 - \psi_2(\hat{\mu})]\mu_0$ …(2)

therefore, the general pre-test single stage shrunken estimator (PSSSE) for the mean will be:

$$\tilde{\mu}_{SS} = \begin{cases} \psi_1(\hat{\mu})\hat{\mu} + [1 - \psi_1(\hat{\mu})]\mu_0, & \text{if } \hat{\mu} \in R, \\ \psi_2(\hat{\mu})\hat{\mu} + [1 - \psi_2(\hat{\mu})]\mu_0, & \text{if } \hat{\mu} \notin R. \end{cases}$$ …(3)

where $\psi_i(\hat{\mu})$, $0 \le \psi_i(\hat{\mu}) \le 1$ [i = 1, 2] is shrinkage weight factor specifying the belive in μ_0 which can be a function of $\hat{\mu}$ or a constant. In this paper we suggest that $\psi_1(\hat{\mu}) = 0$, $\psi_2(\hat{\mu}) = k$ [constant] and (R) is a pre-test region.

Pre-test double stage Shrunken estimator for the mean (μ) that utilize a prior estimate (μ_0) is represents as following steps:-

1.Select two positive integers (n_1) and (n_2).

2.Obtain a random sample of size (n_1) on x [first stage sample]. Compute sample mean $\hat{\mu}_1$ [MLE].

3.Choose a suitable region (R) around μ_0.

In this work we suggest pre-test region.

4.If $\hat{\mu}_1 \in R$, we suggest the shrinkage estimator which is defined in (1) [here, we suggest $\psi_1(\hat{\mu}_1) = \frac{n_1(\hat{\mu}_1 - \mu_0)^2}{c^2\sigma^2}$.

However, if $\hat{\mu}_1 \notin R$, obtain a second stage random sample of size n_2 on x and suggest the estimator of μ as the polling of two samples mean ($\hat{\mu}_p$).

i.e.; $\hat{\mu}_p = [n_1\hat{\mu}_1 + n_2\hat{\mu}_2]/n$ …(4)

where $\hat{\mu}_2$ is the mean of the second sample and $n = n_1 + n_2$.

Thus, the general pre-test double stage shrunken estimator (PDSSE) has the following form:

$$\tilde{\mu}_{DS} = \begin{cases} \psi_1(\hat{\mu}_1)\hat{\mu}_1 + [1 - \psi_1(\hat{\mu}_1)]\mu_0, & \text{if } \hat{\mu}_1 \in R, \\ \hat{\mu}_p & \text{if } \hat{\mu}_1 \notin R, \end{cases}$$ …(5)

Furthermore, in this paper we suggest the following forms of (PSSSE) and (PDSSE) respectively:-

$$\tilde{\mu}_{SS} = \begin{cases} \mu_0, & \text{if } \hat{\mu} \in R, \\ k\hat{\mu} + [1 - k]\mu_0, & \text{if } \hat{\mu} \notin R. \end{cases}$$ …(6)

and

$$\tilde{\mu}_{DS} = \begin{cases} \frac{n_1(\hat{\mu}_1 - \mu_0)^3}{c^2\sigma^2} + \mu_0, & \text{if } \hat{\mu}_1 \in R, \\ [n_1\hat{\mu}_1 + n_2\hat{\mu}_2]/n, & \text{if } \hat{\mu}_1 \notin R. \end{cases}$$ …(7)

where R is the pre-test region for testing the hypothesis $H_0 : \mu = \mu_0$ against $H_A : \mu \ne \mu_0$ with level of significance (α) using test statistic $T(\hat{\mu}/\mu_0) = \frac{\hat{\mu} - \mu_0}{\sigma/\sqrt{n}}$ in $\tilde{\mu}_{SS}$ and $T(\hat{\mu}_1/\mu_0) = \frac{\hat{\mu}_1 - \mu_0}{\sigma/\sqrt{n_1}}$ in $\tilde{\mu}_{DS}$.

i.e.;

$$\left. \begin{aligned} R &= \left[\mu_0 - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \mu_0 + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right] for \tilde{\mu}_{SS} \\ or\ R &= \left[\mu_0 - Z_{\alpha/2}\frac{\sigma}{\sqrt{n_1}}, \mu_0 + Z_{\alpha/2}\frac{\sigma}{\sqrt{n_1}}\right] for \tilde{\mu}_{DS} \end{aligned} \right\}$$ …(8)

$z_{\alpha/2}$ is $100(\alpha/2)$ percentile point of standard normal distribution. For simple notation we shall assume that R=[*a*,*b*].

Several authors have been studied (PSSSE) and (PDSSE) for the mean and variance of different distributions, for example, Thompson [1], Katti [2], Mehta and Srinivasan [3], Pandy [4], Waikar, *et al.* [5], Handa, Kambo and Al-Hemyari [6], Al-Bayyati and Arnold [7], Al-Hemyari and Al-Juboori [8] and Al-Juboori [9].

Pre-Test Single Stage Shrunken Estimator (PSSSE)

In this section, we suggest (PSSSE) defined in (6) which has the following form:

$$\tilde{\mu}_{SS} = \begin{cases} \mu_0, & \text{if } \hat{\mu} \in R, \\ k\hat{\mu} + [1-k]\mu_0, & \text{if } \hat{\mu} \notin R. \end{cases}$$

The expressions for Bias [B(·)] and mean squared error [MSE(·)] of $\tilde{\mu}_{SS}$ are represented respectively as follows:

$$\mathrm{Bias}(\tilde{\mu}_{SS} \mid \mu, R) = E(\tilde{\mu}_{SS} - \mu)$$
$$= \left\{ \int_R [-(\mu-\mu_0)] + \int_{\bar{R}} [k(\hat{\mu}-\mu_0)+(\mu_0-\mu)] \right\} f(\hat{\mu} \mid \mu) d\hat{\mu}$$
$$= \left\{ \int_R [-(\mu-\mu_0)] + \int_{-\infty}^{\infty} [k(\hat{\mu}-\mu_0)+(\mu_0-\mu)] - \int_R [k(\hat{\mu}-\mu_0)+(\mu_0-\mu)] \right\} f(\hat{\mu} \mid \mu) d\hat{\mu}$$

where $\bar{R}$ is the complement region of R=[*a*,*b*] in real sample and $f(\hat{\mu} \mid \mu)$ is p.d.f. of $\hat{\mu} \sim n(\mu, \frac{\sigma^2}{n})$.

The previous expression will result

$$B(\tilde{\mu}_{SS} \mid \mu, R) = \frac{\sigma}{\sqrt{n}}[-\lambda[1-k+k\, j_0(a^*,b^*)] - k\, j_1(a^*,b^*)] \quad \ldots(9)$$

where

$$j_\ell(a^*,b^*) = \frac{1}{\sqrt{2\pi}} \int_{a^*}^{b^*} z^\ell \exp(-z^2/2) dz, \ell = 0,1,2,\ldots \quad \ldots(10)$$

$$z = \frac{\sqrt{n}(\hat{\mu}-\mu)}{\sigma}, \lambda = \frac{\sqrt{n}(\mu-\mu_0)}{\sigma}, \quad \ldots(11)$$

$$a^* = -\lambda - z_{\alpha/2}, \; b^* = -\lambda + z_{\alpha/2}, \quad \ldots(12)$$

and k = 0.1(0.1)0.5.

$$\mathrm{MSE}(\tilde{\mu}_{SS} \mid \mu, R) = E(\tilde{\mu}_{SS} - \mu)^2$$
$$= \left\{ \int_R (\mu-\mu_0)^2 + \int_{-\infty}^{\infty} [k(\hat{\mu}-\mu_0)+(\mu_0-\mu)]^2 - \int_R [k(\hat{\mu}-\mu_0)+(\mu_0-\mu)]^2 \right\} f(\hat{\mu} \mid \mu) d\hat{\mu},$$

we conclude,

$$MSE(\tilde{\mu}_{SS} \mid \mu, R) = \frac{\sigma^2}{n} \begin{bmatrix} K^2(1+\lambda^2) - \lambda^2(2K-1) - \\ K^2[j_2(a^*,b^*) + 2\lambda j_1(a^*,b^*) + \lambda^2 j_0(a^*,b^*)] + \\ 2K\lambda[j_1(a^*,b^*) + \lambda j_0(a^*,b^*)] \end{bmatrix} \ldots(13)$$

The Efficiency of $\tilde{\mu}_{SS}$ relative to $\hat{\mu}$ is given by

$$\mathrm{R.Eff}(\tilde{\mu}_{SS} \mid \mu, R) = \frac{\mathrm{MSE}(\hat{\mu} \mid \mu)}{\mathrm{MSE}(\tilde{\mu} \mid \mu, R)} \quad \ldots(14)$$

Pre-Test Double Stage Shrunken Estimator (PDSSE)

In this section, we suggest (PDSSE) defined in (7) as follows:

$$\tilde{\mu}_{DS} = \begin{cases} \frac{n_1(\hat{\mu}_1-\mu_0)^3}{c^2\sigma^2} + \mu_0, & \text{if } \hat{\mu}_1 \in R, \\ \hat{\mu}_p & \text{if } \hat{\mu}_1 \notin R. \end{cases} \ldots(15)$$

where $\hat{\mu}_p = [n_1\hat{\mu}_1 + n_2\hat{\mu}_2]/n$, n = n_1 + n_2 and c = $(z_{\alpha/2}) \times 10^3$.

The expressions for Bias [B(·)] and mean squared error [MSE(·)] of $\tilde{\mu}_{DS}$ are respectively given below:

$$\mathrm{Bias}(\tilde{\mu}_{DS} \mid \mu, R) = E(\tilde{\mu}_{DS} - \mu)$$
$$= \left\{ \int_{\hat{\mu}_1 \in R} \int_{\hat{\mu}_2=-\infty}^{\infty} [\frac{n(\hat{\mu}_1-\mu_0)^3}{c^2\sigma^2} + \mu_0 - \mu] + \int_{\hat{\mu}_1 \in \bar{R}} \int_{\hat{\mu}_2=-\infty}^{\infty} [\hat{\mu}_p - \mu] \right\} f(\hat{\mu}_1 \mid \mu) d\hat{\mu}_1,$$

and by simple calculations, we get

$$B(\tilde{\mu}_{DS} \mid \mu, R) = \frac{\sigma}{\sqrt{n_1}} \left\{ \frac{1}{c^2}[j_3(a^*,b^*) + 3\lambda j_2(a^*,b^*) + 3\lambda^2 j_1(a^*,b^*) + \lambda^3 j_0(a^*,b^*)] - \lambda j_0(a^*,b^*) - \frac{1}{1+r} j_1(a^*,b^*) \right\}, \quad \ldots(16)$$

and,

$$MSE(\tilde{\mu}_{DS} \mid \mu, R) = \frac{\sigma^2}{n_1} \left\{ \frac{1}{c^4}[j_6(a^*,b^*) + 6\lambda j_5(a^*,b^*) + 15\lambda^2 j_4(a^*,b^*) + 20\lambda^3 j_3(a^*,b^*) + 15\lambda^4 j_2(a^*,b^*) + 6\lambda^5 j_1(a^*,b^*) + \lambda^6 j_0(a^*,b^*)] - \frac{2}{c^2}\lambda[j_3(a^*,b^*) + 3\lambda j_2(a^*,b^*) + 3\lambda^2 j_1(a^*,b^*) + \lambda^3 j_0(a^*,b^*)] + \lambda^2 j_0(a^*,b^*) - [\frac{1}{(1-r)^2} j_2(a^*,b^*) + \frac{r}{(1+r)^2} j_0(a^*,b^*) + \frac{1}{1+r} \right\}, \quad \ldots(17)$$

where $r = n_2/n_1$

Also, the expressions for the Expected sample size $E(n \mid \mu,R)$ and the percentage of the overall sample saved (p.o.s.s.) of $\tilde{\mu}_{DS}$ are respectively given as follows

$E(n \mid \mu,R) = n_1 [1 + r(1 - j_0(a^*,b^*)]$ …(18)

p.o.s.s. = $\frac{n_2}{n} j_0(a^*,b^*) \times 100$ …(19)

The efficiency of $\tilde{\mu}_{DS}$ relative to $\hat{\mu}$ is given by

$$R.Eff(\tilde{\mu}_{DS} \mid \mu, R) = \frac{MSE(\hat{\mu} \mid \mu)}{[MSE(\tilde{\mu}_{DS} \mid \mu, R)][E(n \mid \mu, R)]} \quad \ldots(20)$$

Conclusions and Numerical Results:

1 . From the expressions of Bias and MSE of $\tilde{\mu}_{SS}$, the following could be easily seen

1) i. $B(\tilde{\mu}_{SS} \mid \mu, R)$ is an odd function of λ.

ii. $MSE(\tilde{\mu}_{SS} \mid \mu, R)$ is an even function of λ.

iii. The considered estimator $\tilde{\mu}_{SS}$ is a consistent estimator of μ,

i.e;

$$\lim_{n\to\infty} MSE(\tilde{\mu}_{SS} \mid \mu, R) = 0.$$

iv. The consider estimator $\tilde{\mu}_{SS}$ dominates ($\hat{\mu}$) with large sample size (n) in the term of MSE,

i.e.;

$$\lim_{n\to\infty}[MSE(\tilde{\mu}_{SS}) - MSE(\hat{\mu})] \le 0.$$

v. Practically, the consider estimator $\tilde{\mu}_{SS}$ is unbiased when $\mu = \mu_0$,

i.e.; $\lim_{\lambda\to 0} B(\tilde{\mu} \mid \mu, R) = 0$.

2) The computations of Relative Efficiency [R.Eff($\tilde{\mu}_{SS}$)] and Bais ratio [$\sqrt{n}\, B(\tilde{\mu}_{SS})/\sigma$] of consider estimator $\tilde{\mu}_{SS}$ were made on different constant involved in it, some of these computations are given in tables (1), (2), (3), (4) and (5) for some samples of these constant e.g. α = 0.02, 0.01, 0.135, k = 0.0 (0.1)0.5 and λ = 0.0(0.1)1,2. The following numerical results from the mentioned tables were made:-

i. Relative Efficiency of $\tilde{\mu}_{SS}$ is maximum when $\mu \approx \mu_0$, and decreases with increasing value of λ.

ii. Relative Efficiency of $\tilde{\mu}_{SS}$ is maximum when the value of α is small.

i.e.; the Relative Efficiency of $\tilde{\mu}_{SS}$ decreases with size α of the pre-test region in neighbourhood of $\mu \approx \mu_0$.

iii. The Bias ratio of $\tilde{\mu}_{SS}$ are reasnobely small when $\mu \approx \mu_0$.
i.e.; The Bias ratio decreases as λ decreases.

iv. The Bias ratio of $\tilde{\mu}_{SS}$ increases when α increases.

v. The Relative Efficiency of $\tilde{\mu}_{SS}$ decreases with increasing value of k.

vi. The Effective Interval [the value of λ that makes R.Eff. greater than one] using proposed estimator $\tilde{\mu}_{SS}$ is [-1,1].

3) The consider estimator $\tilde{\mu}_{SS}$ is better than the classical estimator (MLE) and the existing estimators, for example Thompson [1], Hirano [10], Kambo, Handa and Al-Hemyari [6], Al-Hemyari and Al-Juboori [8], Al-Juboori [9] and others in terms of higher Relative Efficiency specially at $\mu \approx \mu_0$.

2. From the expressions of Bias and MSE of $\tilde{\mu}_{DS}$ we can see the following

1) i. The consider estimator $\tilde{\mu}_{DS}$ is consistent of μ.

i.e.; $\lim_{n\to\infty} MSE(\tilde{\mu}_{DS} \mid \mu, R) = 0$.

ii. The consider estimator $\tilde{\mu}_{DS}$ dominates $\hat{\mu}$ with large sample size

i.e.;

$\lim_{n\to\infty}[MSE(\tilde{\mu}_{DS}) - MSE(\hat{\mu})] \le 0$.

iii. Estimator $\tilde{\mu}_{DS}$ is unbiased when $\mu = \mu_0$.

i.e.; $\lim_{n\to 0} B(\tilde{\mu} \mid \mu, R) = 0$.

2) The computations of Relative Efficiency [R.Eff(·)], Bias ratio [B(·)], Expected sample size [E(n | μ,R)], Expected sample size proportion, Percentage of the overall sample saved and probability of a voiding the second sample were used for the estimator $\tilde{\mu}_{DS}$. These computations were performed for n_1 = 12, r =0.5, 1,2,4,8,12, λ = 0.0(0.1)1,2, α = 0.01,0.05,0.1.

Some of these computations are given in the tables (6), (7), (8), (9), (10), (11), (12), (13), (14) and (15).

The observations mentioned in the tables lead to the following results:

i. R.Eff($\tilde{\mu}_{DS}$) are adversely proporational with small value of α.

ii. R.Eff($\tilde{\mu}_{DS}$) are maximum when $\mu \approx \mu_0$ and decreasing with the increasing value of λ.

iii. B($\tilde{\mu}_{DS}$) are reasonably small when $\mu \approx \mu_0$, otherwise B($\tilde{\mu}$) will be maximum.

iv. B($\tilde{\mu}_{DS}$) are reasonably small with small value of α.

v. R.Eff($\tilde{\mu}_{DS}$) and B($\tilde{\mu}_{DS}$) are decreasing function with respect to first sample size n_1.

vi. The Expected values of sample size of $\tilde{\mu}_{DS}$ is closure to n_1 specially when $\mu \approx \mu_0$ and start faraway slowly with increasing of λ.

vii. Percentage of the overall sample saved [$\frac{n_2}{n}$ $j_0(a^*,b^*)\times 100$] is a decreasing function of λ, and has a maximum value when $\mu = \mu_0$.

viii. R.Eff($\tilde{\mu}_{DS}$) is an increasing function with respect to r ($r = n_2/n_1$).

3) The suggested estimator $\tilde{\mu}_{DS}$ is more efficient than the estimators introduced by Waikar *et al.* [5], Al-Hemyari [6] and Al-Nazal [11].

Table (1) Shown $\frac{\sqrt{n}}{\sigma}$ B(·) and R.Eff(·) for $\tilde{\mu}_{SS}$ w.r.t. α, λ, R when k = 0.1

λ / α		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.01	**R.Eff(·) B(·)**	1194.8 (0)	93.449 (0.09916)	24.829 (0.19826)	11.173 (0.29727)	6.3175 (0.39612)	4.0567 (0.49477)	2.8251 (0.59316)	2.0814 (0.69123)	1.5983 (0.78892)	1.2669 (0.88616)	1.0298 (0.98289)	0.27249 (1.910)
0.05	**R.Eff(·) B(·)**	358.31 (0)	81.289 (0.09719)	24.509 (0.1943)	11.34 (0.29123)	6.4812 (0.38791)	4.186 (0.48424)	2.9266 (0.58015)	2.1628 (0.67558)	1.6652 (0.77045)	1.3232 (0.86472)	1.0781 (0.95833)	0.28823 (1.857)
0.1	**R.Eff(·) B(·)**	227.68 (0)	73.527 (0.09559)	24.278 (0.19109)	11.487 (0.28641)	6.6216 (0.38147)	4.2945 (0.47618)	3.0095 (0.57048)	2.2274 (0.6643)	1.7167 (0.75759)	1.365 (0.85031)	1.1126 (0.94245)	0.29543 (1.8348)
0.135	**R.Eff(·) B(·)**	191.51 (0)	70.075 (0.09476)	24.165 (0.18944)	11.567 (0.28395)	6.6966 (0.3782)	4.352 (0.47213)	3.0531 (0.56567)	2.2609 (0.65878)	1.7429 (0.75139)	1.3859 (0.8435)	1.1295 (0.93507)	0.29823 (1.8265)

Table (2) Shown $\frac{\sqrt{n}}{\sigma}$ B(·) and R.Eff(·) for $\tilde{\mu}_{SS}$ w.r.t. α, λ, R when k = 0.2

λ / α		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.01	**R.Eff(·) B(·)**	298.7 (0)	76.092 (0.09831)	23.535 (0.19652)	10.954 (0.29453)	6.2748 (0.39224)	4.0573 (0.48954)	2.8387 (0.58632)	2.0993 (0.68246)	1.6177 (0.77783)	1.2867 (0.87232)	1.0498 (0.96578)	0.29423 (1.8202)
0.05	**R.Eff(·) B(·)**	89.578 (0)	49.19 (0.09439)	20.937 (0.1886)	10.722 (0.28247)	6.3889 (0.37581)	4.2179 (0.46848)	2.9904 (0.56031)	2.233 (0.65116)	1.7343 (0.74091)	1.3891 (0.82944)	1.1404 (0.91666)	0.33091 (1.7139)
0.1	**R.Eff(·) B(·)**	56.921 (0)	38.219 (0.09439)	19.273 (0.1886)	10.58 (0.282471)	6.5047 (0.37581)	4.3655 (0.46848)	3.1254 (0.54095)	2.3487 (0.62859)	1.8322 (0.71518)	1.4721 (0.80063)	1.2114 (0.8849)	0.34953 (1.6697)
0.135	**R.Eff(·) B(·)**	47.877 (0)	34.275 (0.08953)	18.526 (0.17888)	10.516 (0.26789)	6.5706 (0.3564)	4.4474 (0.44426)	3.1996 (0.53135)	2.4117 (0.61755)	1.8848 (0.70278)	1.5162 (0.78699)	1.2484 (0.87014)	0.35715 (1.6531)

Table (3) Shown $\frac{\sqrt{n}}{\sigma}$ B(·) and R.Eff(·) for $\tilde{\mu}_{SS}$ w.r.t. α, λ, R when k = 0.3

λ / α		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.01	**R.Eff(·) B(·)**	132.75 (0)	57.726 (0.09746)	21.437 (0.19478)	10.486 (0.2918)	6.1262 (0.38836)	4.0015 (0.45272)	2.8175 (0.57948)	2.0934 (0.67369)	1.6194 (0.7667)	1.2929 (0.85848)	1.0586 (0.94868)	0.31382 (1.7303)
0.05	**R.Eff(·) B(·)**	39.812 (0)	29.34 (0.09158)	16.425 (0.1829)	9.5023 (0.2737)	5.9994 (0.36372)	4.0886 (0.45272)	2.9574 (0.54046)	2.2402 (0.62674)	1.7598 (0.71136)	1.4234 (0.79416)	1.1792 (0.87498)	0.37626 (1.5709)
0.1	**R.Eff(·) B(·)**	25.298 (0)	20.967 (0.08678)	13.873 (0.17328)	8.897 (0.25924)	5.9475 (0.34441)	4.1905 (0.42854)	3.0954 (0.51143)	2.3783 (0.59289)	1.8874 (0.67276)	1.5384 (0.75094)	1.2821 (0.82735)	0.41182 (1.5045)
0.135	**R.Eff(·) B(·)**	21.279 (0)	18.276 (0.08429)	12.856 (0.16832)	8.628 (0.25184)	5.9315 (0.3346)	4.2514 (0.4164)	3.1746 (0.49702)	2.4567 (0.57633)	1.9592 (0.65418)	1.6024 (0.73049)	1.3385 (0.80522)	0.42724 (1.4796)

Table (4) Shown $\frac{\sqrt{n}}{\sigma}$ B(·) and R.Eff(·) for $\tilde{\mu}_{SS}$ w.r.t. α, λ, R when k = 0.4

λ / α		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.01	**R.Eff(·) B(·)**	74.674 (0)	43.021 (0.09662)	18.959 (0.19304)	9.8306 (0.28907)	5.8864 (0.38449)	3.894 (0.47909)	2.7631 (0.57264)	2.064 (0.66492)	1.6035 (0.75567)	1.2849 (0.84464)	1.0557 (0.93157)	0.32967 (1.6404)

0.05	**R.Eff(·) B(·)**	22.394 (0)	18.67 (0.08878)	12.474 (0.1772)	8.0578 (0.26493)	5.4118 (0.35163)	3.8258 (0.43696)	2.8338 (0.52062)	2.1832 (0.60232)	1.7377 (0.68182)	1.4211 (0.75887)	1.1891 (0.83331)	0.42076 (1.4278)
0.1	**R.Eff(·) B(·)**	14.23 (0)	12.79 (0.08237)	9.8243 (0.16438)	7.108 (0.24566)	5.1485 (0.32588)	3.8234 (0.404731)	2.9277 (0.48191)	2.3092 (0.55718)	1.8705 (0.63035)	1.5507 (0.70126)	1.3118 (0.7698)	0.47922 (1.3393)
0.135	**R.Eff(·) B(·)**	11.969 (0)	11.002 (0.07906)	8.865 (0.15776)	6.7164 (0.23579)	5.0370 (0.3128)	3.8332 (0.38853)	2.9858 (0.4627)	2.3839 (0.5351)	1.9482 (0.60557)	1.6258 (0.67399)	1.3821 (0.74029)	0.5063 (1.3062)

Table (5) Shown $\frac{\sqrt{n}}{\sigma}$ B(·) and R.Eff(·) for $\tilde{\mu}_{SS}$ w.r.t. α, λ, R when k = 0.5

λ / α		**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**2**
0.01	**R.Eff(·) B(·)**	47.791 (0)	32.357 (0.09577)	16.451 (0.1912)	9.0606 (0.28633)	5.5765 (0.38061)	3.743 (0.47386)	2.6792 (0.5658)	2.0131 (0.65615)	1.5708 (0.74458)	1.2632 (0.8308)	1.0414 (0.91446)	0.34024 (1.5505)
0.05	**R.Eff(·) B(·)**	14.332 (0)	12.696 (0.08597)	9.4691 (0.17151)	6.6729 (0.25617)	4.7435 (0.33953)	3.4784 (0.4212)	2.641 (0.50077)	2.0712 (0.5779)	1.6714 (0.65227)	1.3826 (0.72359)	1.1686 (0.79164)	0.45898 (1.2848)
0.1	**R.Eff(·) B(·)**	9.1073 (0)	8.5004 (0.07796)	7.0923 (0.15547)	5.5746 (0.23207)	4.31 (0.30735)	3.3577 (0.38091)	2.6632 (0.45239)	2.1576 (0.52148)	1.7852 (0.58794)	1.5066 (0.65157)	1.2945 (0.71225)	0.54455 (1.1741)
0.135	**R.Eff(·) B(·)**	7.6604 (0)	7.2619 (0.07382)	6.288 (0.1472)	5.1532 (0.21973)	4.1322 (0.29101)	3.3131 (0.36066)	2.6861 (0.42837)	2.2127 (0.49388)	1.8545 (0.55696)	1.5809 (0.61748)	1.3693 (0.67536)	0.58706 (1.1327)

Table (6) Shown $\frac{\sqrt{n}}{\sigma}$ B(·) and R.Eff(·) for $\tilde{\mu}_{DS}$ w.r.t. α, λ, R when r = 0.5

λ / α		**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**2**
0.01	**R.Eff (·) B(·)**	2.105 (0)	1.6536 (0.094024)	1.0066 (0.18764)	0.60965 (0.28045)	0.39313 (0.372)	0.27019 (0.46184)	0.19574 (0.54948)	0.14788 (0.63439)	0.11556 (0.716)	0.092811 (0.79371)	0.076258 (0.86687)	0.023193 (1.2133)
0.05	**R.Eff (·) B(·)**	0.0.60156 (0)	0.5572 (0.079592)	0.45654 (0.15837)	0.35141 (0.23552)	0.26638 (0.31024)	0.20383 (0.38175)	0.159 (0.44927)	0.12677 (0.51207)	0.10327 (0.56945)	0.08582 (0.62078)	0.072644 (0.6655)	0.029141 (0.70238)
0.1	**R.Eff (·) B(·)**	0.36504 (0)	0.34887 (0.06723)	0.30814 (0.13351)	0.25836 (0.19791)	0.2113 (0.25954)	0.17187 (0.31753)	0.14064 (0.3711)	0.11646 (0.41954)	0.09783 (0.46224)	0.08344 (0.49868)	0.072242 (0.52849)	0.034841 (0.47295)

Table (7) Shown $\frac{\sqrt{n}}{\sigma}$ B(·) and R.Eff(·) for $\tilde{\mu}_{DS}$ w.r.t. α, λ, R when r = 1

λ / α		**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**2**
0.01	**R.Eff(·) B(·)**	3.5272 (0)	2.4395 (0.09526)	1.2671 (0.19016)	0.70354 (0.28434)	0.43356 (0.3774)	0.29031 (0.4689)	0.20681 (0.55842)	0.15436 (0.64545)	0.11944 (0.72942)	0.095096 (0.80977)	0.77489 (0.88585)	0.021221 (1.2695)
0.05	**R.Eff(·) B(·)**	0.96468 (0)	0.8564 (0.083415)	0.64079 (0.16605)	0.45159 (0.24711)	0.31975 (0.32583)	0.23272 (0.40142)	0.17487 (0.47313)	0.1354 (0.54019)	0.10765 (0.6019)	0.087602 (0.65758)	0.072745 (0.70661)	0.025098 (0.76879)
0.1	**R.Eff(·) B(·)**	0.56204 (0)	0.52348 (0.07288)	0.43426 (0.1448)	0.33845 (0.21479)	0.25892 (0.28195)	0.19923 (0.34539)	0.15583 (0.40427	0.12434 (0.45783)	0.10124 (0.50542)	0.084038 (0.54645)	0.0713 (0.58048)	0.02937 (0.5353)

Table (8) Shown $\frac{\sqrt{n}}{\sigma}$ B(·) and R.Eff(·) for $\tilde{\mu}_{DS}$ w.r.t. α, λ, R when r = 2

λ / α		**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**2**

0.01	**R.Eff(·) B(·)**	7.1088 (0)	3.769 (0.0965)	1.5629 (0.19268)	0.78991 (0.28823)	0.46583 (0.3828)	0.30428 (0.47597)	0.21317 (0.56737)	0.15703 (0.6565)	0.12009 (0.74284)	0.09453 (0.8258)	0.07613 (0.90482)	0.017835 (1.3257)
0.05	**R.Eff(·) B(·)**	1.7986 (0)	1.4511 (0.08724)	0.91754 (0.17372)	0.56745 (0.2587)	0.36863 (0.34142)	0.2532 (0.4211)	0.18242 (0.49699)	0.13663 (0.56832)	0.10561 (0.63436)	0.083786 (0.69438)	0.067952 (0.74772)	0.01947 (0.8352)
0.1	**R.Eff(·) B(·)**	0.9779 (0)	0.86017 (0.07853)	0.63133 (0.15608)	0.43648 (0.23168)	0.30395 (0.30437)	0.21797 (0.37324)	0.16155 (0.43743)	0.12346 (0.49613)	0.096964 (0.5486)	0.078008 (0.59422)	0.06412 (0.63247)	0.022268 (0.59764)

λ \ α		**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**2**
0.01	**R.Eff(·) B(·)**	7.1088 (0)	3.769 (0.0965)	1.5629 (0.19268)	0.78991 (0.28823)	0.46583 (0.3828)	0.30428 (0.47597)	0.21317 (0.56737)	0.15703 (0.6565)	0.12009 (0.74284)	0.09453 (0.8258)	0.07613 (0.90482)	0.017835 (1.3257)
0.05	**R.Eff(·) B(·)**	1.7986 (0)	1.4511 (0.08724)	0.91754 (0.17372)	0.56745 (0.2587)	0.36863 (0.34142)	0.2532 (0.4211)	0.18242 (0.49699)	0.13663 (0.56832)	0.10561 (0.63436)	0.083786 (0.69438)	0.067952 (0.74772)	0.01947 (0.8352)
0.1	**R.Eff(·) B(·)**	0.9779 (0)	0.86017 (0.07853)	0.63133 (0.15608)	0.43648 (0.23168)	0.30395 (0.30437)	0.21797 (0.37324)	0.16155 (0.43743)	0.12346 (0.49613)	0.096964 (0.5486)	0.078008 (0.59422)	0.06412 (0.63247)	0.022268 (0.59764)

Table (9) Shown $\frac{\sqrt{n}}{\sigma}$ B(·) and R.Eff(·) for $\tilde{\mu}_{DS}$ w.r.t. α, λ, R when r = 4

λ \ α		**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**2**
0.01	**R.Eff(·) B(·)**	16.265 (00	5.3531 (0.09749)	1.7718 (0.1947)	0.83384 (0.29134)	0.47617 (0.3871)	0.3049 (0.48162)	0.21026 (0.57452)	0.15263 (0.66534)	0.11498 (0.75358)	0.08908 (0.8387)	0.070523 (0.92)	0.013345 (1.3706)
0.05	**R.Eff(·) B(·)**	3.6239 (0)	2.3922 (0.090297)	1.1783 (0.17987)	0.6325 (0.26798)	0.3793 (0.3539)	0.24731 (0.43684)	0.17132 (0.51608)	0.12415 (0.59082)	0.09314 (0.72382)	0.07185 (0.7238)	0.056732 (0.7806)	0.013341 (0.8883)
0.1	**R.Eff(·) B(·)**	1.7736 (0)	1.382 (0.083052)	0.82757 (0.16511)	0.4916 (0.24518)	0.31 (0.3223)	0.20771 (0.39553)	0.14634 (0.46396)	0.10737 (0.52676)	0.081439 (0.63244)	0.06353 (6.3244)	0.0507886 (0.67406)	0.014956 (0.64751)

Table (10) Shown $\frac{\sqrt{n}}{\sigma}$ B(·) and R.Eff(·) for $\tilde{\mu}_{DS}$ w.r.t. α, λ, R when r = 8

λ \ α		**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**2**
0.01	**R.Eff(·) B(·)**	38.438 (0)	6.4383 (0.09815)	1.8282 (0.19604)	0.82476 (0.29342)	0.46045 (0.38998)	0.28945 (0.48539)	0.19598 (0.5793)	0.13948 (0.67123)	0.10285 (0.76073)	0.077832 (0.84724)	0.060088 (0.93012)	0.0088181 (1.4006)
0.05	**R.Eff(·) B(·)**	7.1001 (0)	3.2839 (0.09234)	1.2407 (0.18396)	0.59663 (0.27416)	0.33722 (0.3622)	0.21086 (0.44734)	0.14102 (0.5288)	0.098949 (0.60583)	0.072003 (0.6776)	0.053952 (0.74344)	0.041438 (0.80253)	0.008152 (0.92375)
0.1	**R.Eff(·) B(·)**	3.027 (0)	1.879 (0.086066)	0.86867 (0.17113)	0.4492 (0.25418)	0.2619 (0.33426)	0.16652 (0.41039)	0.11268 (0.48165)	0.07992 (0.54718)	0.05884 (0.60617)	0.044692 (0.6579)	0.034878 (0.70179)	0.009001 (0.68076)

Table (11) Shown $\frac{\sqrt{n}}{\sigma}$ B(·) and R.Eff(·) for $\tilde{\mu}_{DS}$ w.r.t. α, λ, R when r = 12

λ \ α		**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**2**
0.01	**R.Eff(·) B(·)**	62.247 (0)	6.668 (0.098404)	1.7936 (0.19656)	0.7957 (0.29422)	0.43881 (0.39108)	0.27242 (0.39108)	0.18192 (0.58112)	0.12751 (0.6735)	0.092464 (0.76349)	0.068746 (0.85053)	0.052105 (0.93402)	0.006576 (1.4121)
0.05	**R.Eff(·) B(·)**	10.013 (0)	3.4937 (0.093121)	1.1605 (0.18554)	0.5334 (0.27654)	0.29368 (0.3654)	0.17983 (0.3654)	0.118 (0.5337)	0.081311 (0.6116)	0.058164 (0.68429)	0.042887 (0.75099)	0.03245 (0.81096)	0.0587 (0.93737)

0.1	R.Eff(·) B(·)	3.9066 (0)	1.9962 (0.087225)	0.79549 (0.17344)	0.3867 (0.25765)	0.21812 (0.33886)	0.13554 (0.33886)	0.09005 (0.58845)	0.062858 (0.55504)	0.04563 (0.61503)	0.34222 (0.66771)	0.026407 (0.71245)	0.00644 (0.69355)

Table (12) Shown Expected Sample Size E(n│μ,R) when α = 0.01 and n_1 = 12

r \ λ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.5	12.059286	12.061524	12.06825	12.079758	12.096426	12.118794	12.147534	12.183444	12.227412	12.280386	12.343362	13.685766
1	12.118572	12.123012	12.1365	12.159516	12.192852	12.237588	12.295068	12.366888	12.45824	12.560772	12.686724	15.371532
2	12.237144	12.246024	12.273	12.319032	12.385704	12.475176	12.590136	12.733776	12.909648	13.121544	13.373448	18.743064
4	12.474288	12.492048	12.546	12.638064	12.771408	12.950352	13.180272	13.467552	13.819296	14.243088	14.746896	25.486128
8	12.948576	12.984096	13..092	13.276128	13.542816	13.900704	14.360544	14.935104	15.638592	16.486176	17.493792	38.972256
12	13.422864	13.476144	13.638	13.914192	14.314224	14.851056	15.540816	16.402656	17.457888	18.729264	20.240688	52.458384

r \ λ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.5	0.66996	0.670084	0.67046	0.671097	0.67202	0.67327	0.67486	0.67686	0.67931	0.68224	0.68574	0.76032
1	0.50494	0.50513	0.50569	0.506645	0.50804	0.5099	0.51229	0.51529	0.51895	0.52337	0.52861	0.64048
2	0.33992	0.34017	0.34092	0.342195	0.34405	0.34653	0.34973	0.35372	0.3586	0.36449	0.37148	0.52064
4	0.20791	0.2082	0.2091	0.21063	0.21286	0.21584	0.21967	0.22446	0.230322	0.23739	0.24578	0.42477
8	0.119894	0.12022	0.12122	0.12293	0.125396	0.12872	0.13297	0.13829	0.144802	0.15265	0.16198	0.36086
12	0.08604	0.08639	0.08742	0.08919	0.09176	0.0952	0.09962	0.10515	0.11191	0.12006	0.12975	0.33627

Table (13) Shown Expected Sample Size Proportion E(n│μ,R)/n when α = 0.01 and n_1 = 12

r \ λ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.5	0.66996	0.670084	0.67046	0.671097	0.67202	0.67327	0.67486	0.67686	0.67931	0.68224	0.68574	0.76032
1	0.50494	0.50513	0.50569	0.506645	0.50804	0.5099	0.51229	0.51529	0.51895	0.52337	0.52861	0.64048
2	0.33992	0.34017	0.34092	0.342195	0.34405	0.34653	0.34973	0.35372	0.3586	0.36449	0.37148	0.52064
4	0.20791	0.2082	0.2091	0.21063	0.21286	0.21584	0.21967	0.22446	0.230322	0.23739	0.24578	0.42477
8	0.119894	0.12022	0.12122	0.12293	0.125396	0.12872	0.13297	0.13829	0.144802	0.15265	0.16198	0.36086
12	0.08604	0.08639	0.08742	0.08919	0.09176	0.0952	0.09962	0.10515	0.11191	0.12006	0.12975	0.33627

Table (14) Shown Probability of Avoiding Second Sample $p(\hat{\mu}_1 \in R)$

α \ λ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.01	0.990119	0.989749	0.988625	0.986707	0.983929	0.980201	0.975411	0.969426	0.962098	0.953269	0.942773	0.719039
0.05	0.950002	0.948856	0.945408	0.93963	0.931481	0.920906	0.907849	0.892256	0.874083	0.853307	0.829932	0.484009
0.1	0.900027	0.898332	0.893249	0.8884797	0.873004	0.857916	0.839602	0.818155	0.7937	0.766398	0.73645	0.361162

Table (15) Shown Percentage of the Overall Sample Saved $\frac{n_2}{n} p(\hat{\mu}_1 \in R) * 100$ when α = 0.01 and n_1 = 12

r \ λ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
0.5	33.00397	32.991633	32.954167	32.890233	32.797633	32.673367	32.5137	32.3142	32.069933	31.775633	31.425767	23.967967
1	49.50595	49.48745	49.43125	49.33535	49.19645	49.01005	48.77055	48.4713	48.1649	47.66345	47.13865	35.95195
2	66.007933	65.983267	65.90833	65.780467	65.595267	65.34733	65.0274	64.6284	64.139867	63.551267	62.851533	47.935933
4	79.20952	79.17992	79.09	78.93656	78.71432	78.41608	78.03288	77.55408	76.96784	76.26152	75.42184	57.52312
8	88.010578	87.977689	87.877777	87.707288	87.460356	87.128978	86.7032	86.1712	85.519822	84.735022	83.802044	63.914578
12	91.3956	91.361446	91.257692	91.080646	90.824215	90.480092	90.037938	89.485476	88.809046	87.994062	87.0252	66.372831

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مقدرات الاختبار الأولي المقلصة ذات المرحلة الواحدة والمرحلتين لمتوسط التوزيع الطبيعي عندما يكون التباين معلوما

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الخلاصة:

يتعلق موضوع هذا البحث بمقدرات الاختبار الأولي المقلصة ذات المرحلة الواحدة والمرحلتين لمتوسط التوزيع الطبيعي (μ) عندما يكون التباين معلوماً وعند توافر المعلومات المسبقة (μ_0) حول القيمة الحقيقية (μ)، باستخدام دالة تقلص موزونة $\psi(\cdot)$ فضلا عن مجال الاختبار الاولي (R).

أشتقت معادلات التحيز $[B(\cdot)]$، متوسط مربعات الخطأ $[MSE(\cdot)]$، الكفاءة $[Eff(\cdot)]$، حجم العينة المتوقع $[E(n/\mu)]$ للمقدرات المقترحة. اعطيت بعض الاستنتاجات والنتائج العددية الخاصة بالمعادلات السابقة

من خلال اختيار بعض القيم للثوابت المتضمنة فيها. أجريت بعض المقارنات للمقدرات المقترحة مع المقدرات الكلاسيكية وبعض البحوث المنجزة حديثاً لبيان فائدة وأفضلية المقدرات المقترحة.