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Comparing Weibull Stress – Strength Reliability Bayesian Estimators for Singly Type II Censored Data under Different loss Functions

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Abstract:

The stress(Y) – strength(X) model reliability Bayesian estimation which defines life of a component with strength X and stress Y (the component fails if and only if at any time the applied stress is greater than its strength) has been studied, then the reliability; R=P(Y<X), can be considered as a measure of the component performance. In this paper, a Bayesian analysis has been considered for R when the two variables X and Y are independent Weibull random variables with common parameter α in order to study the effect of each of the two different scale parameters β and λ; respectively, using three different [weighted, quadratic and entropy] loss functions under two different prior functions [Gamma and extension of Jeffery] and also an empirical Bayes estimator Using Gamma Prior, for singly type II censored sample. An empirical study has been used to make a comparison between the three estimators of the reliability for stress – strength Weibull model, by mean squared error MSE criteria, taking different sample sizes (small, moderate and large) for the two random variables in eight experiments of different values of their parameters. It has been found that the weighted loss function was the best for small sample size, and the entropy and Quadratic were the best for moderate and large sample sizes under the two prior distributions and for empirical Bayes estimation.

Key words: Bayesian estimation, Reliability, Stress-strength model, Type II censored data, Weibull distribution.

Introduction:

Weibull models are used to describe various types of observed failures of components and phenomena. They are widely used in reliability and survival analysis (1). A considerable attention for the problem of making inference about the stress- strength reliability (one component or system) model has been received. If X be the strength of a component and Y be the stress applied to the component, then reliability; R=P(Y<X), can be considered as a measure of the component performance. Various different lifetime distributions are considered to estimate R. (2-5). As an example; Seuba et al (6) analyzed the applicability of the Weibull analysis to unidirectional microporous yttrium-stabilized-zirconia (YSZ) prepared by ice-tempting, performed crush tests on samples with controlled microstructural features with the loading direction parallel to the porosity. The compressive strength data were fitted using two different fitting techniques, ordinary least squares and Bayesian Markov Chain Monte Carlo, to evaluate whether Weibull statistics are an adequate descriptor of the strength distribution. They assess the effect of different microstructural features (volume, size, densification of the walls, and morphology) on Weibull modulus and strength and found that the key microstructural parameter controlling reliability is wall thickness. In contrast, pore volume is the main parameter controlling the strength.

In this paper the reliability Bayesian analysis when stress X and strength Y are two independent Weibull random variables with parameters(α,β) and (α,λ) respectively is done under two prior functions with three loss distributions. A simulation study has been used to compare by

(MSE) the performance of the six different obtained estimators. The results are recorded in Tables (1 to 5).

Table 1. Conclusions Summary

Experiment	Best Estimators Performance
(1)	E for all sample sizes.
(2)	W for n=15 with Jeffrey function, while E and Q are the best for n=30 and n=90
(3) and (4)	Q for all sample sizes, except for Jeffrey with n=15
(5)	E and Q for n=30and n=90, while W is the best for Jeffrey with n=15
(6)	W for n=15 and for Jeffrey with n=30, for the other cases E and Q are the best
(7) and (8)	by E and Q for all sample sizes, except W is the best for Exp.(8) for Jeffrey and gamma when n=15

Table 2. The MSE values of reliability estimators for experiments 1 and 2

		α=0.8	C=1, t=3, a=4, b1=0.4, b2=0.8			
			Exp. 1: λ=0.3, μ=0.9, R=0.2500			
n, r		Criteria	Weighted	Quadratic	Entropy	Best
15,5	Jeffery	Mean	0.2993	0.2193	0.2009	
		MSE	0.0189	0.0151	0.0151	Q, E
	Gamma	Mean	0.3272	0.2946	0.2926	
		MSE	0.0187	0.0145	0.0143	E
	E Gamma	Mean	0.3331	0.3016	0.2997	
		MSE	0.0251	0.0207	0.0204	E
30,8	Jeffery	Mean	0.3162	0.2730	0.2694	
		MSE	0.0156	0.0113	0.0110	E
	Gamma	Mean	0.3277	0.3032	0.3022	
		MSE	0.0154	0.0121	0.0120	E
	E Gamma	Mean	0.3301	0.3060	0.3050	
		MSE	0.0181	0.0147	0.0146	E
90,25	Jeffery	Mean	0.3344	0.3221	0.3219	
		MSE	0.0111	0.0092	0.0091	E
	Gamma	Mean	0.3356	0.3255	0.3253	
		MSE	0.0111	0.0094	0.0093	E
	E Gamma	Mean	0.3361	0.3259	0.3257	
		MSE	0.0114	0.0098	0.0097	E
			Exp. 2: λ=2, μ=3.7, R=0.3509			
15,5	Jeffery	Mean	0.3527	0.2691	0.2482	
		MSE	0.0189	0.0249	0.0275	W
	Gamma	Mean	0.3829	0.3508	0.3488	
		MSE	0.0185	0.0179	0.0179	Q, E
	E Gamma	Mean	0.3890	0.3575	0.3555	
		MSE	0.0217	0.0209	0.0209	Q, E
30,8	Jeffery	Mean	0.3789	0.3352	0.3312	
		MSE	0.0139	0.0137	0.0137	Q, E
	Gamma	Mean	0.3904	0.3663	0.3652	
		MSE	0.0139	0.0129	0.0128	E
	E Gamma	Mean	0.3940	0.3700	0.3689	
		MSE	0.0153	0.0142	0.0141	E
90,25	Jeffery	Mean	0.4003	0.3882	0.3880	
		MSE	0.0070	0.0061	0.0060	E
	Gamma	Mean	0.4011	0.3911	0.3910	
		MSE	0.0070	0.0062	0.0061	E
	E Gamma	Mean	0.4021	0.3922	0.3920	
		MSE	0.0072	0.0064	0.0064	Q,E

Table 3. The MSE values of reliability estimators for experiments 3 and 4

		α=0.8	C=2, t=2, a=4, b1=2.1, b2=1.5			
		Exp. 3: β=0.3, λ=0.9, R=0.2500				
n, r		criteria	Weighted	Quadratic	Entropy	Best
15,9	Jeffery	Mean	0.3097	0.2580	0.2826	
		MSE	0.0135	0.0094	0.0108	Q
	Gamma	Mean	0.2986	0.2761	0.2871	
		MSE	0.0093	0.0074	0.0082	Q
	E Gamma	Mean	0.3309	0.3086	0.3196	
		MSE	0.0172	0.0140	0.0154	Q
30,17	Jeffery	Mean	0.3311	0.3098	0.3202	
		MSE	0.0124	0.0094	0.0108	Q
	Gamma	Mean	0.3184	0.3044	0.3113	
		MSE	0.0094	0.0076	0.0085	Q
	E Gamma	Mean	0.3366	0.3226	0.3295	
		MSE	0.0135	0.0112	0.0123	Q
90,57	Jeffery	Mean	0.3380	0.3325	0.3353	
		MSE	0.0095	0.0086	0.0090	Q
	Gamma	Mean	0.3330	0.3281	0.3306	
		MSE	0.0085	0.0078	0.0082	Q
	E Gamma	Mean	0.3385	0.3336	0.3361	
		MSE	0.0096	0.0088	0.0092	Q
		Exp. 4: β=2, λ=3.7, R=0.3509				
15,9	Jeffery	Mean	0.3706	0.3177	0.3430	
		MSE	0.0119	0.0128	0.0117	E
	Gamma	Mean	0.3750	0.3524	0.3635	
		MSE	0.0107	0.0103	0.0104	Q
	E Gamma	Mean	0.3935	0.3714	0.3822	
		MSE	0.0138	0.0127	0.0132	Q
30,17	Jeffery	Mean	0.3954	0.3743	0.3847	
		MSE	0.0085	0.0072	0.0077	Q
	Gamma	Mean	0.3913	0.3774	0.3843	
		MSE	0.0076	0.0068	0.0071	Q
	E Gamma	Mean	0.4013	0.3876	0.3944	
		MSE	0.0091	0.0080	0.0085	Q
90,57	Jeffery	Mean	0.4050	0.3997	0.4024	
		MSE	0.0049	0.0044	0.0047	Q
	Gamma	Mean	0.4026	0.3979	0.4002	
		MSE	0.0046	0.0042	0.0044	Q
	E Gamma	Mean	0.4056	0.4008	0.4032	
		MSE	0.0050	0.0045	0.0047	Q

Table 4. The MSE values of reliability estimators for experiments 5 and 6

		α=0.8	C=1, t=3, a=4, b1=0.4, b2=0.8			
		Exp. 5: β=0.3, λ=0.9, R=0.2500				
n, r		criteria	Weighted	Quadratic	Entropy	Best
15,5	Jeffery	Mean	0.2592	0.1842	0.1682	
		MSE	0.0151	0.0163	0.0173	W
	Gamma	Mean	0.2906	0.2587	0.2569	
		MSE	0.0131	0.0110	0.0109	E
	E Gamma	Mean	0.2904	0.2598	0.2581	
		MSE	0.0185	0.0164	0.0163	E
30,8	Jeffery	Mean	0.2763	0.2346	0.2313	
		MSE	0.0103	0.0090	0.0090	Q, E
	Gamma	Mean	0.2909	0.2668	0.2659	
		MSE	0.0096	0.0080	0.0079	Q
	E Gamma	Mean	0.2893	0.2656	0.2647	
		MSE	0.0116	0.0100	0.0100	Q, E
90,25	Jeffery	Mean	0.2864	0.2742	0.2740	
		MSE	0.0045	0.0038	0.0037	E
	Gamma	Mean	0.2891	0.2791	0.2789	
		MSE	0.0045	0.0038	0.0038	Q, E
	E Gamma	Mean	0.2879	0.2779	0.2777	
		MSE	0.0047	0.0040	0.0040	Q, E
		Exp. 6: β=2, λ=3.7, R=0.3509				
15,5	Jeffery	Mean	0.3306	0.2482	0.2283	
		MSE	0.0185	0.0271	0.0302	W

	Gamma	Mean	0.3621	0.3301	0.3281	
		MSE	0.0175	0.0180	0.0180	W
	E Gamma	Mean	0.3660	0.3344	0.3324	
		MSE	0.0198	0.0202	0.0202	W
30,8	Jeffery	Mean	0.3530	0.3093	0.3054	
		MSE	0.0126	0.0143	0.0145	W
	Gamma	Mean	0.3656	0.3413	0.3402	
		MSE	0.0123	0.0123	0.0123	---
	E Gamma	Mean	0.3676	0.3435	0.3424	
		MSE	0.0133	0.0132	0.0132	Q, E
90,25	Jeffery	Mean	0.3718	0.3595	0.3593	
		MSE	0.0048	0.0045	0.0045	Q, E
	Gamma	Mean	0.3730	0.3629	0.3627	
		MSE	0.0048	0.0045	0.0045	Q, E
	E Gamma	Mean	0.3735	0.3634	0.3633	
		MSE	0.0049	0.0046	0.0046	Q, E

Table 5. The MSE values of reliability estimators for experiments 7 and 8

		α=0.8 C=2, t=2, a=4, b1=2.1, b2=1.5				
		Exp. 7: β=0.3, λ=0.9, R=0.2500				
n, r		criteria	Weighted	Quadratic	Entropy	Best
15,9	Jeffery	Mean	0.2680	0.2185	0.2419	
		MSE	0.0087	0.0084	0.0080	E
	Gamma	Mean	0.2604	0.2387	0.2493	
		MSE	0.0057	0.0054	0.0054	Q, E
	E Gamma	Mean	0.2876	0.2657	0.2764	
		MSE	0.0105	0.0091	0.0096	Q
30,17	Jeffery	Mean	0.2831	0.2623	0.2725	
		MSE	0.0059	0.0048	0.0052	Q
	Gamma	Mean	0.2736	0.2600	0.2667	
		MSE	0.0044	0.0038	0.0040	Q
	E Gamma	Mean	0.2882	0.2744	0.2812	
		MSE	0.0064	0.0054	0.0058	Q
90,57	Jeffery	Mean	0.2915	0.2874	0.2895	
		MSE	0.0029	0.0025	0.0027	Q
	Gamma	Mean	0.2884	0.2847	0.2865	
		MSE	0.0026	0.0023	0.0024	Q
	E Gamma	Mean	0.2918	0.2881	0.2899	
		MSE	0.0029	0.0026	0.0027	Q
		Exp. 8: β=2, λ=3.7, R=0.3509				
15,9	Jeffery	Mean	0.3441	0.2914	0.3166	
		MSE	0.0108	0.0140	0.0118	W
	Gamma	Mean	0.3528	0.3302	0.3413	
		MSE	0.0098	0.0103	0.0099	W
	E Gamma	Mean	0.3664	0.3441	0.3551	
		MSE	0.0116	0.0115	0.0114	E
30, 17	Jeffery	Mean	0.3660	0.3447	0.3552	
		MSE	0.0065	0.0063	0.0063	Q, E
	Gamma	Mean	0.3646	0.3506	0.3575	
		MSE	0.0060	0.0059	0.0059	Q, E
	E Gamma	Mean	0.3718	0.3578	0.3647	
		MSE	0.0067	0.0064	0.0065	Q
90,57	Jeffery	Mean	0.3772	0.3732	0.3752	
		MSE	0.0022	0.0019	0.0021	Q
	Gamma	Mean	0.3759	0.3722	0.3741	
		MSE	0.0021	0.0020	0.0021	Q
	E Gamma	Mean	0.3775	0.3738	0.3757	
		MSE	0.0023	0.0021	0.0022	Q

Let X ~ Wei (α,β) and Y ~ Wei (α,λ) , where Wei means Weibull distribution under common shape parameter α and different scale parameters Table 1 β and λ (as a special case in our research where the other cases can be done as a future work), then the probability distribution function for two independent Weibull r.v.'s are [2]:

$f(x) = \alpha\beta x^{\alpha-1}e^{-\beta x^{\alpha}}$ x > 0 ; α, β >0

$f(y) = \alpha\lambda y^{\alpha-1}e^{-\lambda y^{\alpha}}$ y > 0; α, λ >0

Where:

$R = P(X > Y) = \int_{y=0}^{\infty}\int_{x=y}^{\infty} f(x)f(y)dxdy$

$= \int_{0}^{\infty}(1 - \int_{0}^{y} f(x)dx)f(y)dy$

$=\int_0^\infty (1-F(y))f(y)dy = \int_0^\infty \overline{F}_x(y)f(y)dy$

$= \int_0^\infty (e^{-\beta y^\alpha})\alpha\lambda y^{\alpha-1}e^{-\lambda y^\alpha}dy$

$=\int_0^\infty \alpha\lambda y^{\alpha-1}\, e^{-(\beta+\lambda)y^\alpha}\, dy$

Let u = $(\beta + \lambda)y^\alpha$ and du = $\alpha(\beta + \lambda)y^{\alpha-1}$ dy, so by transformation, it will be:

$\therefore R = \int_0^\infty \frac{\lambda}{\beta+\lambda} e^{-u}du = \frac{\lambda}{\beta+\lambda}$

Singly Type II Censored Sample

Let $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_m$ be two random samples, and r < n and r < m , such that : $x_r, \ldots, x_n$ and $y_r, \ldots, y_m$. The likelihood function for this type of data is(7) :

$L = \frac{n!}{(n-r)!}[1-F(x_r)]^{n-r}\prod_{i=1}^r f(x_i,\underline{\theta})$

Where:

$[1-F(x_r)]^{n-r} = [1-1+e^{-\beta x_r{}^\alpha}]^{n-r}$

$[1-F(y_r)]^{m-r} = [1-1+e^{-\lambda y_r{}^\alpha}]^{m-r}$

and

$\prod_{i=1}^r f(x_i.\alpha,\beta) = \prod_{i=1}^r \alpha\beta x_i{}^{\alpha-1}\, e^{-\beta x_i{}^\alpha}$

$=\alpha^r\beta^r \prod_{i=1}^r x_i{}^{\alpha-1}\, e^{-\beta\sum_{i=1}^r x_i{}^\alpha}$

$\prod_{i=1}^r f(y_i;\alpha,\lambda) = \prod_{i=1}^r \alpha\lambda y_i{}^{\alpha-1}e^{-\lambda y_i{}^\alpha}$

$=\alpha^r\lambda^r \prod_{i=1}^r y_i{}^{\alpha-1}e^{-\lambda\sum_{i=1}^r y_i{}^\alpha}$

Then:

$L(\beta,\alpha \backslash \underline{x}) =$

$\frac{n!\alpha^r\beta^r}{(n-r)!}\prod_{i=1}^r x_i{}^{\alpha-1}\, e^{-(n-r)\beta x_i{}^\alpha}e^{-\beta\sum_{i=1}^r x_i{}^{\alpha-1}}$ …… (1)

And

$L(\lambda,\alpha \backslash \underline{y}) =$

$\frac{m!\,\alpha^r\lambda^r}{(m-r)!}\prod_{i=1}^r y_i{}^{\alpha-1}\, e^{-(m-r)\lambda y_r{}^\alpha}e^{-\lambda\sum_{i=1}^r y_i{}^\alpha}$ …… (2)

The Posterior Distributions

Under Gamma Prior

The Gama distribution is used as a prior distribution because of its wide importance in Bayesian analysis. Let β, λ be two independent Gamma random variables with common parameter (a), (one can consider the case of uncommon parameter in other papers for recommendation), the pdf is given by(8):

$g(\beta) = \frac{b_1{}^a}{\Gamma a}\beta^{a-1}e^{-b_1\beta}$ β>0; b_1,a >0 …… (3)

$g(\lambda) = \frac{b_2{}^a}{\Gamma a}\lambda^{a-1}e^{-b_2\lambda}$ λ>0; b_2,a >0 …… (4)

The posterior function as:

$$P(\beta,\lambda\ |\underline{x},\underline{y}) = \frac{L\left(\beta,\lambda\middle|\underline{x},\underline{y}\right)\, g(\beta,\lambda)}{\int_0^\infty\int_0^\infty L(\beta,\lambda\,|\underline{x},\underline{y})g(\beta,\lambda)d\beta d\lambda}$$

Using (1), (2), (3) and (4), it will be:

$L(\beta,\lambda\,|\,\underline{x},\underline{y})g(\beta,\lambda) =$

$\frac{n!}{(n-r)!}\cdot\frac{m!}{(m-r)!}\cdot\frac{b_1{}^a b_2{}^a}{\Gamma a\Gamma a}\alpha^{2r}\beta^{r+a-1}\prod_{i=1}^r x_i{}^{\alpha-1}$

$\prod_{i=1}^r y_i{}^{\alpha-1}\, e^{-b_1\beta-(n-r)\beta x_r{}^\alpha-\beta\sum_{i=1}^r x_r{}^\alpha}$

$e^{-b_2\lambda-(m-r)\lambda y_r{}^\alpha-\lambda\sum_{i=1}^r y_i{}^\alpha}$

And

$\int_0^\infty\int_0^\infty L(\beta,\lambda|\,\underline{x},\underline{y})g(\beta,\lambda)d\beta d\lambda =$

$\frac{n!}{(n-r)!}\cdot\frac{m!}{(m-r)!}\cdot\frac{b_1{}^\alpha b_2{}^\alpha}{\Gamma a\Gamma a}\alpha^{2r}\prod_{i=1}^r x_i{}^{\alpha-1}\prod_{i=1}^r y_i{}^{\alpha-1}$

$\int_0^\infty \beta^{r+a-1}e^{-\beta[b_1+(n-r)x_r{}^\alpha+\sum_{i=1}^r x_i{}^\alpha]}d\beta$

$\int_0^\infty \lambda^{r+a-1}e^{-\lambda[b_2+(m-r)y_r{}^\alpha+\sum_{i=1}^r y_i{}^\alpha]}d\lambda$

Since $\int_0^\infty x^{r-1}e^{-cx} = \frac{\Gamma(r)}{c^r}$, then:

$\int_0^\infty \beta^{r+a-1}e^{-\beta[b_1+(n-r)x_r{}^\alpha+\sum_{i=1}^r x_i{}^\alpha]}d\beta = \frac{\Gamma(r+a)}{U_x{}^{r+a}}$

and

$\int_0^\infty \lambda^{r+a-1}e^{-\lambda[b_2+(m-r)y_r{}^\alpha+\sum_{i=1}^r y_i{}^\alpha]}d\lambda = \frac{\Gamma(r+a)}{U_y{}^{r+a}}$

where $U_x = b_1 + (n-r)x_r{}^\alpha + \sum_{i=1}^r x_i{}^\alpha$ and $U_y = b_2 + (m-r)y_r{}^\alpha + \sum_{i=1}^r y_i{}^\alpha$ Then the posterior distribution $P(\beta,\lambda\ |\underline{x},\underline{y})$ under gamma prior, will be :

$P_1(\beta,\lambda|\underline{x},\underline{y}) =$

$\frac{U_x{}^{r+a}}{(r+a-1)!}\cdot\frac{U_y{}^{r+a}}{(r+a-1)!}\beta^{r+a-1}\lambda^{r+a-1}e^{-\beta U_x}e^{-\lambda U_y}$ …… (5)

Under Extension of Jeffry Prior

Regarding the case of non-informational distributions the Extension of Jeffry as prior distribution have been used for the case of common parameter (c) in order to focus on the rest of the parameters, where the functions for (β,λ) are given by(9):

$g(\beta) = k\frac{n^c}{\beta^{2c}}$ β>0; k,n,c >0 ……(6)

$g(\lambda) = k\frac{m^c}{\lambda^{2c}}$ λ>0; k,m,c>0 ……(7)

Using equations (1), (2), (6) and (7), it will be:

$L(\beta,\lambda\,|\underline{x},\underline{y})g(\beta,\lambda) =$

$\frac{n!}{(n-r)!}\cdot\frac{m!}{(m-r)!}\cdot k^2n^{2c}\beta^{r-2c}\lambda^{r-2c}\prod_{i=1}^r x_i{}^{\alpha-1}\prod_{i=1}^r y_i{}^{\alpha-1}$

$e^{-(n-r)\beta x_r{}^\alpha}\, e^{-\beta\sum_{i=1}^r x_i{}^\alpha}e^{-(m-r)\lambda y_r{}^\alpha}e^{-\lambda\sum_{i=1}^r y_i{}^\alpha}$

$\int_0^\infty\int_0^\infty L(\beta,\lambda\,|\underline{x},\underline{y})g(\beta,\lambda)d\beta d\lambda =$

$\frac{n!}{(n-r)!}\cdot\frac{m!}{(m-r)!}\cdot k^2n^{2c}\prod_{i=1}^r x_i{}^{\alpha-1}\prod_{i=1}^r y_i{}^{\alpha-1}\beta$

$\int_0^\infty \beta^{r-2c}\, e^{-\beta[\sum_{i=1}^r x_i{}^\alpha+(n-r)x_r{}^\alpha]}d\beta$

$\int_0^\infty \lambda^{r-2c}e^{-\lambda[\sum_{i=1}^r y_i{}^\alpha+(m-r)y_r{}^\alpha]}d\lambda$

Now let $Z_x = \sum_{i=1}^r x_i{}^\alpha + (n-r)x_r{}^\alpha$ and $Z_y = \sum_{i=1}^r y_i{}^\alpha + (m-r)y_i{}^\alpha$

so by the same procedure:
$\int_0^\infty \beta^{r-2c}\, e^{-\beta[\sum_{i=1}^{r} x_i^{\alpha}+(n-r)x_r^{\alpha}}\, d\beta = \frac{\Gamma(r-2c+1)}{Z_x^{\,r-2c+1}}$
and $\int_0^\infty \lambda^{r-2c}\, e^{-\lambda[\sum_{i=1}^{r} y_i^{\alpha}+(m-r)y_i^{\alpha}]}\, d\lambda = \frac{\Gamma(r-2c+1)}{Z_y^{\,r-2c+1}}$
Then the posterior will be:

$$P_2(\beta,\lambda|\underline{x},\underline{y}) = \frac{Z_x^{\,r-2c+1}}{(r-2c)!}\cdot\frac{Z_y^{\,r-2c+1}}{(r-2c)!}\,\beta^{r-2c}\lambda^{r-2c}e^{-\beta Z_x}e^{-\lambda Z_y} \quad \ldots\ldots (8)$$

The Bayes Estimators

In this section the Bayes estimators for stress-strength Weibull reliability under three loss functions are derived as following(3):

For Gamma Prior function

(i) Under Weighted Loss Function

In this section the Bayesian estimator for R using gamma prior function will be derived under weighted loss function[5]; $\hat{R}_{wG}$, where:

$$\hat{R}_{wG} = \frac{1}{E(R^{-1}|\underline{x},\underline{y})} = [E\left(R^{-1}\middle|\underline{x},\underline{y}\right)]^{-1}$$

$$E\left(R^{-1}\middle|\underline{x},\underline{y}\right) = \int_0^\infty\int_0^\infty R^{-1}P\left(\beta,\lambda\middle|\underline{x},\underline{y}\right)d\beta d\lambda$$
$$= \int_0^\infty\int_0^\infty \left(\frac{\lambda}{\beta+\lambda}\right)^{-1} P\left(\beta,\lambda\middle|\underline{x},\underline{y}\right)d\beta d\lambda$$
$$= \int_0^\infty\int_0^\infty \lambda^{-1}(\beta+\lambda)P\left(\beta,\lambda\middle|\underline{x},\underline{y}\right)d\beta d\lambda$$
$$= \int_0^\infty\int_0^\infty \lambda^{-1}\beta P\left(\beta,\lambda\middle|\underline{x},\underline{y}\right)d\beta d\lambda + \int_0^\infty\int_0^\infty P(\beta,\lambda|\underline{x},\underline{y})d\beta d\lambda$$
$$= A_1 + A_2 \quad \ldots.. (9)$$

Where

$$A_1 = \int_0^\infty\int_0^\infty \lambda^{-1}\beta\,\frac{U_x^{r+a}U_y^{r+a}}{\Gamma(r+a)\Gamma(r+a)}\beta^{r+a-1}\,\lambda^{r+a-1}\,e^{-\beta U_x}e^{-\lambda U_y}d\beta d\lambda$$
$$= \frac{U_x^{r+a}U_y^{r+a}}{\Gamma(r+a)\Gamma(r+a)}\int_0^\infty \beta^{r+a}e^{-\beta U_x}d\beta \int_0^\infty \lambda^{r+a-2}e^{-\lambda U_y}d\lambda$$
$$= \frac{U_x^{r+a}U_y^{r+a}}{(r+a-1)!(r+a-1)!}\cdot\frac{(r+a)!(r+a-2)!}{U_x^{r+a+1}U_y^{r+a-1}} = \frac{(r+a)U_y}{(r+a-1)U_x}$$

and

$$A_2 = \frac{U_x^{r+a}U_y^{r+a}}{\Gamma(r+a)\Gamma(r+a)}\int_0^\infty \beta^{r+a-1}e^{-\beta U_x}d\beta \int_0^\infty \lambda^{r+a-1}e^{-\lambda U_y}d\lambda$$
$$= \frac{U_x^{r+a}U_y^{r+a}}{(r+a-1)!(r+a-1)!}\cdot\frac{(r+a-1)!(r+a-1)!}{U_x^{r+a}U_y^{r+a}} = 1$$

Then $\hat{R}_{wG} = \left[1+\frac{(r+a)U_y}{(r+a-1)U_x}\right]^{-1}$ ….. (10)

(ii) Under Quadratic Loss Function

The derivation of Bayesian estimator for R using gamma prior under quadratic loss function[10] ; $\hat{R}_{QG}$, will be as:

$$\hat{R}_{QG} = \frac{E(R^{-1}|\underline{x},\underline{y})}{E(R^{-2}|\underline{x},\underline{y})}$$

$$E(R^{-2}|\underline{x},\underline{y}) = \int_0^\infty\int_0^\infty \lambda^{-2}\,(\beta+\lambda)^{-2}P\left(\beta,\lambda\,\middle|\underline{x},\underline{y}\right)d\beta d\lambda$$
$$= \int_0^\infty\int_0^\infty(\beta^2\lambda^{-2}+2\beta\lambda\lambda^{-2}+\lambda^2\lambda^{-2})P\left(\beta,\lambda\middle|\underline{x},\underline{y}\right)d\beta d\lambda$$
$$= \int_0^\infty\int_0^\infty(\beta^2\lambda^{-2}+2\beta\lambda^{-1}+1)\,P(\beta,\lambda\,|\underline{x},\underline{y})\,d\beta d\lambda$$

$$= \int_0^\infty\int_0^\infty\Big(\beta^2\lambda^{-2}P(\beta,\lambda\,|\underline{x},\underline{y}) + 2\beta\lambda^{-1}P(\beta,\lambda\,|\underline{x},\underline{y}) + P(\beta,\lambda\,|\underline{x},\underline{y})\Big)\,d\beta d\lambda$$
$$= A_1 + 2A_2 + 1 \quad \ldots.. (11)$$

Where

$$A_1 = \frac{U_x^{r+a}U_y^{r+a}}{\Gamma(r+a)\Gamma(r+a)}\int_0^\infty \beta^{r+a+1}\,e^{-\beta U_x}d\beta \int_0^\infty \lambda^{r+a-3}\,e^{-\lambda U_y}d\lambda$$
$$= \frac{U_x^{r+a}U_y^{r+a}}{(r+a-1)!(r+a-1)!}\cdot\frac{(r+a+1)!(r+a-3)!}{U_x^{r+a+2}U_y^{r+a-2}}$$
$$= \frac{(r+a)(r+a+1)U_y^2}{(r+a-1)(r+a-2)U_x^2}$$

$$A_2 = \frac{U_x^{r+a}\,U_y^{r+a}}{\Gamma(r+a)\Gamma(r+a)}\int_0^\infty \beta^{r+a}\,e^{-\beta U_x}d\beta \int_0^\infty \lambda^{r+a-2}\,e^{-\lambda U_y}d\lambda$$

$$= \frac{U_x^{r+a}U_y^{r+a}}{(r+a-1)!(r+a-1)!}\cdot\frac{(r+a)!(r+a-2)!}{U_x^{r+a+1}U_y^{r+a-1}}$$
$$= \frac{(r+a)U_y}{(r+a-1)U_x}$$

Then:

$$\hat{R}_{QG} = \frac{1+\frac{(r+a)U_y}{(r+a-1)U_x}}{1+2\frac{(r+a)U_y}{(r+a-1)U_x}+\frac{(r+a)(r+a+1)U_y^2}{(r+a-1)(r+a-2)U_x^2}} \quad \ldots\ldots (12)$$

(iii) Under Entropy Loss Function

The derivation of Bayesian estimator for R using gamma prior under entropy loss function**(10)**; $\hat{R}_{tEG}$, as:

$$\hat{R}_{tEG} = \left[E(R^{-t}|\underline{x},\underline{y})\right]^{-\frac{1}{t}} \quad \text{where} \quad t \neq 0$$

If t=1 , then : $\hat{R}_{1EG} = \left[E(R^{-1}|\underline{x},\underline{y})\right]^{-1} = \hat{R}_{wG}$ as in equation (10)

If t=2 , then : $\hat{R}_{2EG} = \left[E(R^{-2}|\underline{x},\underline{y})\right]^{-\frac{1}{2}}$, so from equation (11) , it can be :

$$\hat{R}_{2EG} = \left[1+2\frac{(r+a)}{(r+a-1)}\frac{U_y}{U_x}+\frac{(r+a)(r+a+1)}{(r+a-1)(r+a-2)}\frac{U_y^2}{U_x^2}\right]^{-\frac{1}{2}} \quad \ldots.. (13)$$

and $\hat{R}_{tEG} = \left[E(R^{-t}|\underline{x},\underline{y})\right]^{-\frac{1}{t}}$

then

$$E\left[R^{-t}\middle|\underline{x},\underline{y}\right] = \int_0^\infty\int_0^\infty \lambda^{-t}(\lambda+\beta)^t P(\beta,\lambda|\,\underline{x},\underline{y})d\beta d\lambda$$

$$= \int_0^\infty\int_0^\infty \lambda^{-t}\sum_{i=0}^{t} C_i^t\lambda^{t-i}\beta^i P(\beta,\lambda|\,\underline{x},\underline{y})d\beta d\lambda$$

$$=\frac{U_x^{r+a}U_y^{r+a}}{\Gamma r+a\Gamma r+a}\sum_{i=0}^{t}C_i^t\int_0^\infty \beta^{r+a+i-1}e^{-\beta U_x}d\beta \int_0^\infty \lambda^{r+a-i-1}e^{-\lambda U_y}\,d\lambda$$

$$=\frac{U_x^{r+a}U_y^{r+a}}{\Gamma r+a\Gamma r+a}\sum_{i=0}^{t}C_i^t\,\frac{(r+a+i-1)!}{U_x^{r+a+i}}\cdot\frac{(r+a-i-1)!}{U_y^{r+a-i}}$$

$$=\frac{\sum_{i=0}^{t}C_i^t(r+a+i-1)!(r+a-i)!\left(\frac{U_y}{U_x}\right)^i}{(r+a-1)!(r+a-1)!}$$

Then finally one can get:

$$\hat{R}_{tEG}=\left[\sum_{i=0}^{t}C_i^t\,\frac{(r+a+i-1)!(r+a-i)!}{(r+a-1)!(r+a-1)!}\left(\frac{U_y}{U_x}\right)^i\right]^{-\frac{1}{t}} \quad \text{….. (14)}$$

The Empirical Bayes Estimator for R Using Gamma Prior

The empirical Bayes estimators of reliability R corresponding to Gamma prior distribution are obtained based on different loss functions, where if the prior Gamma parameters (b_1 and b_2) are unknown, then it may use the empirical Bayes approach to get its estimation from likelihood function and probability density function of prior distribution as(11):

$$f\left(\underline{x},\underline{y}\,\middle|\,b_1b_2\right)=\int_0^\infty\int_0^\infty L\left(\beta,\lambda\,\middle|\,\underline{x},\underline{y}\right)g(\beta,\lambda)d\beta d\lambda$$

Precisely have that

$$f\left(\underline{x},\underline{y}\,\middle|\,b_1b_2\right)=A\,\frac{(r+a-1)!}{U_x^{r+a}}\cdot\frac{(r+a-1)!}{U_y^{r+a}}$$

Where $A=\frac{n!}{(n-r)!}\,\frac{m!}{(m-r)!}\,\frac{b_1^a b_2^a}{\Gamma a\Gamma a}\,\alpha^{2r}\prod_{i=1}^{r}x_i^{\alpha-1}\prod_{i=1}^{r}y_i^{\alpha-1}$

Now the ML estimators of (b_1 and b_2) can be obtained by taking the natural log as:

$$\ln L(b_1b_2\mid\underline{x},\underline{y})=ln\left(\frac{n!}{(n-r)!}\right)+ln\left(\frac{m!}{(m-r)!}\right)+alnb_1+alnb_2-2ln\Gamma a+2rln\alpha+\ln\left(\prod_{i=1}^{r}x_i^{\alpha-1}\right)+\ln\left(\prod_{i=1}^{r}y_i^{\alpha-1}\right)+\ln[(r+a-1)!]-(r+a)\ln[b_1+(n-r)x_r^\alpha+\sum_{i=1}^{r}x_i^\alpha]+\ln[(r+a-1)!]-(r+a)\ln[b_2+(m-r)!\,y_r^\alpha+\sum_{i=1}^{r}y_i^\alpha]$$

$$\frac{\partial ln}{\partial b_1}=\frac{a}{b_1}-\frac{r+a}{b_1+(n-r)x_r^\alpha+\sum_{i=1}^{r}x_i^\alpha}\to 0$$

$$\frac{a\hat{b}_1+a(n-r)x_r^\alpha+a\sum_{i=1}^{r}x_i^\alpha-\hat{b}_1r-a\hat{b}_1}{\hat{b}_1[\hat{b}_1+(n-r)x_r^\alpha+\sum_{i=1}^{r}x_i^\alpha]}=0$$

$$\therefore\hat{b}_1=\frac{a}{r}[(n-r)x_r^\alpha+\sum_{i=1}^{r}x_i^\alpha] \quad \text{….. (15)}$$

$$\frac{\partial ln}{\partial b_2}=\frac{a}{b_2}-\frac{r+a}{b_2+(m-r)y_r^\alpha+\sum_{i=1}^{r}y_i^\alpha}\to 0$$

$$a\hat{b}_2+a[(m-r)y_r^\alpha+\sum_{i=1}^{r}y_i^\alpha]-r\hat{b}_2-a\hat{b}_2=0$$

$$\therefore\ \hat{b}_2=\frac{a}{r}\,[(m-r)y_r^\alpha+\sum_{i=1}^{r}y_i^\alpha] \quad \text{….. (16)}$$

Using these two estimators in (15) and (16) in the reliability estimators obtained above under Gamma prior using three loss functions.

For Extension of Jeffery Prior

The Bayesian estimators for R will be derived in this section for extension of Jeffery prior as in equations (6 and 7) under the three loss functions.

(i) Weighted Loss Function

From equations (8), the Bayesian estimators of R for extension of Jeffery prior under the weighted loss functions; $\hat{R}_{wJ}$, from equation (9), will be:

$\hat{R}_{wJ}=[D_1+D_2]^{-1}$ where

$$D_1=\int_0^\infty\int_0^\infty\lambda^{-1}\beta\,\frac{Z_x^{r-2c+1}}{(r-2c)!}\,\frac{Z_y^{r-2c+1}}{(r-2c)!}\,\beta^{r-2c}\lambda^{r-2c}e^{-\beta Z_x}e^{-\lambda Z_y}\,d\beta d\lambda$$

$$=\frac{Z_x^{r-2c+1}}{(r-2c)!}\,\frac{Z_y^{r-2c+1}}{(r-2c)!}\int_0^\infty\beta^{r-2c+1}e^{-\beta Z_x}d\beta\int_0^\infty\lambda^{r-2c-1}e^{-\lambda Z_y}\,d\lambda$$

$$=\frac{Z_x^{r-2c+1}}{(r-2c)!}\,\frac{Z_y^{r-2c+1}}{(r-2c)!}\,\frac{(r-2c+1)!}{Z_x^{r=2c+2}}\,\frac{(r-2c-1)!}{Z_y^{r-2c}}\to D_1=\frac{(r-2c+1)Z_y}{(r-2c)Z_x}$$

$$D_2=\frac{Z_x^{r-2c+1}}{(r-2c)!}\,\frac{Z_y^{r-2c+1}}{(r-2c)!}\int_0^\infty\beta^{r-2c}e^{-\beta Z_x}d\beta\int_0^\infty\lambda^{r-2c}e^{-\lambda Z_y}d\lambda$$

$$=\frac{Z_x^{r-2c+1}}{(r-2c)!}\,\frac{Z_y^{r-2c+1}}{(r-2c)!}\,\frac{(r-2c)!}{Z_x^{r-2c+1}}\,\frac{(r-2c)!}{Z_y^{r-2c+1}}\to D_2=1$$

$$\therefore\hat{R}_{wJ}=\frac{(r-2c)Z_x}{(r-2c+1)Z_y+(r-2c)Z_x} \quad \text{….. (17)}$$

(ii) Under Quadratic Loss Function

Here having $\hat{R}_{QJ}=\frac{E(R^{-1}\,|\underline{x},\underline{y})}{E(R^{-2}|\underline{x},\underline{y})}$

From equations (8) and (11), assuming that $E(R^{-2}\mid x,y)=D_1+2D_2+1$, then:

$$D_1=\frac{Z_x^{r-2c+1}}{(r-2c)!}\,\frac{Z_y^{r-2c+1}}{(r-2c)!}\int_0^\infty\beta^{r-2c+2}\,e^{-\beta Z_x}d\beta\int_0^\infty\lambda^{r-2c-2}\,e^{-\lambda Z_y}d\lambda$$

$$=\frac{Z_x^{r-2c+1}}{(r-2c)!}\,\frac{Z_y^{r-2c+1}}{(r-2c)!}\,\frac{(r-2c+2)!}{Z_x^{r-2c+3}}\,\frac{(r-2c-2)!}{Z_y^{r-2c-1}}$$

$$\therefore D_1=\frac{(r-2c+2)(r-2c+1)}{(r-2c)(r-2c-1)}\,\frac{Z_y^2}{Z_x^2}$$

Now

$$D_2=\frac{Z_x^{r-2c+1}}{(r-2c)!}\,\frac{Z_y^{r-2c+1}}{(r-2c)!}\int_0^\infty\beta^{r-2c+1}e^{-\beta Z_x}d\beta\int_0^\infty\lambda^{r-2c-1}\,e^{-\lambda Z_y}d\lambda$$

$$\therefore D_2=\frac{(r-2c+1)Z_y}{(r-2c)Z_x}$$

Then: $E\left(R^{-2}\,\middle|\underline{x},\underline{y}\right)=\frac{(r-2c+2)(r-2c+1)Z_y^2}{(r-2c)(r-2c-1)Z_x^2}+2\,\frac{(r-2c+1)Z_y}{(r-2c)Z_x}+1$

$$\therefore \hat{R}_{QJ} = \frac{1+\frac{(r-2c+1)Z_y}{(r-2c)Z_x}}{1+2\frac{(r-2c+1)Z_y}{(r-2c)Z_x}+\frac{(r-2c+2)(r-2c+1)Z_y^2}{(r-2c)(r-2c-1)Z_x^2}} \quad \ldots (18)$$

(iii) Under Entropy Loss Function

$$\hat{R}_{tEJ} = \left[E(R^{-t}\,|\underline{x},\underline{y})\right]^{-\frac{1}{t}} \qquad t \neq 0$$

If t = 1 $\rightarrow \hat{R}_t = \hat{R}_w$ (eq. 16)

If t = 2 $\rightarrow \hat{R}_{2EJ} = \left[E(R^{-2}\,|\underline{x},\underline{y})\right]^{-\frac{1}{2}}$, then :

$$\hat{R}_{2EJ} = 1 + 2\frac{(r-2c+1)Z_y}{(r-2c)Z_x} + \frac{(r-2c+2)(r-2c+1)Z_y^2}{(r-2c)(r-2c-1)Z_x^2} \quad \ldots.. (19)$$

and $\hat{R}_{tEJ} = \left[E(R^{-t}\,|\underline{x},\underline{y})\right]^{-\frac{1}{t}}$

$$E(R^{-1}|\underline{x},\underline{y}) = \int_0^\infty\int_0^\infty \left(\frac{\lambda}{\beta+\lambda}\right)^{-1} P_J(\beta,\lambda\,|\underline{x},\underline{y})\,d\beta d\lambda$$

$$= \frac{Z_X^{r-2c+1}}{(r-2c)!}\frac{Z_y^{r-2c+1}}{(r-2c)!}\sum_{i=0}^t C_i^t \int_0^\infty \beta^{r-2c+i}e^{-\beta Z_x}d\beta \int_0^\infty \lambda^{r-2c-i}e^{-\lambda Z_y}d\lambda$$

$$= \sum_{i=0}^t C_i^t \frac{Z_x^{r-2c+1}}{(r-2c)!}\frac{Z_y^{r-2c+1}}{(r-2c)!}\frac{(r-2c+i)!}{Z_x^{r-2c+i+1}}\frac{(r-2c-i)!}{Z_y^{r-2c-i+1}}$$

$$= \sum_{i=0}^t C_i^t \frac{(r-2c+i)!(r-2c-i)!}{(r-2c)!(r-2c)!}\left(\frac{Z_y}{Z_x}\right)^i$$

$$\therefore \hat{R}_{tEJ} = \left[\sum_{i=o}^t C_i^t \frac{(r-2c+i)!(r-2c-i)!}{(r-2c)!(r-2c)!}\left(\frac{Z_y}{Z_x}\right)^i\right]^{-\frac{1}{t}} \quad \ldots. (20)$$

Empirical Study

To compare between estimators for which is the best to estimate the reliability of stress – strength Weibull model; (Since it is not possible to apply real data in our research, recommending doing so in future researches), an empirical study made by simulation procedure using MATLAB program to compare among them by MSE criteria, under different sample sizes (n = m =15) representing the smallest sample size, (n=m=30) for moderate and (n=m=90) for large (which is known to have a range greater than 75) sample sizes, in eight experiments of different parameters values and when α=0.8. The replication done for (q= 5000).

Equation (20) is used to generate different values of the two random variables X and Y by F(x) and F(y) respectively, where U is uniform random variable on interval (0,1), then by the inverse of distribution function technique got from:

$$x = \left[-\frac{1}{\beta}\ln(1-U)\right]^{\frac{1}{\alpha}} \qquad and$$

$$y = [-\frac{1}{\lambda}\ln(1-U)]^{-\frac{1}{\alpha}} \qquad \ldots.. (21)$$

Conclusions:

The results of the simulation study are recorded in Tables (2 to 5) below, where there is a fluctuation in the behavior of the estimated reliability of this system when the sample sizes change using the loss functions. While in Table (1), the best performance of the estimators is recorded as a summary of the experiment conclusions.

As a final result, it is found that for small sample size the best performance was for weighted loss function, and the entropy and Quadratic are the best for moderate and large sample sizes.

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Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Al-Mustansiriyah University.

References:

1. Salim H, Akma N. Bayes Estimator for Exponential Distribution with Extension of Jeffery prior Information. MJMS. 2009;3(2).
2. Al-Dubaicy AR, Karam NS. The Lomax Bayesian estimation under a logarithm loss function . NTICT. 2017:40-45.
3. Karam GS, Abbas FI, Abood ZM, Kadhim KK, Karam NS. An enhanced approach for biomedical image restoration using image fusion techniques. InAIP Conference Proceedings 2018 May 24 (Vol. 1968, No. 1, p. 030028). AIP Publishing LLC.
4. Hussein AY. On the Bayes estimation of Exponentiated Gumbel Shape Parameter. Ms Thesis, Department of Mathematics, College of Education, Al-Mustansiriyah University; 2017.
5. Kasim A. Bayes Estimators of the Shape parameter of Exponentiated Rayleigh Distribution. Ms Thesis, Department of Mathematics, College of Education, Al-Mustansiriyah University; 2014.
6. Feroze N, Aslam M. Bayesian Analysis of Exponentiated Gamma Distribution under Type II Censored Samples. IJAST. 2012; 49.
7. Seuba J, Deville S, Guizard Ch, St A. The effect of wall thickness distribution on mechanical reliability and strength in unidirectional porous ceramics. Sci Technol Adv Mater. 2016; 17(1): 128–135.
8. Mark AN. Parameter Estimation for the Two-Parameter Weibull Distribution. Brigham Young University – Provo: 2013.
9. AL-Noor NH, Saad Sh. Non-Bayes, Bayes and Empirical Bayes estimations for Reliability and

failure rate function of Lomax distribution. MTM. 2015;3(2).
10. Li Ch, Hao H. Reliability of a Stress–Strength Model with Inverse Weibull DistributionIAENG Int. J. Appl. Math. 2017;47(3)10.
11. Asgharzadeh A, Valiollahi R, Raqab M.Z. Stress-strength reliability of Weibull distribution based on progressively censored samples. SORT. 2011;35(2):103-124.

مقارنة مقدرات بيز لمعولية ويبل للاجهاد-المتانه لبيانات الرقابة من نوع II المفرد تحت دوال خساره مختلفة

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الخلاصة:
قمنا بدراسة التقديرالبيزي لمعوليه نموذج الاجهاد (Y) – المتانة (X) الذي يعرف عمر المكونة مع المتانة X والاجهاد Y (تفشل المكونة اذا وفقط في اي وقت يكون الاجهاد اكبر من متانته)، فان المعوليه؛ $R=P(Y<X)$، يمكن اعتبارها مقياسا لأداء المكونة. في هذا البحث، تم حساب التقدير البيزي لدالة المعولية عندما المتغيرين X وY عباره عن متغيرات ويبل العشوائية مع معلمة شكل α مشتركه ومعلمتي قياس مختلفتين β وλ ؛ على التوالي، باستخدام ثلاثة دوال مختلفة للخسارة(الموزون، التربيعي، الانتروبي) ضمن توزيعين سابقين مختلفين(كاما ومعلومات جيفري الموسعة) وكذلك مقدر بيز التجريبي عند توزيع كاما السابق، للعينه الخاضعة للرقابة من النوع الثاني. تم استخدام دراسة تجريبيه للمقارنة بين المقدرات الثلاثة عن طريق معيار متوسط مربعات الخطأ (MSE)، عند احجام مختلفة للعينه(صغيره ومتوسطة وكبيره) في ثماني تجارب لقيم مختلفة لمعلمات المتغيرين العشوائيين. توصل البحث الى ان مقدر المعولية بالاعتماد على دالة الخسارة الموزونة كان الافضل في حجوم العينات الصغير ومقدري دالتي الخسارة التربيعية والانتروبي كانا الافضل لحجوم العينات المتوسطة والكبيرة ضمن التوزيعين السابقين ومقدر بيز التجريبي.

الكلمات المفتاحية: التقدير البيزي، المعوليه، بيانات الرقابة من النوع الثاني، توزيع ويبل، نموذج الاجهاد – المتانة.