A proposed Mathematical Expression for Computer Design of Electrostatic Mirror

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Abstract:

A computational investigation has been carried out on the design and properties of the electrostatic mirror. In this research, we suggest a mathematical expression to represent the axial potential of an electrostatic mirror. The electron beam path under zero magnification condition had been investigated as mirror trajectory with the aid of fourth – order – Runge – Kutta method. The spherical and chromatic aberration coefficients of mirror has computed and normalized in terms of the focal length. The choice of the mirror depends on the operational requirements, i.e. each optical element in optical system has suffer from the chromatic aberration, for this case, it is use to operate the mirror in optical system at various values of chromatic aberration to correct it in that system.

Key words: Electrostatic mirror – mirror aberration.

Introduction:

Electron mirror is used to correct the aberration of lenses, this idea are back to the middle of twentieth century [1]. An electron mirror is creating when an electrode with sufficiently high negative potential is placed in the path of an electron beam. The negative electrode forms a potential hill that decelerates the incident electrons, these electrons are lose their kinetic energy before reaching the electrode and are back to re–accelerated in the reverse direction. (i.e. the electrons towards away from the electrode) [2].

Unlike a light optics mirror, where the reflection occurs at the physical surface, the electron mirror represents a "soft" mirror, which allows the electrons to penetrate into the inhomogeneous reflection medium formed by the electrostatic potential [3]. Early in the development of the electron optics theory, Scherzer showed that under the assumption of static field, rotational symmetric lens, a pace charge free beam or a beam in which the velocity component does not reverse direction, any lens or lens system always suffers from chromatic and spherical aberrations. By introducing a reflection in the electron path using an electron mirror, the electron beam direction reverses and the electron velocity changes sign, thus the Scherzer theorem no longer applies [4].

In case of ion mirror, there is a potential hill in the ion mirror in which a time shift occurs as ion enters the mirror, are accelerated, turn around, and finally accelerate in the reverse direction [5]. In ion mirror there is one hard reflecting point, this surface dose not necessarily coincide with either the physical location of the turn around point of the ion mirror or a physical electrode surface [6]. The field inside a parabolic mirror (reflection) is curved along the axis and according to the Laplace equation it also has a curvature

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in a radial (or transverse) direction [7]. In electrostatic ion mirror the electrostatic field continuously changes in the interface region between the retarding and repelling fields, and electric field extends out from the retarding field region into the field – free drift path [8].

Materials and Methods:

The Axial Potential Equation

A potential distribution function has been suggested to represents an electrostatic mirror in the following form:

$$U(z) = \frac{a}{1+(z^2/b)} \quad \dots \quad (1)$$

whereas *a* is the maximum high potential field, and *b* is the area of field. Figure (1.a) shows the axial potential distribution at the mirror length and figure (1.b) shows the first and second derivatives of the potential. This potential computed at *a*= 50 V and *b*= 55 mm^2 because at this values, we got the minimum spherical and chromatic aberrations coefficients. The second derivatives of the potential has one infliction point, hence the mirror has two electrodes [9].

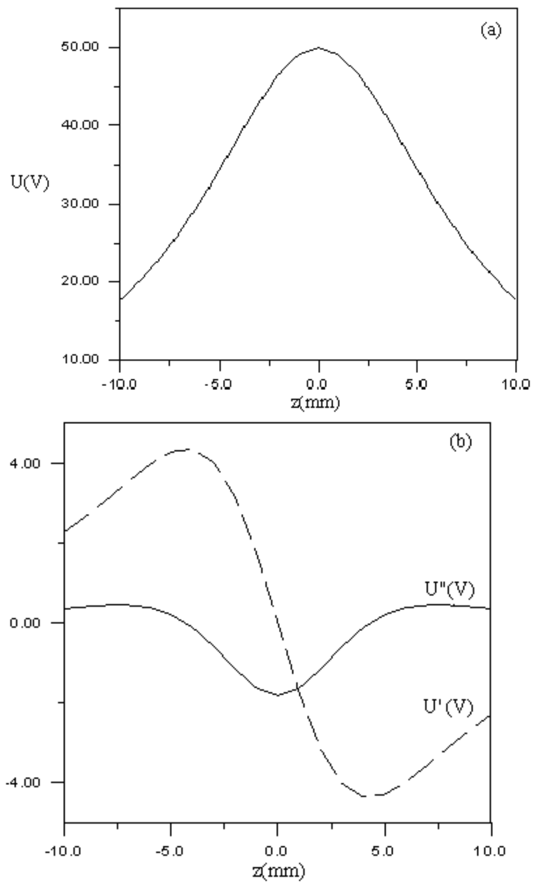

Fig. (1): The potential and its first and second derivatives.
a- The axial potential distribution.
b- The first and second derivatives of the potential.

The Trajectory Equation and Mirror Aberration

The equation of motion of charge particle traveling at a non relativistic velocity in the electrostatic field can be reduced to the following paraxial ray equation [10]

$$\frac{d^2R}{dz^2} + \frac{U'}{2U}\frac{dR}{dz} + \frac{U''}{4U}R = 0 \quad \dots \quad (2)$$

whereas U' and U" are the first and second derivatives of the axial potential U respectively. R represents the radial displacement of the beam form the axis z and the primes denote a derivative with respect to z.

The most important aberrations in an electron-optical system are spherical and chromatic aberration.

The spherical aberration Cs and chromatic aberration Cc referred to the image/ object side is calculated from the following equations [11].

$$Cs = \frac{U_{\circ}^{-1/2}}{16\ R'^{4}_{\circ}} \int_{z\circ}^{zi} \left[\frac{5}{4}\left(\frac{U''}{U}\right)^{2} + \frac{5}{24}\left(\frac{U'}{U}\right)^{2} + \frac{14}{3}\left(\frac{U'}{U}\right)^{3}\frac{R'}{R} - \frac{3}{2}\left(\frac{U'}{U}\right)^{2}\frac{R'^{2}}{R^{2}} \right] \sqrt{U}\ R^{4}\ dz$$

…(3)

$$C_c = \frac{\sqrt{U_{\circ}}}{R'^{2}_{\circ}} \int_{z\circ}^{zi} \left[\frac{1}{2}\left(\frac{U'}{U}\right) R' + \frac{1}{4}\left(\frac{U''}{U}\right) R \right] \frac{R}{\sqrt{U}}\ dz$$

... (4)

where U=U(z) is the axial potential, the primes denote derivatives with respect to z, and U = U (zo) is the potential at the object side where z = zo. where zo and zi are the object and image coordinates, respectively.

Results and Discussion:

The ion beam path along the electrostatic mirror field under zero magnification condition represented by equation (2) and accelerating mode of operation has been considered. Taking various values of the constant a under consideration, it has been found that the relative aberration coefficients are not change. Thus the value a= 50 V has been maintained as a constant one in computing the field and the trajectory at various values of b. Figure (2) shows the trajectory of an electron beam in the electrostatic mirror field at various values of b. The trajectory normalized in terms of electrode length. These trajectories realized the beam reflection at various values of b= (55, 65, 75, 85, 95, 105) mm^2.

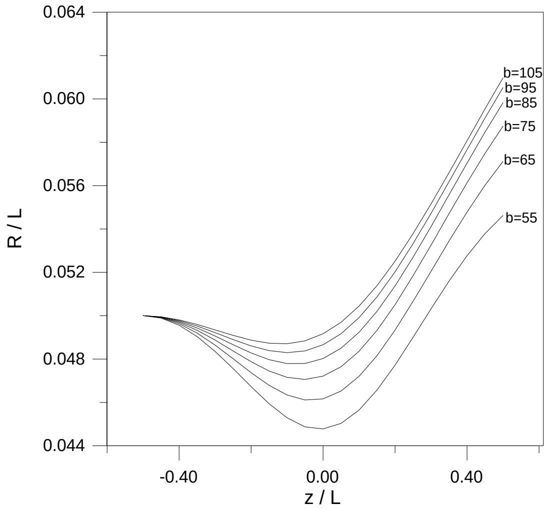

Fig. (2): The reflection of electron beam trajectory

Like a light optical mirror, where the reflection occurs at the physical surface, the beam reflection point at nearly in the middle of the axis of electrodes. It is seen that, the trajectory is more deflected from the axis where b increased.

The aberration coefficients of electrostatic mirror have been computed with the aid of the corresponding trajectory of the electron beam. We realized the reflection of electron beam trajectory at values range of constant b which determined between 55 mm^2 to 105 mm^2. Under this range, we computed the aberration coefficients of electrostatic mirror. Figure (3) shows the relative spherical aberration coefficient Cs/fi of the electrostatic mirror as a function of the constant b under zero magnification condition.

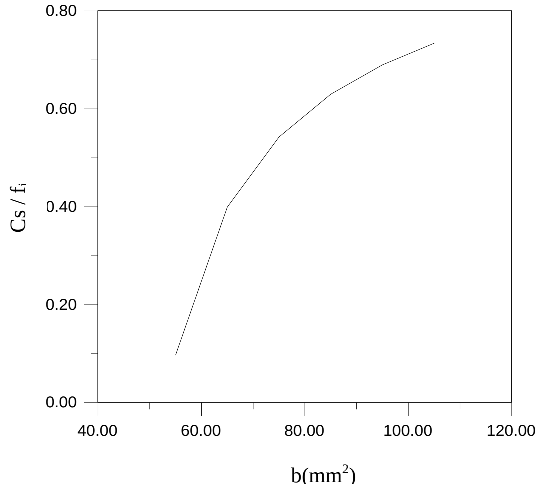

Fig. (3): The relative spherical aberration coefficient

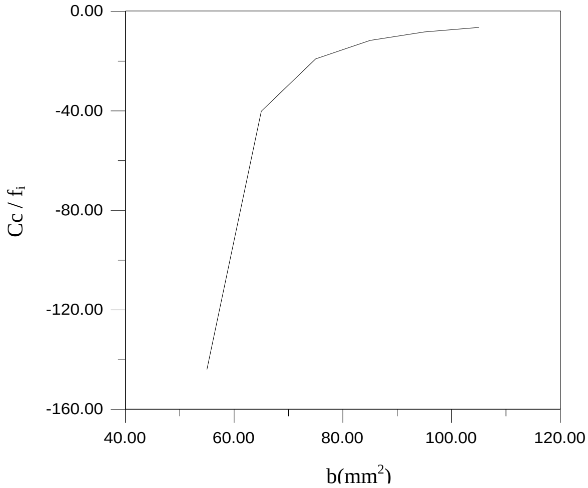

Fig. (4): The relative chromatic aberration coefficient

The spherical aberration coefficient has been computed with aid of equation (3). The trajectory shown in figure (2) has been used for computing the relative spherical aberration coefficient as a function of constant b. The spherical aberration coefficient increase when the values of b increase too. It is seen that Cs/fi has a minimum value at b= 55 mm^2. At this value of constant b, the value of $(Cs/fi)_{min}$ is equal to 0.097.

The chromatic aberration coefficients have been computed with the aid of equation (4). The relative chromatic aberration coefficient Cc/fi has been computed as a function of constant b. Figure (4) shows that Cc/fi increase with increasing b.

The minus value of the chromatic aberration Cc indicated beam direction reverses and the electron velocity changes sign. If we increased in value of b above from 105mm^2, the chromatic aberration towards to positive value because the electron beam trajectory deflect not reflect from the axis. In this case, the electron motion in one direction and it has positive value of velocity.

Conclusion:

It appears that concepts put forward in the present investigation with regard to the mathematical expression for the axial potential and the electron beam to produce a good result from the electron – optical point of view was significant. We conclude that the chromatic aberration coefficient is always in negative sign, while the spherical aberration coefficient, excitation parameter, and the focal length are in positive sign. Then we can use such lenses to correct aberration. The maximum value of axial potential distribution cannot change in the aberrations values.

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اقتراح تعبير رياضي لتصميم حاسوبي لمرآة كهروسكونية

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الخلاصة:

اجريَ بحث حاسوبي عن تصميم وخواص مرآة كهروسكونية. في هذا البحث، اقترحنا معادلة رياضية تمثل الجهد المحوري لمرآة الكهروسكونية. درس مسار الحزمة الالكترونية تحت شرط التكبير الصفري على انها مسار مرآتي بالاستعانة بطريقة fourth – order – Runge – Kutta. وحسبت معادلات الزيغ الكروي واللوني للمرآة وتم تعبيرها بدلالة البعد البؤري. ان اختيار المرآة يعتمد على مستلزمات التشغيل وذلك كل عنصر بصري في اي نظام يعاني من زيغ لوني، في هذه الحالة يمكن تشغيل المرآة في منظومة بصرية عند قيم مختلفة للزيغ اللوني لتصحيحها في ذلك النظام.