Evaluation of the Compton (Incoherent) and Rayleigh (Coherent) Differential Cross Sections of Scattering for Rhodium $^{103}Rh_{45}$ and Tantalum $^{181}Ta_{73}$ by Employing CSC model

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Abstract:

The differential cross section for the Rhodium and Tantalum has been calculated by using the Cross Section Calculations (CSC) in range of energy(1keV-1MeV) . This calculations based on the programming of the Klein-Nashina and Rayleigh Equations. Atomic form factors as well as the coherent functions in Fortran90 language Machine proved very fast an accurate results and the possibility of application of such model to obtain the total coefficient for any elements or compounds.

Key words: Cross section , coherent (incoherent) scattering.

Introduction :

There has been more interest in obtaining reliable values of cross section for elements and compounds as well as alloys because of its required in varity of applications in radiography, tomography ,space physics ,plasma physics etc [1].

A review of literature showed that the studies concern Geswar [2] measured the total cross section for various elements in energy range of 4.5 to 20 keV. Theoretical photoelectric cross section for Z=13 are calculated by Brysk and Zerby [3] in energy range 1 to 150 keV , they employed bound-state wave function that's obtained by Lieberman and Cromer [4] .Hubbell and Berger [5] reported pair production cross sections in the fields of atomic nucleus for 11 elements over range 15 to 100 MeV these values were calculated by using Bon approximation with Bethe –Hitler high energy approximation. In the present research, we use the mathematical model Cross Section Calculation (CSC) which has more applications.Rhodium $^{103}Rh_{45}$ is a member of the platinum group of metals. It has a higher melting point than platinum, but a lower density. It is alloyed with platinum and palladium in electrodes for spark plugs, advanced laboratory equipment and in thermocouples. Rhodium compounds also have catalytic uses in automotive catalytic converters. Rhodium is used as a plating metal in jewelry production to enhance the whiteness of white gold. Rhodium is available as metal and compounds with purities from 99% to 99.999% (ACS grade to ultra-high purity); metals in the form of foil, sputtering target, and rod, and compounds.Tantalum $^{181}Ta_{73}$ is a greyish silver, heavy, and very hard metal. When pure, it is ductile and can be drawn into fine wire, which can be used as a filament for evaporating metals such as aluminum. Tantalum is almost completely immune to chemical attack at temperatures below

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150°C, and is attacked only by hydrofluoric acid, acidic solutions containing the fluoride ion, and free sulphur trioxide. The element has a melting point exceeded only by tungsten and rhenium.

Theoretical Aspects of CSC model:

The various methods of photons interactions with matters allows to utilizes the scattered photons or transmitted through material to determine and calculating the information about the material itself [6] .The probabilities of photons interactions with matter are functions for both incident energy and atomic number Z for material; these probabilities are expressed as a parameter calls cross section express in barns unit($1barn=10^{-28}$ $meter^2$) and the Mechanisms of photon Interaction are described as following:

I- Rayleigh (Coherent) Differential Scattering Cross Section:

In Rayleigh scattering event, a photon scattered off atomic electrons where energies of incident and scattered photons are identical, in this process the recoil energy of absorber atom is negligible and the process occurs at small angle, thus the cross section of Rayleigh depends upon the photon energy E and atomic number of the absorber Z [7]:

$$\sigma_{Rayleigh} \alpha\, Z^{8/3} / E^2 \quad \ldots (1)$$

and differential cross section is given by [8]:

$$\frac{d\sigma_{Rayleigh}}{d\Omega} = \frac{d\sigma_{Thomson}}{d\Omega}\left[F(x,Z)\right]^2 \quad \ldots(2)$$

where $d\Omega$ is the solid angle and the term $\frac{d\sigma_{Thomson}}{d\Omega}$ is defined as the differential cross section of free electron or Thomson cross section [8]:

$$\frac{d\sigma_{Thomson}}{d\Omega} = \frac{1}{2} r_e^2 \left(1 + \cos^2\phi\right) \quad \ldots(3)$$

and r_e is the classical radius of electron ($r_e = 2.817\times10^{-15}$m) and F(x,Z) is the atomic form factor will be discussed in next section .So equation (3) can be written as following [9]:

$$\frac{d\sigma_{Rayleigh}}{d\Omega} = \frac{r_e^2}{2}(1+\cos^2\phi)[(F(x,Z)]^2 \quad \ldots (4)$$

II-Compton (Incoherent) Differential Cross section

Compton scattering is an inelastic scattering process in which a photon imparts some of its energy to atomic electrons and is deflected through angle of scattering. .The differential cross section of Compton is proportional to atomic number Z of the absorber and given by Klein-Nashina equation [10]:

$$\frac{d\sigma_{incoherent}}{d\Omega} = \frac{d\sigma_{Klien-Nishina}}{d\Omega} S(q,Z) \quad \ldots.. (5)$$

Where: $\frac{d\sigma_{Klein-Nashina}}{d\Omega}$ is defined as :

$$\frac{d\sigma_{Klein-Nashina}}{d\Omega} = Zr_e^2\left(\frac{1}{1+\alpha(1-\cos\phi)}\right)\left(\frac{1+\cos^2\phi}{2}\right)\left[1+\frac{\alpha(1-\cos\phi)^2}{(1+\cos^2\phi)(1+\alpha(1-\cos\theta)}\right] \ldots(6)$$

This equation gives the probability that a photon is deflected at given angle and transfer some momentum to the free electron , were $\alpha=hf/m_oc^2$ ((h is Planks constant, f photon frequency and m_oc^2 is the rest mass energy of electron equals to 0.511MeV)) and the term S(q, Z)represents the incoherent scattering function will discussed in next section .

III Differential Cross Section Concept

The cross section **σ** is the total area represented by a scattering center to the incident photon, but the deflection of the photon (if it is not absorbed) depends on the distance from the interaction center at which it is incident. The annular area d**σ** within which the deflection will be within a solid angle dΩ, centered on deflection angle **θ**, is the differential cross section d**σ**/d**Ω**. The relationship between an increment in **θ** and an increment in Ω is:

$$\frac{d\sigma}{d\Omega}=\frac{1}{2\pi\sin\theta}\frac{d\sigma}{d\Omega} \quad ...(7)$$

The differential cross section concept is illustrated in the following fig.(1).

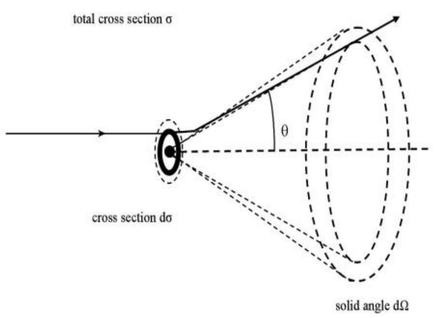

Fig. (1) Shows the solid angle

IV-The Atomic Form Factor F(x, Z):

The Atomic Form Factor F(x, Z) defined as the amplitude of electron oscillation when interact with incident radiation, the electron in turn oscillates with the same frequency of the incoming wave train and since an accelerating electric charge emits electromagnetic radiation, scattered photon appears to emerged from the interaction site with the same frequency of the oscillating electron, consequently the scattered amplitude (as function of the scattering angle of electron, is not equals to Z times the free electron amplitude; but its modified by atomic form factor F(x, Z) where X is the momentum transfer parameter measured in units of $1/A^0$ defined as :

$$X=(1/\lambda)\sin(\theta/2) \quad \ldots (8)$$

Where λ is the incident wavelength and is θ the scattering angle. The function F(x, Z) is related to the ratio of the amplitude of scattered wave form to scattered amplitude from free electron and given by:

$$F(x,Z)=\frac{\dfrac{d\sigma}{d\Omega_{atom}}}{\dfrac{d\sigma}{d\Omega_{\text{Thomson}}}} \quad .....(9)$$

The values of F(x,Z) for atoms up to Z=100 published by Hubbell et als [11].By employing the relativistic Hartee-Fock wave function [12] to constructing a table of atomic form factors.

V-The Coherent Scattering Function S(q,Z):

The function S(q ,Z) represents the probability that an atom will be raised to the excited state or ionized state when a photon imparts a recoil momentum (q) to any one of the atomic electrons. Cromer and Mann[13] have calculated the incoherent scattering function S(q,Z) for all spherically symmetric free electrons by using Hartree –Fock - Slater wave function with exchanging term .The values of S(q,Z) and F(x, Z) are feed into series subroutine of Fortran 90 program which all the equations are written in Fortran machine language which gives the results in a very short time in of running

VI-Results and Discussion

This work never contains a comparison the obtained results with any experimental works for similar elements ,this due to that in scattering process the atoms are assumed to be isolated from the influences of the neighbors atoms which is in contrast to the reality because there are unavoidable interactions between various atoms. As the molecular and chemical effects are not taken into account, thus, for example in Compton scattering ,the electron assumed to be bound and not free also initially at rest. However in reality the bound electrons in material their momentum gives raise to range of possible energies which is referred as Doppler Broadening and the quantum theory refers that even at zero Kelvin the atoms vibrates in their equilibrium positions. Hubbell et als [14] estimated the magnitude of the discrepancy between theoretical and experimental K-edge cross section for various elements and compound from Ti to Zn to be in the range of 3% to 12%.

The coherent differential cross section has been calculated for the two elements from eq.(2) the results have been shown in figures (2,4)for Rh and Tantalum respectively by comparison these values we conclude increasing the values as Z increases and decreasing as photons energy increases in accordance with eq.(1) This means that the Coherent scattering differential cross section of Rayleigh never dominates the total cross section but at small angles scattering (θ less than 10^0)) and at photon energies less than 100keV because the scattering angle become greater and for the incoherent differential cross section which obtained from eq.(5) also proportional to the atomic Z and the photon incident energy. Coherent cross section of both elements is begin from very small photon energy less than 1 keV as shown in figures (2,4). Incoherent scattering cross section for both elements begin from very small of photon energy near 1 keV and full dominates in about 500 keV then the cross section decreases when energy be more than this value as shown in figures (3,5).

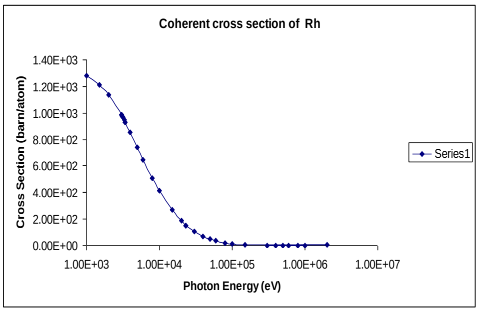

Fig.(2): Coherent differential cross section versus the photon energies for Rhodium Rh.

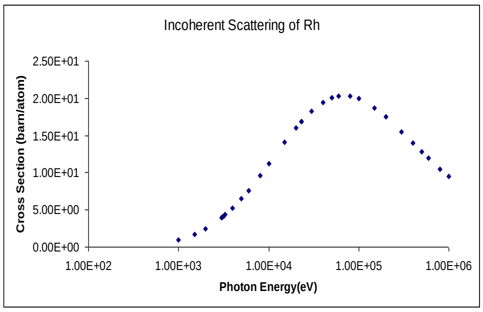

Fig.(3): Incoherent differential cross section versus the photon energies for Rhodium (Rh).

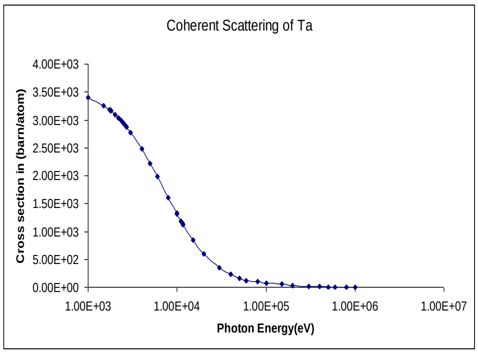

Fig.(4):Coherent differential cross section versus the photon energies for Tantalum Ta .

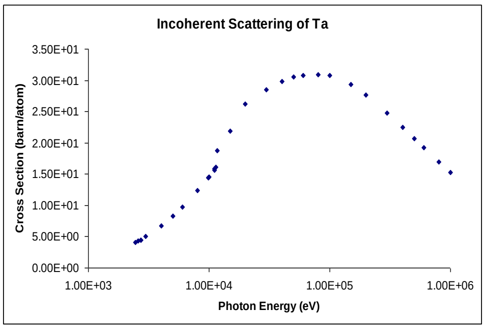

Fig.(5): Incoherent differential cross section versus the photon energies for Tantalum Ta.

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حساب مساحة المقطــع العرضي التفاضلي لاستطارتي كومـــــبتن (المتشاكه) لعنصري الروهيديوم (غير المتشاكهة) ورايــلـي $^{103}Rh_{45}$ والتانتالوم $^{181}Ta_{73}$ باستخدام نموذج CSCالرياضي

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الخلاصة :

في هذا البحث تم حساب مساحات المقطع العرضي لتفاضلي لعنصري الروهيديوم $^{103}Rh_{45}$ و التانتالوم $^{181}Ta_{73}$ من خلال ستخدام النموذج الرياضي (حسابات مساحات المقطع العرضي) ولمدى طاقي (1KeV-1MeV)وبالاعتماد على برمجة معادلتي استطارة كلين-ناشينية و رايلي ومعاملي التركيب الذري و الدوال المتشاكهة بلغة فورتران 90 والذي اثبت دقة وسرعة عالية في الحصول على النتائج وامكانية تطبيقه لحصول على معاملات التوهين الكلية لاي عنصر اوسبيكة .