The construction of Complete (k_n,n)-arcs in The Projective Plane PG(2,5) by Geometric Method, with the Related Blocking Sets and Projective Codes

*Amal SH.Al-Mukhtar** *Umniyat A. Hassan**

Received 20, December, 2012
Accepted 20, June, 2013

Abstract:

A (k,n)-arc is a set of k points of PG(2,q) for some n, but not n + 1 of them, are collinear.

A (k,n)-arc is complete if it is not contained in a (k + 1,n)-arc.

In this paper we construct complete (k_n,n)-arcs in PG(2,5), n = 2,3,4,5, by geometric method, with the related blocking sets and projective codes.

Key words: Complete arcs, blocking set, projective cod.

1- Introduction:

Let PG(2,q) be the projective plane over Galois field GF(q). The points of PG(2,q) are the non-zero vectors of the vector space V(3,q) with the rule that X(x_1,x_2,x_3) and Y(λx_1,λx_2,λx_3) are the same point, where $\lambda \in$ GF(q)\{0}.

Similarly, x[x_1,x_2,x_3] and y[(λx_1,λx_2,λx_3] are the same line, where $\lambda \in$ GF(q)\{0}.

The point X(x_1,x_2,x_3) is on the line Y[y_1,y_2,y_3] if and only if $x_1 y_1 + x_2 y_2 + x_3 y_3 = 0$.

In PG(2,q), there are $q^2 + q + 1$ points and $q^2 + q + 1$ lines, every line contains exactly q + 1 points and every point is on exactly q + 1 lines.

Definition 1.1:[1]

A (k,n)-arc K is in PG(2,q) is a set of k points such that some lines of the plane meet K in n points but no line meets K in more than n points, where $n \geq 2$.

Definition 1.2:[2]

A (k,n)-arc is complete if it is not contained in a (k+1,n)-arc. The maximum number of points that can a (k,2)-arc can have is m(2,q) and this arc is an oval.

Theorem 1.3:[3]

In PG(2,q),

$$m(2,q) = \begin{cases} q+1 & \text{for } q \text{ odd} \\ q+2 & \text{for } q \text{ even} \end{cases}$$

Definition 1.4:[1]

A line ℓ in PG(2,q) is an i-secant of a (k,n)-arc K if $|\ell \cap K| = i$.

Definition 1.5:[1]

A variety V(F) of PG(2,q) is a subset of PG(2,q) such that

V(F) = {P(A) ∈ PG(2,q) | F(A) = 0}.

Definition 1.6:[1]

Let Q(2,q) be the set quadrics in PG(2,q), that is the varieties V(F), where:

$$F = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 \quad \ldots(1)$$

If V(F) is non singular, then the quadric is a conic, that is, if

*Department of Mathematics, College of Education for Pure Science/Ibn-Al-Haitham, University of Baghdad

$$A = \begin{bmatrix} a_{11} & \frac{a_{12}}{2} & \frac{a_{13}}{2} \\ \frac{a_{12}}{2} & a_{22} & \frac{a_{23}}{2} \\ \frac{a_{13}}{2} & \frac{a_{23}}{2} & a_{33} \end{bmatrix}$$

Is non singular, then the quadric (1) is a conic.

Theorem 1.8:[3]

In PG(2,q), with q odd, every oval is a conic.

Definition 1.9:[3]

A point N which is not on a (k,n)-arc has index i if there exactly i(n-secants) of the arc through N, the number of the points N of index i is denoted by N_i.

Remark 1.10:[3]

The (k,n)-arc is complete if and only if $N_0 = 0$. Thus the arc is complete if and only if every point of PG(2,q) not on the arc lies on some n-secant of the arc.

Definition 1.11:[4]

An (b,t)-blocking set B in PG(2,q) is a set of b points such that every line of PG(2,q) intersects B in at least t points, and there is a line intersecting B in exactly t points.

If B contains a line, it is called trivial, thus B is a subset of PG(2,q) which meets every line but contains no line completely; that is $t \leq |B \cap \ell| \leq q$ for every line ℓ in PG(2,q). We may note that a blocking set is merely a (k,n)-arc with $n \leq q$ and no 0-secants. A blocking set B is minimal if B\{p} is not blocking set for every $p \in B$.

1.12 The Relation Between the Blocking (b,t)-set and the (k,n)-arc: [4]

The (k,n)-arcs and the (b,t)-blocking sets are each complement to the other in the projective plane PG(2,q), that is, $n + t = q + 1$ and $k + b = q^2 + q + 1$. Thus the complement of the (b,t)-blocking set is the set of points that intersects every line in at most n points which represents the (k,n)-arc. Also finding minimal (b,t)-blocking set is equivalent to finding maximal (k,n)-arc in PG(2,q).

Definition 1.13:[3]

In PG(2,q), let B contains a line ℓ minus a point P plus a set of q points one on each of the q lines through P other than ℓ but not all collinear; then B is minimal (2q,1)-blocking set. Blocking sets of this kind are called rédéi-type studied by [Bruen, A.A. and Thas, J.A. (1977)] and in [Blockhuis, A.A. and Brouwer, E. and S.Z. "onyi, T. (1995)].

Definition 1.14:[5,6]

Let V(n,q) denote the vector space of all ordered n-tuples over GF(q). A linear code C over GF(q) of length n and dimension k is a k-dimensional subspace of V(n,q). The vectors of C are called codewords. The Hamming distance between two codewords is defined to be the number of coordinate places in which they differ. The minimum distance of a code is the smallest distances between distinct codewords. Such a code is called an $[n,k,d]_q$ code if its minimum hamming distance is d.

There exists a relationship between complete (n,r)-arcs in PG(2,q) and $[n,3,d]_q$ codes, given by the next theorem.

Theorem 1.15:[5]

There exists a projective $[n,3,d]_q$ code if and only if there exists an (n,n – d)-arc in PG(2,q).

2- The Projective Plane PG(2,5)

In this paper we consider the case q = 5 and the elements of GF(5) are denoted by 0,1,2,3,4.

A projective plane π = PG(2,5) over GF(5) consists of 31 points, 31 lines, each line contains 6 points and through each point there are 6 lines.

Let P_i and L_i be the points and lines of PG(2,5), respectively. Let i stands for the point P_i, i = 1,2,…, 31. The points and the lines of PG(2,5) are given in the table (1).

Table (1)Points and Lines of PG(2,5)

i		P_i					L_i		
1	1	0	0	2	7	12	17	22	27
2	0	1	0	1	7	8	9	10	11
3	1	1	0	6	7	16	20	24	28
4	2	1	0	4	7	14	21	23	30
5	3	1	0	5	7	15	18	26	29
6	4	1	0	3	7	13	19	25	31
7	0	0	1	1	2	3	4	5	6
8	1	0	1	2	11	16	21	26	31
9	2	0	1	2	9	14	19	24	29
10	3	0	1	2	10	15	20	25	30
11	4	0	1	2	8	13	18	23	28
12	0	1	1	1	27	28	29	30	31
13	1	1	1	6	11	15	19	23	27
14	2	1	1	4	9	16	18	25	27
15	3	1	1	5	10	13	21	24	27
16	4	1	1	3	8	14	20	26	27
17	0	2	1	1	17	18	19	20	21
18	1	2	1	5	11	14	17	25	28
19	2	2	1	6	9	13	17	26	30
20	3	2	1	3	10	16	17	23	29
21	4	2	1	4	8	15	17	24	31
22	0	3	1	1	22	23	24	25	26
23	1	3	1	4	11	13	20	22	29
24	2	3	1	3	9	15	21	22	28
25	3	3	1	6	10	14	18	22	31
26	4	3	1	5	8	16	19	22	30
27	0	4	1	1	12	13	14	15	16
28	1	4	1	3	11	12	18	24	30
29	2	4	1	5	9	12	20	23	31
30	3	4	1	4	10	12	19	26	28
31	4	4	1	6	8	12	21	25	29

2- The Constructions of (k,n)-arcs in PG(2,5) :[1]

Let A ={1,2,7,13} be the set of reference and unit points in π = PG(2,5), where 1 (1,0,0), 2 (0,1,0), 7 (0,0,1), 13 (1,1,1).

A is a (4,2)-arc since no three points of A are collinear, the points of A are the vertices of a quadrangle whose sides are the lines:

[1,1] = {1,2,3,4,5,6}
[1,7] = {1,7,8,9,10,11}
[1,13] = {1,12,13,14,15,16}
[2,7] = {2,7,12,17,22,27}
[2,13] = {2,8,13,18,23,28}
[7,13] = {3,7,13,19,25,31}

The diagonal points of A are the points {3,8,12} where [1,2] $\cap$ [7,13] = 3, [1,7] $\cap$ [2,13] = 8, [1,13] $\cap$ [2,7] = 12 which are the intersections of the pairs of the opposite sides. Then there are 25 points on the sides of the quadrangle four of them are the points of the arc A and three of them are the diagonal points of A. So there are six points not on the sides of the quadrangle which are the points of index zero for A, these points are:

20,21,24,26,29,30

Hence A is incomplete (6,2)-arc.

2.1 The Conics in PG(2,5) through the Reference and Unit Points

The general equation of the conic is:

$$a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 = 0 \quad \ldots(1)$$

By substituting the points of A in (1), we get;

$a_{12} + a_{13} + a_{23} = 0$ and $a_{11} = a_{22} = a_{33} = 0$, so (1) becomes:

$$a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 = 0 \quad \ldots(2)$$

If $a_{12} = 0$, then the conic is degenerated, therefore $a_{12} \neq 0$, similarly, $a_{13} \neq 0$ and $a_{23} \neq 0$.

Dividing equation (2) by a_{12}, we get:

$$x_1x_2 + \alpha x_1x_3 + \beta x_2x_3 = 0 \quad \ldots(3)$$

where $\alpha = \frac{a_{13}}{a_{12}}$, $\beta = \frac{a_{23}}{a_{12}}$, then $\beta = -(1 + \alpha)$ since $1 + \alpha + \beta = 0 \pmod 5$.

Then (3) can be written as:

$$x_1x_2+\alpha x_1x_3-(1+\alpha)x_2x_3=0 \quad \ldots(4)$$

where $\alpha \neq 0$ and $\alpha \neq 4$, for if $\alpha = 0$ or $\alpha = 4$ we get a degenerated conic, that is, α = 1,2,3.

2.2 The Equations and the Points of the Conics in PG(2,5) through the Reference and Unit Points [1]

For any value of α, there is a unique conic contains 6 points, 4 of them are the reference and unit points

1. If $\alpha = 1$, then the equation of the conic C_1 is

$$x_1x_2 + x_1x_3 + 3x_2x_3 = 0$$

The points of C_1 are : 1,2,7,13,20,26.

2. If $\alpha = 2$, then the equation of the conic C_2 is

$$x_1x_2 + 2x_1x_3 + 2x_2x_3 = 0$$

The points of C_2 are : 1,2,7,13,21,29.

3. If $\alpha = 3$, then the equation of the conic C_3 is

$$x_1x_2 + 3x_1x_3 + x_2x_3 = 0$$

The points of C_3 are : 1,2,7,13,24,30.

Thus we found five conics two of them are degenerated and the remaining three conics C_1, C_3, C_3 are non-degenerated, which are complete (k,2)-arcs.

2.3 The Construction of Complete (k_n,n)-arcs in PG(2,5) and the Related Blocking Sets and Projective Codes

The complete (k,n)-arcs in PG(2,5) can be constructed by eliminating the conics given above from PG(2,5) as follows:

2.3.1 The Construction of Complete (k_5,5)-arc

Let π = PG(2,5), we take a conic, say C_1, where C_1 = {1,2,7,13,20,26}.

Let K = π – C_1 = {3,4,5,6,8,9,10,11,12,14,15,16,17,18,19,21,22,23,24,25,27,28,29,30,31}

The construction of complete (k_5,5)-arc must satisfies the following:

(1) Any line of π must intersects the arc in at most 5 points.

(2) Every point not in the arc is on at least one 5-secant of the arc.

We eliminate five points from K which are: 6,12,17,22,25 to satisfy (1). The point 20 is of index zero we add it to K to satisfy (2), then

K_5=K ∪ {20}\{6,12,17,22,25}= {3,4,5,8,9,10,11,14,15,16,18,19,20,21,23,24,27,28,29,30,31}. Then K_5 is a complete (21,5)-arc as shown in table (2).

Let β_1 = π – K_5 = {1,2,6,7,12,13,17,22,25,26}. β_1 is (10,1)-blocking set of size (2q) which is of Rédei-type(figure-1-) contains the line L_1\{27} = {2,7,12,17,22} and one point on each line through the point 27 other than L_1, which are non-collinear points: 1,6,13,25,26. Note that each line in π intersects β_1 in at least one point as shown in table (2). By theorem 1.15, there exists a projective [21,3,16] code which is equivalent to the complete (21,5)-arc K_5.

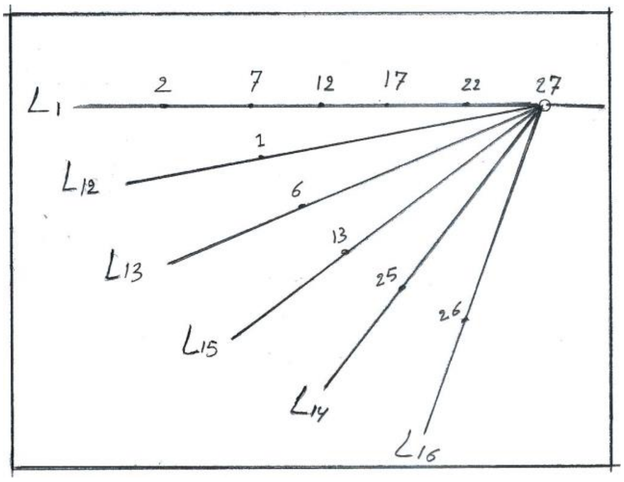

Fig.-1-

Table (2)

i	$K_5 \cap L_i$	$\beta_1 \cap L_i$	$\lvert K_5 \cap L_i \rvert$	$\lvert \beta_1 \cap L_i \rvert$
1	{27}	{2,7,12,17,22}	1	5
2	{8,9,10,11}	{1,7}	4	2
3	{16,24,28,20}	{6,7}	4	2
4	{4,14,21,23,30}	{7}	5	1
5	{5,15,18,29}	{7,26}	4	2
6	{3,19,31}	{7,13,25}	3	3
7	{3,4,5}	{1,2,6}	3	3
8	{11,16,21,31}	{2,26}	4	2
9	{9,14,19,24,29}	{2}	5	1
10	{10,15,20,30}	{2,25}	4	2
11	{8,18,23,28}	{2,13}	4	2
12	{27,28,29,30,31}	{1}	5	1
13	{11,15,19,23,27}	{6}	5	1
14	{4,9,16,18,27}	{25}	5	1
15	{5,10,21,24,27}	{13}	5	1
16	{3,8,14,20,27}	{26}	5	1
17	{18,19,20,21}	{1,17}	4	2
18	{5,11,14,28}	{17,25}	4	2
19	{9,30}	{6,13,17,26}	2	4
20	{3,10,16,23,29}	{17}	5	1
21	{4,8,15,24,31}	{17}	5	1
22	{23,24}	[1,22,25,26}	2	4
23	{4,11,20,29}	{13,22}	4	2
24	{3,9,15,21,28}	{22}	5	1
25	{10,14,18,31}	{6,22}	4	2
26	{5,8,16,19,30}	{22}	5	1
27	{14,15,16}	{1,12,13}	3	3
28	{3,11,18,24,30}	{12}	5	1
29	{5,9,23,20,31}	{12}	5	1
30	{4,10,19,28}	{12,26}	4	2
31	{8,21,29}	{6,12,25}	3	3

2.3.2 The Construction of Complete (k_4,4)-arc

We take the union of two conics, say C_1 and C_2, where C_1 = {1,2,7,13,20,26} and C_2 = {1,2,7,13,21,29}.

Let K = $\pi - (C_1 \cup C_2)$

= {3,4,5,6,8,9,10,11,12,14,15,16,17,18,19,22,23,24,25,27,28, 30,31}

The construction must satisfies the following:

(1) Any line of π intersects K in at most 4 points.

(2) Any point not in K is on at least one 4-secant.

We eliminate seven points of K which are: 11,12,15,16,17,18,22 to satisfy (1). There are no points of index zero for K then

K_4=K \{11,12,15,16,17,18,22} = {3,4,5,6,8,9,10,14,19,23,24,25,27,28,30,31}. Then K_4 is a complete (16,4)-arc as shown in table (3).

By theorem 1.15, there exists a projective [16,3,12] code which is equivalent to the complete (16,4)-arc.

Let $\beta_2 = \pi - K_4$ ={1,2,7,11,12,13,15,16,17,18,20,21,22, 26,29} is a (15,2)-blocking set of size (3q), note that each line in π intersects β_2 in at least two points of π as shown in table (3).

Table (3)

i	$K_4 \cap L_i$	$\beta_2 \cap L_i$	$\lvert K_4 \cap L_i \rvert$	$\lvert \beta_2 \cap L_i \rvert$
1	{27}	{2,7,12,17,22}	1	5
2	{8,9,10}	{1,7,11}	3	3
3	{6,24,28}	{7,16,20}	3	3
4	{4,14,23,30}	{7,21}	4	2
5	{5}	{7,15,18,26,29}	1	5
6	{3,19,25,31}	{7,13}	4	2
7	{3,4,5,6}	{1,2}	4	2
8	{31}	{2,11,16,21,26}	1	5
9	{9,14,19,24}	{2,29}	4	2
10	{10,25,30}	{2,15,20}	3	3
11	{8,23,28}	{2,13,18}	3	3
12	{27,28,30,31}	{1,29}	4	2
13	{6,19,23,27}	{11,15}	4	2
14	{4,9,25,27}	{16,18}	4	2
15	{5,10,24,27}	{13,21}	4	2
16	{3,8,14,27}	{20,26}	4	2
17	{19}	{1,17,18,20,21}	1	5
18	{5,14,25,28}	{11,17}	4	2
19	{6,9,30}	{13,17,26}	3	3
20	{3,10,23}	{16,17,29}	3	3
21	{4,8,24,31}	{15,17}	4	2
22	{23,24,25}	[1,22,26}	3	3
23	{4}	{11,13,20,22,29}	2	4
24	{3,9,28}	{15,21,22}	3	3
25	{6,10,14,31}	{18,22}	4	2
26	{5,8,19,30}	{16,22}	4	2
27	{14}	{1,12,13,15,16}	1	5
28	{3,24,30}	{11,12,18}	3	3
29	{5,9,23,31}	{12,20}	4	2
30	{4,10,19,28}	{12,26}	4	2
31	{6,8,25}	{12,21,29}	3	3

2.3.3 The Construction of Complete (k_3,3)-arc

We take the union of three conics, say C_1 C_2 and C_3, where C_1 = {1,2,7,13,20,26}, C_2 =

{1,2,7,13,21,29} and C_3 = {1,2,7,13,24,30}

Let K = $\pi - (C_1 \cup C_2 \cup C_3)$ = {3,4,5,6,8,9,10,11,12,14,15,16,17,18,19,22,23,25,27,28,31}

The construction must satisfies the following:

(1) Any line of π intersects K in at most three points.

(2) Any point not in K is on at least one 3-secant of K.

We eliminate 10 points of K which are: 4,6,11,16,22,23,25,27,28,31 to satisfy (1). There are no points of index zero for K then

K_3 = K \ {4,6,11,16,22,23,25,27,28,31} ={3,5,8,9,10,12,14,15,17,18,19} is a complete (11,3)-arc as shown in table (4).

By theorem 1.15, there exists a projective [11,3,8] code which is equivalent to the complete (11,3)-arc K_3.

Let $\beta_3 = \pi - K_3$ = {1,2,4,6,7,11,13,16,20,21,22,23,24,25,26,27,28,29,30,31}, then β_3 is a (20,3)-blocking set of size (4q)which is a trivial since β_3 contains some lines of π completely as shown in table (4), note that each line intersects β_3 in at least three points of π.

Table (4)

i	$K_3 \cap L_i$	$\beta_3 \cap L_i$	$\lvert K_3 \cap L_i \rvert$	$\lvert \beta_3 \cap L_i \rvert$
1	{12,17}	{2,7,22,27}	2	4
2	{8,9,10}	{1,7,11}	3	3
3	ϕ	{6,7,16,20,24,28}	0	6
4	{14}	{4,7,21,23,30}	1	5
5	{5,15,18}	{7,26,29}	3	3
6	{3,19}	{7,13,25,31}	2	4
7	{3,5}	{1,2,4,6}	2	4
8	ϕ	{2,11,16,21,26,31}	0	6
9	{9,14,19}	{2,24,29}	3	3
10	{10,15}	{2,20,25,30}	2	4
11	{8,18}	{2,13,23,28}	2	4
12	ϕ	{1,27,28,29,30,31}	0	6
13	{15,19}	{6,11,23,27}	2	4
14	{9,18}	{4,16,25,27}	2	4
15	{5,10}	{13,21,24,27}	2	4
16	{3,8,14}	{20,26,27}	3	3
17	{17,18,19}	{1,20,21}	3	3
18	{5,14,17}	{11,25,28}	3	3
19	{9,17}	{6,13,26,30}	2	6
20	{3,10,17}	{16,23,29}	3	3
21	{8,15,17}	{4,24,31}	3	3
22	ϕ	[1,22,23,24,25,26}	0	6
23	ϕ	{4,11,13,20,22,29}	0	6
24	{3,9,15}	{21,22,28}	3	3
25	{10,14,18}	{6,22,31}	3	3
26	{5,8,19}	{16,22,30}	3	3
27	{12,14,15}	{1,13,16}	3	3
28	{3,12,18}	{11,24,30}	3	3
29	{5,9,12}	{20,23,31}	3	3
30	{10,12,19}	{4,26,28}	3	3
31	{8,12}	{6,21,25,29}	2	4

2.3.4 The Construction of Complete (k_2,2)-arc

The construction must satisfies the following:

(1) The complete arc intersects every line in π in at most 2 points.

(2) every point not in the arc is on at least one 2-secant of the arc.

We eliminate 5 points from K_3 to satisfy (1), which are: 8,9,10,14,18. There are no points of index zero for K_3 then K_2=K_3\ {8,9,10,14,18}= {3,5,12,15,17,19} K_2 is a complete (6,2)-arc as shown in table (5).

By theorem 1.15, there exists a projective [6,3,4] code which is equivalent to the complete (6,2)-arc.

Let $\beta_4 = \pi - K_2$ = {1,2,4,6,7,8,9,10,11,13,14,16,18,20,21,…,31} β_4 is a (25,4)-blocking set of size (5q) which is a trivial since β_4 contains some lines completely as shown in table (5), note that each line intersects β_4 in at least four points.

Table (5)

i	$K_2 \cap L_i$	$\beta_4 \cap L_i$	$\lvert K_2 \cap L_i \rvert$	$\lvert \beta_4 \cap L_i \rvert$
1	{12,17}	{2,7,22,27}	2	4
2	ϕ	{1,7,8,9,10,11}	0	6
3	ϕ	{6,7,16,20,24,28}	0	6
4	ϕ	{4,7,14,21,23,30}	0	6
5	{5,15}	{7,18,26,29}	2	4
6	{3,19}	{7,13,25,31}	2	4
7	{3,5}	{1,2,4,6}	2	4
8	ϕ	{2,11,16,21,26,31}	0	6
9	{19}	{2,9,14,24,29}	1	5
10	{15}	{2,10,20,25,30}	1	5
11	ϕ	{2,8,13,18,23,28}	0	6
12	ϕ	{1,27,28,29,30,31}	0	6
13	{15,19}	{6,11,23,27}	2	4
14	ϕ	{4,9,16,18,25,27}	0	6
15	{5}	{10,13,21,24,27}	1	5
16	{3}	{8,14,20,26,27}	1	5
17	{17,19}	{1,18,20,21}	2	4
18	{5,17}	{11,14,25,28}	2	4
19	{17}	{6,9,13,26,30}	1	5
20	{3,17}	{10,16,23,29}	2	4
21	{15,17}	{4,8,24,31}	2	4
22	ϕ	[1,22,23,24,25,26}	0	6
23	ϕ	{4,11,13,20,22,29}	0	6
24	{3,15}	{9,21,22,28}	2	4
25	ϕ	{6,10,14,18,22,31}	0	6
26	{5,19}	{8,16,22,30}	2	4
27	{12,15}	{1,13,14,16}	2	4
28	{3,12}	{11,18,24,30}	2	4
29	{5,12}	{9,20,23,31}	2	4
30	{12,19}	{4,10,26,28}	2	4
31	{12}	{6,8,21,25,29}	1	5

References

1. Kadhum, S.J., (2001), Construction of (k,n)-arcs from (k,m)-arcs in PG(2,p) for 2 ≤ m < n, M.Sc. Thesis, University of Baghdad, Iraq.
2. Hirschfeld, J. W. P., (2001), Complete arcs Discrete Math., North Holland Mathematics Studies 123, North-Holland, Amesterdam, 243-250.
3. Hirschfeld, J. W. P., (1979), Projective Geometries Over Finite Fields, Second Edition, Oxford University Press, Oxford.
4. Hirschfeld, J. W. P. and Storme, L., (1998), The Packing Problem in Statistics, Coding Theory and Finite Projective Spaces, J.Statistical Planning and Inference, 72, pp.355-380.
5. R.Hill, (1992), Optimal Linear Codes in: C.Mitichell (Ed.) Crytography and Coding, Oxford University Press:Oxford, pp.75-104.
6. Rumen Daskalov, (2008), Discrete Mathematics 308 (1341-1345) , AGeometric Construction of (38,2)-Blocking Set in PG(2,13) and the Related $[14,5,3,133]_{13}$ code,Technical University of Gabrovo, Bulgaria.

بطريقة هندسية مع PG(2,5) في المستوي إلاسقاطي (k_n,n)بناء الاقواس المجموعات القالبية والشفرات الاسقاطية المرتبطة بها

آمال شهاب المختار   *امنيات عدنان حسن*

*قسم الرياضيات ، كلية التربية للعلوم الصرفة/ابن الهيثم ، جامعة بغداد

الخلاصة:

القوس (k,n) هو مجموعة من k من النقاط في PG(2,q) بحيث ان توجد n ولا توجد n + 1 منها على استقامة واحدة، فيكون القوس – (k,n) كاملاً اذا لم يكن محتوى في قوس-(k + 1,n).

في هذا البحث نقوم ببناء اقواس – (k_n,n) كاملة حيث n = 2,3,4,5 في المستوي الاسقاطي PG(2,5) بطريقة هندسية، مع المجموعات القالبية والشفرات الاسقاطية المرتبطة بها.