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On Skew Left n-Derivations with Lie Ideal Structure

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Abstract

In this paper the centralizing and commuting concerning skew left n-derivations and skew left n-derivations associated with antiautomorphism on prime and semiprime rings were studied and the commutativity of Lie ideal under certain conditions were proved.

Key words: Centralizing mapping, Commuting mapping, Prime ring, Skew left n-derivation.

Introduction:

Throughout this paper $\mathcal{R}$ represents an associative ring with center $\mathcal{Z}(\mathcal{R})$ and α^* an antiautomorphism of $\mathcal{R}$. A ring $\mathcal{R}$ is said to be n-torsion free if na=0 with $a\in\mathcal{R}$ then a=0, where n is nonzero integer (1). For any $\upsilon,\gamma\in\mathcal{R}$, the commutator $\upsilon\gamma$-$\gamma\upsilon$ is denoted by $[\upsilon,\gamma]$ (2). Recall that a ring $\mathcal{R}$ is said to be prime if $a\mathcal{R}b$=0 implies that either a=0 or b=0 for all $a,b\in\mathcal{R}$ (3) and it is semiprime if $a\mathcal{R}a$=0 implies that a=0 for all $a\in\mathcal{R}$ (1). An additive mapping $\xi:\mathcal{R}\rightarrow\mathcal{R}$ is called a derivation if $\xi(\upsilon\gamma)=\xi(\upsilon)\gamma+\upsilon\,\xi(\gamma)$ for all $\upsilon,\gamma\in\mathcal{R}$ (4), and it is called a skew derivation (α^*-derivation) of $\mathcal{R}$ associated with the antiautomorphism α^* if $\xi(\upsilon\gamma)=\xi(\upsilon)\alpha^*(\gamma)+\upsilon\,\xi(\gamma)$ for all $\upsilon,\gamma\in\mathcal{R}$ (5). An additive mapping $\xi:\mathcal{R}\rightarrow\mathcal{R}$ is called a left derivation if $\xi(\upsilon\gamma)=\gamma\xi(\upsilon)+\upsilon\,\xi(\gamma)$ for all $\upsilon,\gamma\in\mathcal{R}$ (6), and it is called a skew left derivation of $\mathcal{R}$ associated with antiautomorphism α^* if $\xi(\upsilon\gamma)=\alpha^*(\gamma)\xi(\upsilon)+\upsilon\,\xi(\gamma)$ for all $\upsilon,\gamma\in\mathcal{R}$ (7), it is clear that the concepts of derivation and left derivation are identical whenever $\mathcal{R}$ is commutative. A map $\mathcal{F}:\mathcal{R}\rightarrow\mathcal{R}$ is said to be commuting (resp. centralizing) on $\mathcal{R}$ if $[\mathcal{F}(\upsilon),\upsilon]$ =0 (resp. $[\mathcal{F}(\upsilon),\upsilon]\in\mathcal{Z}(\mathcal{R})$) for all $\upsilon\in\mathcal{R}$ (2). An additive subgroup $\mathcal{U}$ of $\mathcal{R}$ is called Lie ideal if whenever $u\in\mathcal{U}$, $\mathfrak{r}\in\mathcal{R}$ then $[\mathcal{U},\mathfrak{r}]\in\mathcal{U}$ (1). A Lie ideal $\mathcal{U}$ of $\mathcal{R}$ is called a square closed Lie ideal of $\mathcal{R}$ if $u^2\in\mathcal{U}$, for all $u\in\mathcal{U}$ (6). A square closed Lie ideal $\mathcal{U}$ of $\mathcal{R}$ such that $\mathcal{U}\nsubseteq\mathcal{Z}(\mathcal{R})$ is called an admissible Lie ideal of $\mathcal{R}$ (4). In 2009, Park introduced the concept of symmetric n-derivation and he studied the concept as centralizing and commuting (2). The history of centralizing and commuting mapping is due to Divinsky in 1955 (8).

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Several authors have studied the concept as commuting and centralizing derivations like J. Vukman who investigated symmetric bi-derivations on prime and semiprime rings (9). We obtain the similar results of Jung and Park ones for permuting 3-derivations on prime and semiprime rings (10) and more results in (11, 12, 13, 14, 15). In the present paper, we have introduced the notion of skew left n-derivation and skew left n-derivationn associated with the antiautomorphism α^* and studied the commuting and centralizing of this notion and commutativity of Lie ideal under certain conditions.

Throughout this paper n is considered as a fixed positive integer.

Preliminaries

Definition (2.1) (2)

A map $\xi:\mathcal{R}^n\rightarrow\mathcal{R}$ is called permuting (or symmetric) if the equation $\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)$ $=\xi(\upsilon_{\pi(1)},\upsilon_{\pi(2)},\dots,\upsilon_{\pi(n)})$ holds, for all $\upsilon_i\in\mathcal{R}$ and for every permutation $\{\pi(1),\pi(2),\dots,\pi(n)\}$.

Definition (2.2) (2)

An n-additive mapping $\xi:\mathcal{R}^n\rightarrow\mathcal{R}$ is said to be a symmetric n-derivation if the following equations are identical:

$$\xi(\upsilon_1\gamma,\upsilon_2,\dots,\upsilon_n)=\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\gamma+\upsilon_1\xi(\gamma,\upsilon_2,\dots,\upsilon_n)$$

$$\xi(\upsilon_1,\upsilon_2\gamma,\dots,\upsilon_n)=\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\gamma+\upsilon_2\xi(\upsilon_1,\gamma,\dots,\upsilon_n)$$

$$\vdots$$

$$\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n\gamma)=\xi(\upsilon_1,\upsilon_2,\dots,\upsilon_n)\gamma+\upsilon_n\xi(\upsilon_1,\upsilon_2,\dots,\gamma)$$

For all $\upsilon_1,\gamma,\upsilon_2,\dots,\upsilon_n\in\mathcal{R}$.

Definition (2.3) (2)

A map $\delta:\mathcal{R}\to\mathcal{R}$ is defined as $\delta(\upsilon)=\Omega(\upsilon,\upsilon,\ldots,\upsilon)$ for all $\upsilon\in\mathcal{R}$, where $\Omega:\mathcal{R}^n\to\mathcal{R}$ is called the trace of the symmetric mapping Ω.

It is clear that the trace function δ is an odd function if n is an odd number and is an even function if n is an even number.

Note (2.4) (2)

Let δ be a trace of an n-additive symmetric map $\mathcal{D}:\mathcal{R}^n\to\mathcal{R}$, then δ satisfies the relation $\delta(\upsilon+\gamma)=\delta(\upsilon)+\delta(\gamma)+\sum_{k=1}^{n-1}\binom{n}{k}h_k(\upsilon,\gamma)$ for all $\upsilon,\gamma\in\mathcal{R}$ such that $h_k(\upsilon,\gamma)=\Omega(\upsilon,\upsilon,\ldots,\upsilon,\gamma,\gamma,\ldots,\gamma)$ where υ appears $(n-k)$-times and γ appear k-times and $\binom{n}{k}=\frac{n!}{k!(n-k)!}$.

Now, we introduce new concept which is called skew left n-derivation is defined as follows.

Definition (2.5):

An n-additive symmetric mapping $\xi:\mathcal{R}^n\to\mathcal{R}$ is said to be a skew left n-derivation if

$\xi(\upsilon_1\gamma,\upsilon_2,\ldots,\upsilon_n)=\gamma\xi(\upsilon_1,\upsilon_2,\ldots,\upsilon_n)+\upsilon_1\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)$

$\xi(\upsilon_1,\upsilon_2\gamma,\ldots,\upsilon_n)=\gamma\xi(\upsilon_1,\upsilon_2,\ldots,\upsilon_n)+\upsilon_2\xi(\upsilon_1,\gamma,\ldots,\upsilon_n)$

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$\xi(\upsilon_1,\upsilon_2,\ldots,\upsilon_n\gamma)=\gamma\xi(\upsilon_1,\upsilon_2,\ldots,\upsilon_n)+\upsilon_n\xi(\upsilon_1,\upsilon_2,\ldots,\gamma)$

For all $\upsilon_1\gamma,\upsilon_2,\ldots,\upsilon_n\in\mathcal{R}$, it is clear that the concepts of derivation and left derivation are identical whenever $\mathcal{R}$ is commutative

Example (2.6):

Let $\mathcal{R}=\left\{\begin{pmatrix}0 & a\\ 0 & 0\end{pmatrix}\mid a\in\mathbb{Z}\right\}$ be a ring, and $\mathbb{Z}$ be a ring of integer numbers. A map $\xi:\mathcal{R}^n\to\mathcal{R}$ is defined by

$\xi\left(\begin{pmatrix}0 & a_1\\ 0 & 0\end{pmatrix},\begin{pmatrix}0 & a_2\\ 0 & 0\end{pmatrix},\ldots,\begin{pmatrix}0 & a_n\\ 0 & 0\end{pmatrix}\right)=\begin{pmatrix}0 & a_1a_2\ldots a_n\\ 0 & 0\end{pmatrix}$, for all $\begin{pmatrix}0 & a_1\\ 0 & 0\end{pmatrix},\begin{pmatrix}0 & a_2\\ 0 & 0\end{pmatrix},\ldots,\begin{pmatrix}0 & a_n\\ 0 & 0\end{pmatrix}\in\mathcal{R}$.

Then it easy to check that ξ is skew left n-derivation.

Example (2.7):

Let $\mathcal{R}=\left\{\begin{pmatrix}a & b\\ 0 & 0\end{pmatrix}\mid a,b\in\mathbb{R}\right\}$, where $\mathbb{R}$ a ring of real numbers and $\mathcal{R}$ is a non-commutative ring under addition and multiplication of matrices. A map $\xi:\mathcal{R}^n\to\mathcal{R}$ is defined by

$\xi\left(\begin{pmatrix}a_1 & b_1\\ 0 & 0\end{pmatrix},\begin{pmatrix}a_2 & b_2\\ 0 & 0\end{pmatrix},\ldots,\begin{pmatrix}a_n & b_n\\ 0 & 0\end{pmatrix}\right)=\begin{pmatrix}0 & b_1b_2\ldots b_n\\ 0 & 0\end{pmatrix}$, for all $\begin{pmatrix}a_1 & b_1\\ 0 & 0\end{pmatrix},\begin{pmatrix}a_2 & b_2\\ 0 & 0\end{pmatrix},\ldots,\begin{pmatrix}a_n & b_n\\ 0 & 0\end{pmatrix}\in\mathcal{R}$.

Then ξ is an n-derivation but it's not a left n-derivation.

Example (2.8):

Let $\mathcal{R}$ be a non-commutative ring. Define a map $\xi:\mathcal{R}^n\to\mathcal{R}$ by $\xi(\upsilon_1,\upsilon_2,\ldots,\upsilon_n)=\xi(\upsilon_1)\,\xi(\upsilon_2)\ldots\xi(\upsilon_n)$, for all $\upsilon_1,\upsilon_2,\ldots,\upsilon_n\in\mathcal{R}$. Then ξ is a skew left n-derivation but it's not n-derivation.

Lemma (2.9) (4): Let $\mathcal{R}$ be a prime ring and $\xi:\mathcal{R}\to\mathcal{R}$ be a derivation such that $a\in\mathcal{R}$. If $a\xi(\upsilon)=0$ holds for all $\upsilon\in\mathcal{R}$, then either $a=0$ or $\xi=0$.

Lemma (2.10) (3): Let $\mathcal{R}$ be a $n!$-torsion free ring and $\lambda\gamma_1+\lambda^2\gamma_2+\ldots+\lambda^n\gamma_n=0$ where $\gamma_1,\gamma_2,\ldots,\gamma_n\in\mathcal{R}$ with $\lambda=1,2,\ldots,n$. Then $\gamma_i=0$, for all $i=1,2,\ldots,n$.

Lemma (2.11) (2): Let $\mathcal{R}$ be a $n!$-torsion free ring and $\lambda\gamma_1+\lambda^2\gamma_2+\ldots+\lambda^n\gamma_n\in\mathcal{Z}(\mathcal{R})$ where $\gamma_1,\gamma_2,\ldots,\gamma_n\in\mathcal{R}$ with $\lambda=1,2,\ldots,n$. Then $\gamma_i\in\mathcal{Z}$, for all $i=1,2,\ldots,n$.

The Main Results

In 2009, K.H. Park (2) studied the concept of symmetric n-derivation as centralizing and commuting, we have studied the concept of skew left n-derivations in rings and introduced the term of skew left n-derivation associated with an antiautomorphism α^*.

In the following results, $\mathcal{U}$ is considered as a non-central Lie ideal of $n!$-torsion free prime ring $\mathcal{R}$.

Theorem3.1: Let $\Omega:\mathcal{U}^n\to\mathcal{R}$ be a skew left n-derivation such that the trace δ of Ω is commuting on $\mathcal{U}$. Then $\Omega=0$.

Proof:

$[\delta(\upsilon),\upsilon]=0,\forall\upsilon\in\mathcal{U}$. … (1)

Substituting $\upsilon=\upsilon+\mu\gamma$ in equation (1) and using it and let $\mu(1\le\mu\le n)$ be any integer, then

$0=[\delta(\upsilon+\mu\gamma),\upsilon+\mu\gamma]$

$=[\delta(\upsilon)+\delta(\mu\gamma)+\sum_{k=1}^{n-1}C_kh_k(\upsilon,\mu\gamma),\ \upsilon+\mu\gamma]$

$=\mu\{[\delta(\upsilon),\gamma]+[c_1h_1(\upsilon,\gamma),\upsilon]\}+\mu^2\{[c_2h_2(\upsilon,\gamma),\upsilon]+[c_1h_1(\upsilon,\gamma),\gamma]\}+\ldots+\mu^n\{[\delta(\gamma),\upsilon]+[c_{n-1}h_{n-1}(\upsilon,\gamma),\gamma]$ … (2)

From lemma (2.10) and equation (2), to have

$[\delta(\upsilon),\gamma]+n[h_1(\upsilon,\gamma),\upsilon]=0$ ... (3)

Replacing $\gamma=2\upsilon\gamma$ in equation (3) and using it, then

$0=[\delta(\upsilon),2\upsilon\gamma]+n[h_1(\upsilon,2\upsilon\gamma),\upsilon]$

$=2\{\upsilon\{[\delta(\upsilon),\gamma]+n[h_1(\upsilon,\gamma),\upsilon]\}+n[\gamma,\upsilon]\delta(\upsilon)\}$

$=2n[\gamma,\upsilon]\delta(\upsilon)$, and by using $n!$-torsion to have $[\gamma,\upsilon]\delta(\upsilon)=0$. … (4)

From equation (4) and Lemma (2.9), to get

A map $\gamma\to[\gamma,\upsilon]$ is a derivation on $\mathcal{U}$. Then $\delta(\upsilon)=0$ … (5)

For each k=1,2,…, n let $p_k(\upsilon)$=$\Omega(\upsilon,\ldots,\upsilon,\upsilon_{k+1},\upsilon_{k+2},\ldots,\upsilon_n)$ where υ appears k-times and $\upsilon,\upsilon_i\in\mathcal{U}$, $i=k+1,k+2,\ldots,n$
Let τ ($1\leq\tau\leq n-1$) be any integer. By equation (5) the relation

$0=\delta(\tau\upsilon+\upsilon_n)=p_n(\tau\upsilon+\upsilon_n)$
$=\tau^n\delta(\upsilon)+\delta(\upsilon_n)+\sum_{k=1}^{n-1}\tau^k c_k p_k(\upsilon)$
$=\sum_{k=1}^{n-1}\tau^k c_k p_k(\upsilon)$
... (6)

By lemma (2.10) and equation (6), to have
$c_{n-1}p_{n-1}(\upsilon)=p_{n-1}(\upsilon)=0$
... (7)

Let ζ ($1\leq\zeta\leq n-2$) be any integer. By equation (7) the relation
$p_{n-1}(\zeta\upsilon+\upsilon_{n-1})=0,\ \forall\ \upsilon,\upsilon_{n-1}\in\mathcal{U}$
$\zeta^{n-1}p_{n-1}(\upsilon)+p_{n-1}(\upsilon_{n-1})+\sum_{k=1}^{n-2}\zeta^k c_k p_k(\upsilon)=0$
$\sum_{k=1}^{n-2}\zeta^k c_k p_k(\upsilon)=0$
... (8)

Using lemma (2.10) and equation (8) to get $c_{n-2}p_{n-2}(\upsilon)=p_{n-2}(\upsilon)=0$, hence $c_1p_1(\upsilon)=0$ and then $p_1(\upsilon)=0$, which means $\Omega(\upsilon_1,\upsilon_2,\ldots,\upsilon_n)=0, \forall\upsilon_i\in\mathcal{U}$, where $i=1,2,\ldots,n$.

Theorem3.2: Let $\Omega:\mathcal{U}^n\rightarrow\mathcal{R}$ be a skew left n-derivation such that the trace δ of Ω is centralizing on $\mathcal{U}$. Then δ is commuting on non-zero ideal I of $\mathcal{U}$.

Proof:
$[\delta(\upsilon),\upsilon]\in\mathcal{Z}(\mathcal{R}), \forall\upsilon\in\mathcal{U}$.
… (1)

Substituting $\upsilon=\upsilon+\mu\gamma$ in equation (1) and using it and let μ($1\leq\mu\leq n$) be any integer, to obtain
$\mathcal{Z}(\mathcal{R})\ni[\delta(\upsilon+\mu\gamma),\upsilon+\mu\gamma]$
$=[\delta(\upsilon)+\delta(\mu\gamma)+\sum_{r=1}^{n-1}C_r h_r(\upsilon,\mu\gamma),\ \upsilon+\mu\gamma]$
$=\mu\{[\delta(\upsilon),\gamma]+[c_1h_1(\upsilon,\gamma),\upsilon]\}+\mu^2\{[c_2h_2(\upsilon,\gamma),\upsilon]+[c_1h_1(\upsilon,\gamma),\gamma]\}+\cdots+\mu^n\{[\delta(\gamma),\upsilon]+[c_{n-1}h_{n-1}(\upsilon,\gamma),\gamma]\}$
... (2)

From lemma (2.11) and equation (2), to have
$[\delta(\upsilon),\gamma]+n[h_1(\upsilon,\gamma),\upsilon]\in\mathcal{Z}(\mathcal{R}),\quad \forall\upsilon,\gamma\in\mathcal{U}$
... (3)

Taking $\gamma=2\upsilon^2$ in equation (3) and using it, to get
$\mathcal{Z}(\mathcal{R})\ni[\delta(\upsilon),2\upsilon^2]+n[h_1(\upsilon,2\upsilon^2),\upsilon]$
$=(2n+2)[\delta(\upsilon),\upsilon]\upsilon$
… (4)

Commuting equation (4) with $\delta(\upsilon)$ gives
$0=(2n+2)[\delta(\upsilon),\upsilon]^2$
... (5)

Substituting $\gamma=2\upsilon\gamma$ in equation (3) to have
$\mathcal{Z}(\mathcal{R})\ni[\delta(\upsilon),2y]+n[h_1(\upsilon,2\upsilon y),\upsilon]$
$=(n+1)[\delta(\upsilon),\upsilon]\gamma+\upsilon\{[\delta(\upsilon),\gamma]+n[h_1(\upsilon,\gamma),\upsilon]\}+n[\gamma,\upsilon]\delta(\upsilon)$

Commuting the last equation with υ, and using equation (3) then
$[(n+1)[\delta(\upsilon),\upsilon]\gamma+n[\gamma,\upsilon]\delta(\upsilon),\upsilon]+[\upsilon\{[\delta(\upsilon),\gamma]+n[h_1(\upsilon,\gamma),\upsilon]\},\upsilon]=0$ … (6)

It follows equation (6) that
$0=(n+1)[[\delta(\upsilon),\upsilon],\upsilon]\gamma+(n+1)[\delta(\upsilon),\upsilon][\gamma,\upsilon]+n[[\gamma,\upsilon],\upsilon]\delta(\upsilon)+n[\gamma,\upsilon][\delta(\upsilon),\upsilon]$
$=(2n+1)[\delta(\upsilon),\upsilon][\gamma,\upsilon]+n[[\gamma,\upsilon],\upsilon]\delta(\upsilon)$
... (7)

Since $\mathcal{U}$ is a non-central Lie ideal then there exists a non-zero ideal I of $\mathcal{U}$. Replacing $\gamma=\delta(\upsilon)\gamma$ in equation (7) for all $\upsilon\in\mathcal{U}$, $\gamma\in I$ and by using equation (1), to have:
$0=(2n+1)[\delta(\upsilon),\upsilon][\delta(\upsilon)\gamma,\upsilon]+n[[\delta(\upsilon)\gamma,\upsilon],\upsilon]\delta(\upsilon)$
$=(2n+)[\delta(\upsilon),\upsilon]^2\gamma+\delta(\upsilon)\{(2n+1)[\delta(\upsilon),\upsilon][\gamma,\upsilon]+n[[\gamma,\upsilon],\upsilon]\delta(\upsilon)\}+2n[\delta(\upsilon),\upsilon][\gamma,\upsilon]\delta(\upsilon)$

According to equation (7), to get
$(2n+1)[\delta(\upsilon),\upsilon]^2\gamma+2n[\delta(\upsilon),\upsilon][\gamma,\upsilon]\delta(\upsilon)=0$
… (8)

Taking $\gamma=[\delta(z),z]$ and $\upsilon=z$ in equation (8) where $z\in I$, to have
$0=(2n+1)[\delta(z),z]^3+2n[\delta(z),z][[\delta(z),z],z]\delta(z)$
$=(2n+1)[\delta(z),z]^3=0,\quad\forall\ z\in I$ and so we have$(2n+1)[\delta(z),z]^2U(2n+1)[\delta(z),z]^2=0$. By the semiprimeness of $\mathcal{R}$, to get
$(2n+1)[\delta(z),z]^2=0$
... (9)

Combining equation (9) with (5) then
$[\delta(z),z]^2=0, \forall z\in I$.

As the center of a semiprime ring contains no non-zero nilpotent elements, then we conclude that $[\delta(z),z]$, $\forall z\in I$.

Theorem 3.3: Let $\Omega:\mathcal{U}^n\rightarrow\mathcal{R}$ be a non-zero skew left n-derivation such that the trace δ of Ω is centralizing $\mathcal{U}$.Then $\mathcal{U}$ is commutative.

Proof:
Suppose that $\mathcal{U}$ is a non-commutative prime ring. Then from theorem (3.2) we have δ which is commuting on $\mathcal{U}$. And from theorem (3.1) we have Ω=0, which is contradiction hence, $\mathcal{U}$ must be commutative prime ring.

Now, the pervious results can be generalized by introducing the concept of skew left n-derivation associated with antiautomorphism as follows:

Definition 3.4:
An n-additive mapping $\xi:\mathcal{R}^n\rightarrow\mathcal{R}$ is called a skew left n-derivation associated with an antiautomorphism α^*if
$\xi(\upsilon_1\gamma,\upsilon_2,\ldots,\upsilon_n)=\alpha^*(\gamma)\xi(\upsilon_1,\upsilon_2,\ldots,\upsilon_n)+\upsilon_1\xi(\gamma,\upsilon_2,\ldots,\upsilon_n)$
$\xi(\upsilon_1,\upsilon_2\gamma,\ldots,\upsilon_n)=\alpha^*(\gamma)\xi(\upsilon_1,\upsilon_2,\ldots,\upsilon_n)+\upsilon_2\xi(\upsilon_1,\gamma,\ldots,\upsilon_n)$
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$\xi(\upsilon_1,\upsilon_2,\ldots,\upsilon_n\gamma)=\alpha^*(\gamma)\xi(\upsilon_1,\upsilon_2,\ldots,\upsilon_n)+\upsilon_n\xi(\upsilon_1,\upsilon_2,\ldots,\gamma)$, for all $\upsilon_1,\gamma,\upsilon_2,\ldots,\upsilon_n\in\mathcal{R}$.

Examples 3.5:

(1) Let $\mathcal{F}$ be a field and let α^* be an antiautomorphism of $\mathcal{F}$. Assume that $\mathcal{R}=\left\{\begin{pmatrix} a & 0 \\ b & 0\end{pmatrix} \mid a,b\in\mathcal{F}\right\}$, where $\mathcal{R}$ is a non-commutative ring under addition and multiplication of matrices. Define a map $\alpha^*:\mathcal{R}\to\mathcal{R}$ as $\begin{pmatrix} a & 0 \\ b & 0\end{pmatrix}=\begin{pmatrix} 0 & 0 \\ 0 & \alpha^*(\text{a})\end{pmatrix}$ for all $a,b\in\mathcal{F}$. Now let us define a map $\xi:\mathcal{R}^n\to\mathcal{R}$ as

$$\xi\left(\begin{pmatrix} a_1 & 0 \\ b_1 & 0\end{pmatrix},\begin{pmatrix} a_2 & 0 \\ b_2 & 0\end{pmatrix},\ldots,\begin{pmatrix} a_n & 0 \\ b_n & 0\end{pmatrix}\right)=\begin{pmatrix} 0 & 0 \\ \alpha^*(a_1)\alpha^*(a_2)\ldots\alpha^*(a_n) & 0\end{pmatrix}$$

for all $\begin{pmatrix} a_1 & 0 \\ b_1 & 0\end{pmatrix},\begin{pmatrix} a_2 & 0 \\ b_2 & 0\end{pmatrix},\ldots,\begin{pmatrix} a_n & 0 \\ b_n & 0\end{pmatrix}\in\mathcal{R}$.

This means that ξ is a skew left n-derivation associated with antiautomorphism α^*, but it is not n-derivation.

(2) Let $\mathbb{C}$ be a complex field and let α^* be an antiautomorphism of $\mathbb{C}$. Assume that $\mathcal{R}=\left\{\begin{pmatrix} a & 0 \\ b & 0\end{pmatrix} \mid a,b\in\mathbb{C}\right\}$, where $\mathcal{R}$ is a non-commutative ring under addition and multiplication of matrices. Define a map $\alpha^*:\mathcal{R}\to\mathcal{R}$ as $\begin{pmatrix} a & 0 \\ b & 0\end{pmatrix}=\begin{pmatrix} 0 & 0 \\ 0 & \alpha^*(\text{a})\end{pmatrix}$ for all $a,b\in\mathbb{C}$. Now let us define a map $\xi:\mathcal{R}^n\to\mathcal{R}$ as $\xi\left(\begin{pmatrix} a_1 & 0 \\ b_1 & 0\end{pmatrix},\begin{pmatrix} a_2 & 0 \\ b_2 & 0\end{pmatrix},\ldots,\begin{pmatrix} a_n & 0 \\ b_n & 0\end{pmatrix}\right)=\begin{pmatrix} 0 & 0 \\ 0 & \alpha^*(a_1)\alpha^*(a_2)\ldots\alpha^*(a_n)\end{pmatrix}$, for all $\begin{pmatrix} a_1 & 0 \\ b_1 & 0\end{pmatrix},\begin{pmatrix} a_2 & 0 \\ b_2 & 0\end{pmatrix},\ldots,\begin{pmatrix} a_n & 0 \\ b_n & 0\end{pmatrix}\in\mathcal{R}$.

This means that ξ is a skew left n-derivation associated with antiautomorphism α^*.

In the following results, $\mathcal{U}$ is assumed as an admissible Lie ideal of $n!$-torsion free ring $\mathcal{R}$ with $n\geq 2$.

Theorem 3.6: Let $\mathcal{R}$ be a prime ring and $\Omega:\mathcal{U}^n\to\mathcal{R}$ be a skew left n-derivation associated with an antiautomorphism α^*. If the trace δ of Ω satisfies $[\delta(\upsilon),\alpha^*(\upsilon)]=0$, for all $\upsilon\in\mathcal{U}$ then $\Omega(\upsilon_1,\upsilon_2,\ldots,\upsilon_n)=0$, for all $\upsilon_i\in\mathcal{U}$ where $i=1,2,\ldots,n$.

Proof:

$[\delta(\upsilon),\alpha^*(\upsilon)]=0,\forall\upsilon\in\mathcal{U}$ … (1)

Substituting $\upsilon=\upsilon+\mu\gamma$ in equation (1) and using it and let $\mu(1\leq\mu\leq n)$ be any integer, to obtain

$0=[\delta(\upsilon+\mu\gamma),\alpha^*(\upsilon+\mu\gamma)]=[\delta(\upsilon)+\delta(\mu\gamma)+\sum_{s=1}^{n-1}C_s f_s(\upsilon,\mu\gamma),\ \alpha^*(\upsilon)+\mu\alpha^*(\gamma)]$
$=\mu\{[\delta(\upsilon),\alpha^*(\gamma)]+[c_1f_1(\upsilon,\gamma),\alpha^*(\upsilon)]\}+\mu^2\{[c_2f_2(\upsilon,\gamma),\alpha^*(\upsilon)]+[c_1f_1(\upsilon,\gamma),\alpha^*(\gamma)]\}+\ldots+\mu^n\{[\delta(\gamma),\alpha^*(\upsilon)]+[c_{n-1}f_{n-1}(\upsilon,\gamma),\alpha^*(\gamma)]\}$ … (2)

Applying lemma (2.10) to equation (2)

$[\delta(\upsilon),\alpha^*(\gamma)]+[c_1f_1(\upsilon,\gamma),\alpha^*(\upsilon)]=0$ … (3)

Replacing $\gamma=2\gamma\upsilon$ in equation (3), to get

$0=[\delta(\upsilon),\alpha^*(2\gamma\upsilon)]+[c_1f_1(\upsilon,2\gamma\upsilon),\alpha^*(\upsilon)]$
$=2[\delta(\upsilon),\alpha^*(\upsilon)]\alpha^*(\gamma)+2\alpha^*(\upsilon)[\delta(\upsilon),\alpha^*(\gamma)]+2c_1[\alpha^*(\upsilon)f_1(\upsilon,\gamma),\alpha^*(\upsilon)]+2c_1[\gamma\delta(\upsilon),\alpha^*(\upsilon)]$
$=2\alpha^*(\upsilon)\{[\delta(\upsilon),\alpha^*(\gamma)]+c_1[f_1(\upsilon,\gamma),\alpha^*(\upsilon)]\}+2c_1[\gamma,\alpha^*(\upsilon)]\delta(\upsilon)$

By using equation (3), the above equation becomes $2c_1[\gamma,\alpha^*(\upsilon)]\delta(\upsilon)=0$, using $n!$-torsion free, to have

$[\gamma,\alpha^*(\upsilon)]\delta(\upsilon)=0,\quad\forall\upsilon,\gamma\in\mathcal{U}$ … (4)

Replacing $\gamma=2\gamma w$ in equation (4), for all $w\in\mathcal{U}$ to have

$0=[2\gamma w,\alpha^*(\upsilon)]\delta(\upsilon)$
$=2[\gamma,\alpha^*(\upsilon)]w\delta(\upsilon)+2\gamma[w,\alpha^*(\upsilon)]\delta(\upsilon)$

By using equation (4) the above equation becomes

$[\gamma,\alpha^*(\upsilon)]w\delta(\upsilon)=0$ … (5)

By using lemma (2.9), $\gamma\to[\gamma,\alpha^*(\upsilon)]$ is a derivation on $\mathcal{U}$. Then $\delta(\upsilon)=0$ … (6)

Now, for each value $l=1,2,\ldots,n$, let us denote $T_l(\upsilon)=\Omega(\upsilon,\upsilon,\ldots,\upsilon_{l+1},\upsilon_{l+2},\ldots,\upsilon_n)$, where $\upsilon,\upsilon_i\in\mathcal{U}$, $i=l+1,l+2,\ldots,n$. $T_n(\upsilon)=\delta(\upsilon)=0$ … (7)

Let $\eta(1\leq\eta\leq n)$ be any positive integer. From equation (7) to have

$0=T_n(\eta\upsilon+\upsilon_n)=T_n(\upsilon_n)+T_n(\eta\upsilon)+\sum_{l=1}^{n-1}\eta^l c_l T_l(\upsilon)=\delta(\upsilon_n)+\eta^n\delta(\upsilon)+\sum_{l=1}^{n-1}\eta^l c_l T_l(\upsilon)=\sum_{l=1}^{n-1}\eta^l c_l T_l(\upsilon)=\eta^1c_1T_1(\upsilon)+\eta^2c_2T_2(\upsilon)+\ldots+\eta^{n-1}c_{n-1}T_{n-1}(\upsilon)$ … (8)

Applying lemma (2.10) to equation (8) then

If $c_1T_1(\upsilon)=0$ then $T_1(\upsilon)=0$ which implies that $\Omega(\upsilon,\upsilon_2,\upsilon_3,\ldots,\upsilon_n)=0$

If $c_2T_2(\upsilon)=0$ then $T_2(\upsilon)=0$ which implies that $\Omega(\upsilon,\upsilon,\upsilon_3,\ldots,\upsilon_n)=0$

If $c_{n-1}T_{n-1}(\upsilon)=0$ then $T_{n-1}(\upsilon)=0$ which implies that $\Omega(\upsilon,\upsilon,\upsilon,\ldots,\upsilon_n)=0$

Hence from above, we have $T_{n-1}(\upsilon)=0$ … (9)

Again let $\tau(1\leq\tau\leq n-1)$ be any positive integer. Then from equation (9) to get

$0=T_{n-1}(\tau\upsilon+\upsilon_{n-1})=T_{n-1}(\tau\upsilon)+T_{n-1}(\upsilon_{n-1})+\sum_{t=1}^{n-2}\tau^t C_t T_t(\upsilon)$

$=\tau^1c_1T_1(\upsilon)+\tau^2c_2T_2(\upsilon)+\cdots+\tau^{n-2}c_{n-2}T_{n-2}(\upsilon)$ … (10)

Again applying lemma (2.10) to equation (10) then

$\Omega(\upsilon,\upsilon,\ldots,\upsilon,\upsilon_{n-1},\upsilon_n)=T_{n-2}(\upsilon)=0.$ … (11)

Continuing the above process, finally we obtain $T_1(\upsilon)$=0, then
$\Omega(\upsilon_1,\upsilon_2,\upsilon_3,\dots,\upsilon_{n-1},\upsilon_n)$=0
… (12)
Replacing υ_1=$2\upsilon_1 p_1$, where $p_1 \in \mathcal{U}$ in equation (12) to get
$0=\Omega(2\upsilon_1 p_1,\upsilon_2,\upsilon_3,\dots,\upsilon_{n-1},\upsilon_n)=$
$\alpha(p_1)\,\Omega(\upsilon_1,\upsilon_2,\upsilon_3,\dots,\upsilon_{n-1},\upsilon_n)+$
$\upsilon_1\Omega(p_1,\upsilon_2,\upsilon_3,\dots,\upsilon_{n-1},\upsilon_n)=$
$\upsilon_1\Omega(p_1,\upsilon_2,\upsilon_3,\dots,\upsilon_{n-1},\upsilon_n)$ … (13)
Applying lemma (2.9) to equation (13) to have
$\Omega(p_1,\upsilon_2,\upsilon_3,\dots,\upsilon_{n-1},\upsilon_n)=0, \forall p_1,\upsilon_i \in \mathcal{U}$.
Replacing υ_2=$\upsilon_2 p_2$, $p_2 \in \mathcal{U}$ in equation (13) to obtain
$0=\Omega(p_1,\upsilon_2 p_2,\upsilon_3,\dots,\upsilon_{n-1},\upsilon_n)=$
$\alpha(p_2)\,\Omega(p_1,\upsilon_2,\upsilon_3,\dots,\upsilon_{n-1},\upsilon_n)+$
$\upsilon_2\Omega(p_1,p_2,\dots,\upsilon_{n-1},\upsilon_n)=\upsilon_2\Omega(p_1,p_2,\dots,\upsilon_{n-1},\upsilon_n)=$
$\Omega(p_1,p_2,\dots,\upsilon_{n-1},\upsilon_n), \forall p_1,p_2,\upsilon_i \in \mathcal{U}$
Repeating the above process we finally obtain $\Omega(p_1,p_2,\dots,p_{n-1},p_n)=0, \forall p_i \in \mathcal{U}$.

Theorem 3.7: Let $\mathcal{R}$ be a semiprime ring and $\Omega:\mathcal{U}^n \to \mathcal{R}$ be a skew left n-derivation associated with an antiautomorphism α^*. If the trace δ of Ω δ is commuting on $\mathcal{U}$ and $[\delta(\upsilon),\alpha^*(\upsilon)] \in \mathcal{Z}(\mathcal{R})$, then $[\delta(\upsilon),\alpha^*(\upsilon)]$=0 for all $\upsilon \in \mathcal{U}$.

Proof:
$[\delta(\upsilon),\alpha^*(\upsilon)] \in \mathcal{Z}(\mathcal{R}), \forall \upsilon \in \mathcal{U}$.
… (1)
Substituting υ=$\upsilon+\mu\gamma$ in equation (1) and using it and let $\mu(1 \le \mu \le n)$ be any integer, then
$\mathcal{Z}(\mathcal{R}) \ni [\delta(\upsilon+\mu\gamma),\alpha^*(\upsilon+\mu\gamma)]$
$=[\delta(\upsilon)+\delta(\mu\gamma)+\sum_{s=1}^{n-1} C_s f_s(\upsilon,\mu\gamma),\ \alpha^*(\upsilon)+\mu\alpha^*(\gamma)]$
$=[\delta(\upsilon),\alpha^*(\upsilon)]+\mu\{[\delta(\upsilon),\alpha^*(\gamma)]+[c_1 f_1(\upsilon,\gamma),\alpha^*(\upsilon)]\}+\mu^2\{[c_2 f_2(\upsilon,\gamma),\alpha^*(\upsilon)]+[c_1 f_1(\upsilon,\gamma),\alpha^*(\gamma)]\}+\dots+\mu^n\{[\delta(\gamma),\alpha^*(\upsilon)]+[c_{n-1} f_{n-1}(\upsilon,\gamma),\alpha^*(\gamma)]\}+\mu^{n+1}[\delta(\gamma),\alpha^*(\gamma)]$ … (2)
Commuting equation (2) with $\delta(\upsilon)$, to have
$\big[[\delta(\upsilon),\alpha^*(\upsilon)],\delta(\upsilon)\big]+\mu\{[[\delta(\upsilon),\alpha^*(\gamma)]+[c_1 f_1(\upsilon,\gamma),\alpha^*(\upsilon)],\delta(\upsilon)]\}+\mu^2\{[[c_2 f_2(\upsilon,\gamma),\alpha^*(\upsilon)]+[c_1 f_1(\upsilon,\gamma),\alpha^*(\gamma)],\delta(\upsilon)]\}+\dots+\mu^n\{[[\delta(\gamma),\alpha^*(\upsilon)]+[c_{n-1} f_{n-1}(\upsilon,\gamma),\alpha^*(\gamma)],\delta(\upsilon)]\}+\mu^{n+1}\big[[\delta(\gamma),\alpha^*(\gamma)],\delta(\upsilon)\big]$=0 … (3)
Applying lemma (2.10) to equation (3), then
$0=[[\delta(\upsilon),\alpha^*(\gamma)],\delta(\upsilon)]+[[c_1 f_1(\upsilon,\gamma),\alpha^*(\upsilon)],\delta(\upsilon)]$
... (4)
Replacing γ=$2\upsilon^2$ in equation (4), to obtain
$0=[[\delta(\upsilon),\alpha^*(2\upsilon^2)],\delta(\upsilon)]+[[c_1 f_1(\upsilon,2\upsilon^2),\alpha^*(\upsilon)],\delta(\upsilon)]$
$=[[\delta(\upsilon),\alpha^*(\upsilon)],\delta(\upsilon)]\alpha^*(\upsilon)+[\delta(\upsilon),\alpha^*(\upsilon)][\alpha^*(\upsilon),\delta(\upsilon)]+[\alpha^*(\upsilon),\delta(\upsilon)][\delta(\upsilon),\alpha^*(\upsilon)]+\alpha^*(\upsilon)\big[[\delta(\upsilon),\alpha^*(\upsilon)],\delta(\upsilon)\big]+c_1[\alpha^*(\upsilon),\delta(\upsilon)][\delta(\upsilon),\alpha^*(\upsilon)]+c_1\alpha^*(\upsilon)\big[[\delta(\upsilon),\alpha^*(\upsilon)],\delta(\upsilon)\big]+c_1[[\upsilon,\alpha^*(\upsilon)],\delta(\upsilon)]\delta(\upsilon)+c_1[\upsilon,\alpha^*(\upsilon)][\delta(\upsilon),\delta(\upsilon)]+c_1[\upsilon,\delta(\upsilon)][\delta(\upsilon),\alpha^*(\upsilon)]+c_1\upsilon[[\delta(\upsilon),\alpha^*(\upsilon)],\delta(\upsilon)]$
$=-(c_1+2)[\delta(\upsilon),\alpha^*(\upsilon)]^2+c_1[[\upsilon,\alpha^*(\upsilon)],\delta(\upsilon)]\,\delta(\upsilon)$
$=-(c_1+2)[\delta(\upsilon),\alpha^*(\upsilon)]^2+c_1[\{\upsilon\alpha^*(\upsilon)-\alpha^*(\upsilon)\upsilon\},\delta(\upsilon)]\,\delta(\upsilon)$
$=-(c_1+2)[\delta(\upsilon),\alpha^*(\upsilon)]^2+c_1[\upsilon,[\alpha^*(\upsilon),\delta(\upsilon)]]\delta(\upsilon)$
$=(c_1+2)[\delta(\upsilon),\alpha^*(\upsilon)]^2$
... (5)
Commuting equation (2) with $\alpha^*(\upsilon)$ and by using lemma (2.10), then
$0=[[\delta(\upsilon),\alpha^*(\gamma)],\alpha^*(\upsilon)]+[c_1 f_1(\upsilon,\gamma),\alpha^*(\upsilon)],\alpha^*(\upsilon)]$
… (6)
Replacing γ=$2\gamma\upsilon$ in equation (6), to obtain
$0=[[\delta(\upsilon),\alpha^*(2\gamma\upsilon)]+[c_1 f_1(\upsilon,2\gamma\upsilon),\alpha^*(\upsilon)],\alpha^*(\upsilon)]$
$=\big[[\delta(\upsilon),\alpha^*(\upsilon)],\alpha^*(\upsilon)\big]\alpha^*(\gamma)+[\delta(\upsilon),\alpha^*(\upsilon)][\alpha^*(\gamma),\alpha^*(\upsilon)]+[\alpha^*(\upsilon),\alpha^*(\upsilon)]\,[\delta(\upsilon),\alpha^*(\gamma)]+\alpha^*(\upsilon)\,\big[[\delta(\upsilon),\alpha^*(\gamma)],\alpha^*(\upsilon)\big]+c_1[\alpha^*(\upsilon),\alpha^*(\upsilon)][f_1(\upsilon,\gamma),\alpha^*(\upsilon)]+c_1\alpha^*(\upsilon)[[f_1(\upsilon,\gamma),\alpha^*(\upsilon)],\alpha^*(\upsilon)]+c_1[[\gamma,\alpha^*(\upsilon)],\alpha^*(\upsilon)]\delta(\upsilon)+c_1[\gamma,\alpha^*(\upsilon)][\delta(\upsilon),\alpha^*(\upsilon)]+c_1[\gamma,\alpha^*(\upsilon)][\delta(\upsilon),\alpha^*(\upsilon)]+c_1\gamma[[\delta(\upsilon),\alpha^*(\upsilon)],\alpha^*(\upsilon)]$
$=[\delta(\upsilon),\alpha^*(\upsilon)][\alpha^*(\gamma),\alpha^*(\upsilon)]+\alpha^*(\upsilon)\{\big[[\delta(\upsilon),\alpha^*(\gamma)],\alpha^*(\upsilon)\big]+c_1[[f_1(\upsilon,\gamma),\alpha^*(\upsilon)],\alpha^*(\upsilon)]+c_1[[\gamma,\alpha^*(\upsilon)],\alpha^*(\upsilon)]\delta(\upsilon)+2c_1[\gamma,\alpha^*(\upsilon)][\delta(\upsilon),\alpha^*(\upsilon)]$
By using equation (6), the last equation becomes
$[\delta(\upsilon),\alpha^*(\upsilon)][\alpha^*(\gamma),\alpha^*(\upsilon)]+c_1\big[[\gamma,\alpha^*(\upsilon)],\alpha^*(\upsilon)\big]\delta(\upsilon)+2c_1[\gamma,\alpha^*(\upsilon)][\delta(\upsilon),\alpha^*(\upsilon)]$=0 …(7)
Replacing γ= $\delta(\upsilon)[\delta(\upsilon),\alpha^*(\upsilon)]$ in equation (7), to get
$0=[\delta(\upsilon),\alpha^*(\upsilon)][\alpha^*(\delta(\upsilon)[\delta(\upsilon),\alpha^*(\upsilon)]),\alpha^*(\upsilon)]+c_1\big[[\delta(\upsilon)[\delta(\upsilon),\alpha^*(\upsilon)],\alpha^*(\upsilon)],\alpha^*(\upsilon)\big]\delta(\upsilon)+2c_1[\delta(\upsilon)[\delta(\upsilon),\alpha^*(\upsilon)],\alpha^*(\upsilon)][\delta(\upsilon),\alpha^*(\upsilon)]$
$=[\delta(\upsilon),\alpha^*(\upsilon)][\alpha^*[\delta(\upsilon),\alpha^*(\upsilon)]\alpha^*(\delta(\upsilon)),\alpha^*(\upsilon)]+c_1[[\delta(\upsilon),\alpha^*(\upsilon)]^2+\delta(\upsilon)[[\delta(\upsilon),\alpha^*(\upsilon)],\alpha^*(\upsilon)],\alpha^*(\upsilon)]\delta(\upsilon)+2c_1[\delta(\upsilon),\alpha^*(\upsilon)]^3+2c_1\delta(\upsilon)\big[[\delta(\upsilon),\alpha^*(\upsilon)],\alpha^*(\upsilon)\big][\delta(\upsilon),\alpha^*(\upsilon)]$
$=[\delta(\upsilon),\alpha^*(\upsilon)]\alpha^*[[\delta(\upsilon),\alpha^*(\upsilon)],\upsilon]\alpha^*(\delta(\upsilon))+[\delta(\upsilon),\alpha^*(\upsilon)]\alpha^*[\delta(\upsilon),\alpha^*(\upsilon)]+\alpha^*[\delta(\upsilon),\upsilon]+2c_1[\delta(\upsilon),\alpha^*(\upsilon)]^3$
$=2c_1[\delta(\upsilon),\alpha^*(\upsilon)]^3$
... (8)
Then $2c_1[\delta(\upsilon),\alpha^*(\upsilon)]^2\,\mathcal{U}\,2c_1[\delta(\upsilon),\alpha^*(\upsilon)]^2=0$
Since $\mathcal{R}$ is a semiprime, then $2c_1[\delta(\upsilon),\alpha^*(\upsilon)]^2=0$, for all $\upsilon \in \mathcal{U}$ … (9)
Combining equation (5) and (9), we have

$[\delta(\upsilon), \alpha^*(\upsilon)]^2$=0, for all $\upsilon \in \mathcal{U}$
As the center of the semiprime ring contains no non-zero nilpotent elements, then we have $[\delta(\upsilon), \alpha^*(\upsilon)]$=0, $\forall \upsilon \in \mathcal{U}$.

Theorem 3.8: Let $\mathcal{R}$ be a prime ring and $\Omega: \mathcal{U}^n \to \mathcal{R}$ be a non-zero skew left n-derivation associated with an antiautomorphism α^*. If the trace δ of Ω is commuting on $\mathcal{U}$ and $[\delta(\upsilon), \alpha^*(\upsilon)] \in \mathcal{Z}(\mathcal{R})$ for all $\upsilon \in \mathcal{U}$, then $\mathcal{U}$ must be commutative.

Proof:
Suppose that $\mathcal{U}$ is anon commutative prime ring. From theorem (3.7), we have $[\delta(\upsilon),\alpha^*(\upsilon)]$=0 for all $\upsilon \in \mathcal{U}$. And from theorem (3.6) we have Ω=0 which is contradiction hence, $\mathcal{U}$ is commutative prime ring.

Conflicts of Interest: None.

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حول مشتقات الالتواء اليسارية من النمط n مع تركيبة مثالي لي

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الخلاصة:
في هذا البحث درست التمركزات و التبادل لمشتقات الالتواء اليسارية من النمط n و كذلك مشتقات الالتواء اليسارية من النمط n المرتبطة مع ضد التشاكلات التقابلية للحلقات الاولية وتم برهنة ابدالية مثالي لي تحت شروط معينة.

الكلمات المفتاحية: دالة التمركز، دالة التبادل، حلقة اولية، مشتقة الالتواء اليسرى من المعيار n.