DOI: http://dx.doi.org/10.21123/bsj.2019.16.2.0395

Estimating the Reliability Function of (2+1) Cascade Model

Ahmed H. Khaleel [1*] *Nada S. Karam* [2]

Received 20/5/2018, Accepted 20/12/2018, Published 2/6/2019

Abstract:

This paper discusses reliability R of the (2+1) Cascade model of inverse Weibull distribution. Reliability is to be found when strength-stress distributed is inverse Weibull random variables with unknown scale parameter and known shape parameter. Six estimation methods (Maximum likelihood, Moment, Least Square, Weighted Least Square, Regression and Percentile) are used to estimate reliability. There is a comparison between six different estimation methods by the simulation study by MATLAB 2016, using two statistical criteria Mean square error and Mean Absolute Percentage Error, where it is found that best estimator between the six estimators is Maximum likelihood estimation method.

Key words: Attenuation factor, Least Square, Scale parameter, Simulation, Strength-Stress .

Introduction:

In standby systems one component work's and the other components are standby. Cascade is a special kind of strength and stress reliability model. Special (2+1) of cascade model contain three components (comp1, comp2 and comp3), where comp1 and comp2 are activate while comp3 remains standby. Suppose that X_1, Y_1 indicate the strength and stress of comp1 and X_2, Y_2 indicate the strength and stress of comp2. Now, if comp1 or comp2 fail's the standby component (comp3) is activated. Let X_3, Y_3 indicate the strength and stress of comp3. Now, if comp1 fail's: $X_3 = mX_1$ and $Y_3 = kY_1$ and if comp2 fail's: $X_3 = mX_2$ and $Y_3 = kY_2$, where (m) is stress attenuation factor and (k) is strength attenuation factor and $(0 < m < 1$, $k > 1)$.

Ozler and Gurler (2012) (1) considered reliability of the k-out-of-n: F systems and its conditional shape with exchangeable components in stress-strength setup. Gogoi and Borah (2012) (2) studied two cases to obtain system reliability for the cascade model. Sandhya and Umamaheswari (2013) (3) worked on a multi-unit standby the model of strength and stress. Umamaheswari and Swathi (2013) (4) studied generalized exponential distribution for cascade model. Singh (2013) (5) considered n of cascade model by strength is exponential distribution and stress is normal distribution to find reliability. Reddy (2016)(6) presented estimation of $R = (X > Y)$ by considering the cascade stress-strength model. Devi, Umamaheswari and Swathi (2016) (7) studied general expression for reliability by n cascade system is derivative when stress and strength follow the lindley distribution and numerical values $R_{(1)}$, $R_{(2)}$, $R_{(3)}$ and R_3 have been computed for the some specific values of parameters. Mutkekar and Munoli (2016) (8) endeavored to the (1 + 1) cascade model by exponential distribution under joint action of stress strength. Karam and Husieen (2017) (9) studied n cascade model while strength-stress are Frechet distributed r.v to find the reliability, and used Marginal Reliabilities values of cascade model to find the reliabilities R_2, R_3 and R_4. Karam and Khaleel (2018) (10) discussed reliability (2+1) cascade model where strength and stress are WD, and used (Maximum likelihood, Moment, Least Square and Weighted Least Square) methods to estimate reliability of this model.

In this paper we refer to the derivation of mathematical formula of reliability in the (2+1) cascade model where strength and stress are IWD by using (Maximum likelihood, Moment, Least Square, Weighted Least Square, Regression and Percentile) estimation methods and compare the results of estimation of the different six methods by using MSE and MAPE that we get from the simulation study.

[1]Department of Statistics, College of Administration & Economics, University of Sumer, Dhi Qar, Iraq
[2] Department of Mathematics, College of Education, Al-Mustansiriyah University, Baghdad, Iraq
[*] Corresponding author: ahmedharoon885@gmail.com

The Mathematical Formula Reliability:

In this model strength-stress random variables of (comp1 and comp2 are basics and comp3 is stand by) to be $(X_i; i = 1,2,3)$ and $(Y_j; j = 1,2,3)$ respectively , where X_i and Y_j are independently identically distributed Inverse Weibull random variables with common known shape parameter σ and unknown scale parameters ρ_i; i=1,2,3 , θ_j; j=1,2,3 .

The CDF of IW(σ, ρ) is :

$$F(x) = e^{-\rho x^{-\sigma}} \qquad x > 0; \sigma, \rho > 0 \qquad .....(1)$$

The PDF of IW(σ, ρ) is :

$$f(x) = \sigma\rho x^{-(\sigma+1)} e^{-\rho x^{-\sigma}} \quad x > 0; \sigma, \rho > 0 \quad .....(2)$$

and

The CDF of IW(σ, θ) is :

$$G(y) = e^{-\theta y^{-\sigma}} \qquad y > 0; \sigma, \theta > 0 \qquad .....(3)$$

The PDF of IW(σ, θ) is :

$$g(y) = \sigma\theta y^{-(\sigma+1)} e^{-\theta y^{-\sigma}} \quad y > 0; \sigma, \theta > 0 \quad .....(4)$$

Reliability function R for (2+1) the cascade model:

$$R = p[X_1 \geq Y_1, X_2 \geq Y_2] + p[X_1 < Y_1, X_2 \geq Y_2, X_3 \geq Y_3] + p[X_1 \geq Y_1, X_2 < Y_2, X_3 \geq Y_3]$$

$$R = R_1 + R_2 + R_3 \qquad .....(5)$$

$$R_1 = p[X_1 \geq Y_1, X_2 \geq Y_2]$$
$$= \int_0^\infty p[X_1 \geq Y_1] g(y_1) dy_1 \int_0^\infty p[X_2 \geq Y_2] g(y_2) dy_2$$
$$= \int_0^\infty \left[\int_{y_1}^\infty f(x_1)\, dx_1\right] g(y_1) dy_1 \int_0^\infty \left[\int_{y_2}^\infty f(x_2)\, dx_2\right] g(y_2) dy_2$$
$$= \int_0^\infty \left[\overline{F}_{x_1}(y_1)\right] g(y_1) dy_1 \int_0^\infty \left[\overline{F}_{x_2}(y_2)\right] g(y_2) dy_2$$
$$= \int_0^\infty \left[1 - e^{-\rho_1 {y_1}^{-\sigma}}\right] \sigma\theta_1 {y_1}^{-(\sigma+1)} e^{-\theta_1 {y_1}^{-\sigma}} dy_1$$
$$.\int_0^\infty \left[1 - e^{-\rho_2 {y_2}^{-\sigma}}\right] \sigma\theta_2 {y_2}^{-(\sigma+1)} e^{-\theta_2 {y_2}^{-\sigma}} dy_2$$

Then it becomes as

$$R_1 = \left[\frac{\rho_1}{\rho_1+\theta_1}\right]\left[\frac{\rho_2}{\rho_2+\theta_2}\right] \qquad ....(6)$$

For R_2 we have that :

$$R_2 = p[X_1 < Y_1, X_2 \geq Y_2, X_3 \geq Y_3] = p[X_1 < Y_1, mX_1 \geq kY_1] p[X_2 \geq Y_2]$$

Where from above we have that:

$$p[X_2 \geq Y_2] = \int_0^\infty \left(\overline{F}_{x_2}(y_2)\right) g(y_2) dy_2 = \left[\frac{\rho_2}{\rho_2+\theta_2}\right]$$

and

$$p[X_1 < Y_1, mX_1 \geq kY_1] = \int_0^\infty \left(F_{x_1}(y_1)\right)\left(\overline{F}_{x_1}\left(\frac{k}{m}y_1\right)\right) g(y_1) dy_1$$
$$= \int_0^\infty \left[e^{-\rho_1 {y_1}^{-\sigma}}\right]\left[1 - e^{-\rho_1 (\frac{k}{m}y_1)^{-\sigma}}\right] \sigma_1\theta_1 {y_1}^{-(\sigma+1)} e^{-\theta_1 {y_1}^{-\sigma}} dy_1$$
$$= \left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\rho_1\theta_1}{\left(\rho_1+\left(\frac{k}{m}\right)^{-\sigma}\rho_1+\theta_1\right)(\rho_1+\theta_1)}\right]$$

So

$$R_2 = \left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\rho_1\theta_1}{\left(\rho_1+\left(\frac{k}{m}\right)^{-\sigma}\rho_1+\theta_1\right)(\rho_1+\theta_1)}\right]\left[\frac{\rho_2}{\rho_2+\theta_2}\right] \quad .....(7)$$

By using the same steps above we can find R_3 :

$$R_3 = p[X_1 \geq Y_1, X_2 < Y_2, X_3 \geq Y_3] =$$
$$= \left[\frac{\rho_1}{\rho_1+\theta_1}\right]\left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\rho_2\theta_2}{\left(\rho_2+\left(\frac{k}{m}\right)^{-\sigma}\rho_2+\theta_2\right)(\rho_2+\theta_2)}\right] \quad .....(8)$$

substitution (6),(7) and ,(8) in (5) the reliability function; R, finally will be as:

$$R = \left[\frac{\rho_1}{\rho_1+\theta_1}\right]\left[\frac{\rho_2}{\rho_2+\theta_2}\right] + \left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\rho_1\theta_1}{\left(\rho_1+\left(\frac{k}{m}\right)^{-\sigma}\rho_1+\theta_1\right)(\rho_1+\theta_1)}\right]\left[\frac{\rho_2}{\rho_2+\theta_2}\right]$$
$$+ \left[\frac{\rho_1}{\rho_1+\theta_1}\right]\left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\rho_2\theta_2}{\left(\rho_2+\left(\frac{k}{m}\right)^{-\sigma}\rho_2+\theta_2\right)(\rho_2+\theta_2)}\right] \qquad ....(9)$$

Methods of Estimating the Reliability Function

Maximum likelihood Estimation method (ML):

Let $x_1, x_2, \dots, x_n$ a random sample from IW(σ, ρ) distribution. The general form is:

$$L(\sigma, \rho; x_1, x_2, \dots x_n) = (\sigma\rho)^n \prod_{i=1}^n x_i^{-(\sigma+1)} e^{-\rho \sum_{i=1}^n x_i^{-\sigma}} \qquad ....(10)$$

The log - equation (10) :

$$LnL = nLn(\sigma) + nLn(\rho) - (\sigma+1)\sum_{i=1}^n ln(x_i) - \rho\sum_{i=1}^n x_i^{-\sigma} \qquad .....(11)$$

Taking partial derivative equation (11) :

$$\frac{\partial\, ln\, L(\sigma, \rho; x_1, x_2, \dots, x_n)}{\partial\rho} = \frac{n}{\rho} - \sum_{i=1}^n x_i^{-\sigma}$$

Then the maximum likelihood estimator for ρ is :

$$\hat{\rho}_{(ML)} = \frac{n}{\sum_{i=1}^n x_i^{-\sigma}} \qquad .....(12)$$

By using the same manner, we can obtain the maximum likelihood estimators for the unknown scale parameters ρ_1, ρ_2, ρ_3 of the strength random variables $X_1 \sim IW(\sigma, \rho_1), X_2 \sim IW(\sigma, \rho_2)$, $X_3 \sim IW(\sigma, \rho_3)$ with sample sizes n_1 , n_2 and n_3, respectively, and $\theta_1, \theta_2, \theta_3$ to the stress random variables $Y_1 \sim IW(\sigma, \theta_1), Y_2 \sim IW(\sigma, \theta_2), Y \sim IW(\sigma, \theta_3)$ with sample sizes m_1 , m_2 and m_3 then the formulas will be as

$$\hat{\rho}_{\xi(ML)} = \frac{n_\xi}{\sum_{i_\xi=1}^{n_\xi} x_{\xi_{i_\xi}}^{-\sigma}}, \ \xi = 1,2,3 \qquad .....(13)$$

and

$$\hat{\theta}_{\xi(ML)} = \frac{m_\xi}{\sum_{j_\xi=1}^{m_\xi} y_{\xi_{j_\xi}}^{-\sigma}}, \ \xi = 1,2,3 \qquad ......(14)$$

Substitution (13) and (14) in (9), the ML estimator for reliability R ; says $\hat{R}_{(ML)}$:

$$\hat{R}_{(ML)} = \left[\frac{\hat{\rho}_{1(ML)}}{\hat{\rho}_{1(ML)}+\hat{\theta}_{1(ML)}}\right]\left[\frac{\hat{\rho}_{2(ML)}}{\hat{\rho}_{2(ML)}+\hat{\theta}_{2(ML)}}\right]$$
$$+ \left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{1(ML)}\hat{\theta}_{1(ML)}}{\left(\hat{\rho}_{1(ML)}+\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{1(ML)}+\hat{\theta}_{1(ML)}\right)\left(\hat{\rho}_{1(ML)}+\hat{\theta}_{1(ML)}\right)}\right]\left[\frac{\hat{\rho}_{2(ML)}}{\hat{\rho}_{2(ML)}+\hat{\theta}_{2(ML)}}\right]$$
$$+ \left[\frac{\hat{\rho}_{1(ML)}}{\hat{\rho}_{1(ML)}+\hat{\theta}_{1(ML)}}\right]\left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{2(ML)}\hat{\theta}_{2(ML)}}{\left(\hat{\rho}_{2(ML)}+\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{2(ML)}+\hat{\theta}_{2(ML)}\right)\left(\hat{\rho}_{2(ML)}+\hat{\theta}_{2(ML)}\right)}\right] \qquad .....(15)$$

Moment Estimation method (Mo):

In this method of IW(σ, ρ) **:**

$$E(x) = \rho^{\frac{1}{\sigma}}\Gamma\left(1 - \frac{1}{\sigma}\right) \qquad .....(16)$$

According to the method of moment ,equating the samples means with the corresponding populations mean

$$\frac{\sum_{i=1}^n x_i}{n} = \hat{\rho}^{\frac{1}{\sigma}}\Gamma\left(1 - \frac{1}{\sigma}\right) \qquad .....(17)$$

Then $\hat{\rho}_{(Mo)}$ becomes as :

$$\hat{\rho}_{(Mo)} = \left[\frac{\bar{x}}{\Gamma\left(1-\frac{1}{\sigma}\right)}\right]^{\sigma} \quad \text{for } \sigma > 1 \qquad \text{..... (18)}$$

The Mo estimators of unknown scale parameters (ρ_1, ρ_2, ρ_3) and $(\theta_1, \theta_2, \theta_3)$ are :

$$\hat{\rho}_{\xi(Mo)} = \left[\frac{\bar{x}_\xi}{\Gamma\left(1-\frac{1}{\sigma}\right)}\right]^{\sigma}, \xi = 1,2,3 \qquad \text{.....(19)}$$

and

$$\hat{\theta}_{\xi(Mo)} = \left[\frac{\bar{y}_\xi}{\Gamma\left(1-\frac{1}{\sigma}\right)}\right]^{\sigma}, \xi = 1,2,3 \qquad \text{.....(20)}$$

Substitution (19) and (20) in (9), the Mo estimator for reliability R ; says $\hat{R}_{(Mo)}$:

$$\hat{R}_{(Mo)} = \left[\frac{\hat{\rho}_{1(Mo)}}{\hat{\rho}_{1(Mo)} + \hat{\theta}_{1(Mo)}}\right]\left[\frac{\hat{\rho}_{2(Mo)}}{\hat{\rho}_{2(Mo)} + \hat{\theta}_{2(Mo)}}\right] + \left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{1(Mo)}\hat{\theta}_{1(Mo)}}{\left(\hat{\rho}_{1(Mo)}+\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{1(Mo)}+\hat{\theta}_{1(Mo)}\right)\left(\hat{\rho}_{1(Mo)}+\hat{\theta}_{1(Mo)}\right)}\right]\left[\frac{\hat{\rho}_{2(Mo)}}{\hat{\rho}_{2(Mo)}+\hat{\theta}_{2(Mo)}}\right] + \left[\frac{\hat{\rho}_{1(Mo)}}{\hat{\rho}_{1(Mo)}+\hat{\theta}_{1(Mo)}}\right]\left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{2(Mo)}\hat{\theta}_{2(Mo)}}{\left(\hat{\rho}_{2(Mo)}+\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{2(Mo)}+\hat{\theta}_{2(Mo)}\right)\left(\hat{\rho}_{2(Mo)}+\hat{\theta}_{2(Mo)}\right)}\right] \qquad \text{.....(21)}$$

Least Square Estimation Method (LS):

In least square method the minimize function of $IW(\sigma, \rho)$:

$$S(\sigma,\rho) = \sum_{i=1}^{n}\left(\hat{F}(x_i) - F(x_i)\right)^2 = \sum_{i=1}^{n}\left(\hat{F}(x_i) - e^{-\rho x^{-\sigma}}\right)^2 \qquad \text{.....(22)}$$

Since CDF of inverse Weibull distribution does not have a linear formula giving to the parameters, so we get the linear formula the following :

$$F(x_i) = e^{-\rho x_i^{-\sigma}}$$

$$-ln\left(F(x_i)\right) = \rho x_i^{-\sigma} \qquad \text{......(23)}$$

Since $\hat{F}(x_i)$ is unknown, so we use (11) :

$$\hat{F}\left(x_{(i)}\right) = \frac{i}{n+1}$$

$$S(\sigma,\rho) = \sum_{i=1}^{n}\left(q_{(i)} - \rho x_{(i)}^{-\sigma}\right)^2 \qquad \text{......(24)}$$

Where $q_{(i)} = -ln\left(\hat{F}\left(x_{(i)}\right)\right) = -ln(P_i)$ and P_i is the plotting position

By derivative equation (24) with respect to parameter ρ (12) :

$$\frac{\partial S(\sigma,\rho)}{\rho} = \sum_{i=1}^{n} 2\left(q_{(i)} - \rho x_{(i)}^{-\sigma}\right) x_{(i)}^{-\sigma}$$

$$\sum_{i=1}^{n} q_{(i)} x_{(i)}^{-\sigma} - \rho \sum_{i=1}^{n} x_{(i)}^{-2\sigma} = 0$$

Then we get the least square estimator of ; say $\hat{\rho}_{(LS)}$:

$$\hat{\rho}_{(LS)} = \frac{\sum_{i=1}^{n} q_{(i)} x_{(i)}^{-\sigma}}{\sum_{i=1}^{n} x_{(i)}^{-2\sigma}} \qquad \text{.....(25)}$$

The LS estimators of unknown scale parameters (ρ_1, ρ_2, ρ_3) and $(\theta_1, \theta_2, \theta_3)$ are :

$$\hat{\rho}_{\xi(LS)} = \frac{\sum_{i_\xi=1}^{n_\xi} q_{\xi_{(i_\xi)}} x_{\xi_{(i_\xi)}}^{-\sigma}}{\sum_{i_\xi=1}^{n_\xi} x_{\xi_{(i_\xi)}}^{-2\sigma}} \qquad \text{......(26)}$$

and

$$\hat{\theta}_{\xi(LS)} = \frac{\sum_{j_\xi=1}^{m_\xi} q_{\xi_{(j_\xi)}} y_{\xi_{(j_\xi)}}^{-\sigma}}{\sum_{j_\xi=1}^{m_\xi} y_{\xi_{(j_\xi)}}^{-2\sigma}} \qquad \text{......(27)}$$

Where $\hat{G}\left(y_{(j)}\right) = \frac{j}{m+1}$ and

$$q_{(j)} = -ln\left(\hat{G}\left(y_{(j)}\right)\right) = -ln\left(P_j\right)$$

Substitution (26) and (27) in (9), the LS estimator for reliability R ; says $\hat{R}_{(LS)}$:

$$\hat{R}_{(LS)} = \left[\frac{\hat{\rho}_{1(LS)}}{\hat{\rho}_{1(LS)}+\hat{\theta}_{1(LS)}}\right]\left[\frac{\hat{\rho}_{2(LS)}}{\hat{\rho}_{2(LS)}+\hat{\theta}_{2(LS)}}\right] + \left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{1(LS)}\hat{\theta}_{1(LS)}}{\left(\hat{\rho}_{1(LS)}+\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{1(LS)}+\hat{\theta}_{1(LS)}\right)\left(\hat{\rho}_{1(LS)}+\hat{\theta}_{1(LS)}\right)}\right]\left[\frac{\hat{\rho}_{2(LS)}}{\hat{\rho}_{2(LS)}+\hat{\theta}_{2(LS)}}\right] + \left[\frac{\hat{\rho}_{1(LS)}}{\hat{\rho}_{1(LS)}+\hat{\theta}_{1(LS)}}\right]\left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{2(LS)}\hat{\theta}_{2(LS)}}{\left(\hat{\rho}_{2(LS)}+\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{2(LS)}+\hat{\theta}_{2(LS)}\right)\left(\hat{\rho}_{2(LS)}+\hat{\theta}_{2(LS)}\right)}\right] \qquad \text{.....(28)}$$

Weighted Least Square Estimation Method (WLS):

In this method we can use the minimizing equation to find estimator of unknown scale parameter of IWD [1] :

$$S = \sum_{i=1}^{n} w_i \left(\hat{F}(x_i) - F(x_i)\right)^2 \qquad \text{.....(29)}$$

Where $w_i = \frac{1}{Var[F(x_i)]} = \frac{(n+1)^2(n+2)}{i(n-i+1)}, i = 1,2,\dots,n$

As steps in equations (23) and (24) we get :

$$S = \sum_{i=1}^{n} w_i\left(q_{(i)} - \rho x_{(i)}^{-\sigma}\right)^2 \qquad \text{.....(30)}$$

By derivative equation (30) :

$$\frac{\partial S}{\partial \rho} = \sum_{i=1}^{n} 2w_i\left(q_{(i)} - \rho x_{(i)}^{-\sigma}\right)x_{(i)}^{-\sigma}$$

$$\sum_{i=1}^{n} w_i q_{(i)} x_{(i)}^{-\sigma} - \hat{\rho}\sum_{i=1}^{n} w_i x_{(i)}^{-2\sigma} = 0$$

Then we get the weighted least square estimator of ρ ; say $\hat{\rho}_{(WLS)}$:

$$\hat{\rho}_{(WLS)} = \frac{\sum_{i=1}^{n} w_i q_{(i)} x_{(i)}^{-\sigma}}{\sum_{i=1}^{n} w_i x_{(i)}^{-2\sigma}} \qquad \text{.....(31)}$$

The WLS estimators of unknown scale parameters (ρ_1, ρ_2, ρ_3) and $(\theta_1, \theta_2, \theta_3)$ are :

$$\hat{\rho}_{\xi(WLS)} = \frac{\sum_{i_\xi=1}^{n_\xi} w_{i_\xi} q_{\xi_{(i_\xi)}} x_{\xi_{(i_\xi)}}^{-\sigma}}{\sum_{i_\xi=1}^{n_\xi} w_{i_\xi} x_{\xi_{(i_\xi)}}^{-2\sigma}}, \xi = 1,2,3 \qquad \text{.....(32)}$$

and

$$\hat{\theta}_{\xi(WLS)} = \frac{\sum_{j_\xi=1}^{m_\xi} w_{j_\xi} q_{\xi_{(j_\xi)}} y_{\xi_{(j_\xi)}}^{-\sigma}}{\sum_{j_\xi=1}^{m_\xi} w_{j_\xi} y_{\xi_{(j_\xi)}}^{-2\sigma}}, \xi = 1,2,3 \qquad \text{.....(33)}$$

Where $w_j = \frac{1}{Var\left[G\left(y_{(j)}\right)\right]} = \frac{(m+1)^2(m+2)}{j(m-j+1)}, j = 1,2,\dots,m$

Substitution (32) and (33) in (9), the WLS estimator for reliability R ; says $\hat{R}_{(WLS)}$:

$$\hat{R}_{(WLS)} = \left[\frac{\hat{\rho}_{1(WLS)}}{\hat{\rho}_{1(WLS)}+\hat{\theta}_{1(WLS)}}\right]\left[\frac{\hat{\rho}_{2(WLS)}}{\hat{\rho}_{2(WLS)}+\hat{\theta}_{2(WLS)}}\right] + \left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{1(WLS)}\hat{\theta}_{1(WLS)}}{\left(\hat{\rho}_{1(WLS)}+\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{1(WLS)}+\hat{\theta}_{1(WLS)}\right)\left(\hat{\rho}_{1(WLS)}+\hat{\theta}_{1(WLS)}\right)}\right]\left[\frac{\hat{\rho}_{2(WLS)}}{\hat{\rho}_{2(WLS)}+\hat{\theta}_{2(WLS)}}\right] + \left[\frac{\hat{\rho}_{1(WLS)}}{\hat{\rho}_{1(WLS)}+\hat{\theta}_{1(WLS)}}\right]\left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{2(WLS)}\hat{\theta}_{2(WLS)}}{\left(\hat{\rho}_{2(WLS)}+\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{2(WLS)}+\hat{\theta}_{2(WLS)}\right)\left(\hat{\rho}_{2(WLS)}+\hat{\theta}_{2(WLS)}\right)}\right] \qquad \text{.....(34)}$$

Regression Estimation Method (Rg):
The method of regression can be obtained by regression equation:

$$z_i = a + bu_i + e_i \qquad ......(35)$$

By taking the log - equation (1) :

$$Ln\Big(F\big(x_{(i)}\big)\Big) = -\rho x_{(i)}^{-\sigma}$$

Replacing $F\big(x_{(i)}\big)$ by P_i get as :

$$Ln(P_i) = -\rho x_{(i)}^{-\sigma} \qquad .....(36)$$

Equating (36) with (35) :

$z_i = ln(P_i)\,, a = 0, b = \rho, u_i = -x_{(i)}^{-\sigma}$ where $; i = 1,2,\dots,n$ ....(37)

Where b can be estimated by minimizing summation of the squared error with respect to b :

$$\hat{b} = \frac{n\sum_{i=1}^{n} z_i u_i - \sum_{i=1}^{n} z_i \sum_{i=1}^{n} u_i}{n\sum_{i=1}^{n}(u_i)^2 - \left(\sum_{i=1}^{n} u_i\right)^2} \qquad .....(38)$$

Then estimator Regression of ρ get as by equations replacement (37) in (38) :

$$\hat{\rho}_{(Rg)} = \frac{-n\sum_{i=1}^{n} ln(P_i) x_{(i)}^{-\sigma} + \sum_{i=1}^{n} ln(P_i) \sum_{i=1}^{n} x_{(i)}^{-\sigma}}{n\sum_{i=1}^{n}\left(x_{(i)}^{-\sigma}\right)^2 - \left(\sum_{i=1}^{n} x_{(i)}^{-\sigma}\right)^2} \qquad .....(39)$$

The Rg estimators of unknown scale parameters $(\rho_1, \rho_2\,, \rho_3)$ and $(\theta_1, \theta_2\,, \theta_3)$ are :

$$\hat{\rho}_{\xi(Rg)} = \frac{-n_\xi \sum_{i_\xi=1}^{n_\xi} \ln\left(P_{i_\xi}\right) x_{\xi_{(i_\xi)}}^{-\sigma} + \sum_{i_\xi=1}^{n_\xi} \ln\left(P_{i_\xi}\right) \sum_{i_\xi=1}^{n_\xi} x_{\xi_{(i_\xi)}}^{-\sigma}}{n_\xi \sum_{i_\xi=1}^{n_\xi}\left(x_{\xi_{(i_\xi)}}^{-\sigma}\right)^2 - \left(\sum_{i_\xi=1}^{n_\xi} x_{\xi_{(i_\xi)}}^{-\sigma}\right)^2} \; ; \xi = 1,2,3 \qquad .....(40)$$

and

$$\hat{\theta}_{\xi(Rg)} = \frac{-m_\xi \sum_{j_\xi=1}^{m_\xi} \ln\left(P_{j_\xi}\right) y_{\xi_{(j_\xi)}}^{-\sigma} + \sum_{j_\xi=1}^{m_\xi} \ln\left(P_{j_\xi}\right) \sum_{j_\xi=1}^{m_\xi} y_{\xi_{(j_\xi)}}^{-\sigma}}{m_\xi \sum_{j_\xi=1}^{m_\xi}\left(y_{\xi_{(j_\xi)}}^{-\sigma}\right)^2 - \left(\sum_{j_\xi=1}^{m_\xi} y_{\xi_{(j_\xi)}}^{-\sigma}\right)^2} \; ; \xi = 1,2,3 \qquad .....(41)$$

As in equation (38) where $z_j = ln\big(P_j\big)\,, a = 0, b = \theta, u_j = -y_{(j)}^{-\sigma}$ where $; j = 1,2,\dots,m$

Substitution (40) and (41) in (9), the Rg estimator for reliability R ; says $\hat{R}_{(Rg)}$:

$$\hat{R}_{(Rg)} = \left[\frac{\hat{\rho}_{1(Rg)}}{\hat{\rho}_{1(Rg)} + \hat{\theta}_{1(Rg)}}\right]\left[\frac{\hat{\rho}_{2(Rg)}}{\hat{\rho}_{2(Rg)} + \hat{\theta}_{2(Rg)}}\right] + \left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{1(Rg)}\hat{\theta}_{1(Rg)}}{\left(\hat{\rho}_{1(Rg)} + \left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{1(Rg)} + \hat{\theta}_{1(Rg)}\right)\left(\hat{\rho}_{1(Rg)} + \hat{\theta}_{1(Rg)}\right)}\right]\left[\frac{\hat{\rho}_{2(Rg)}}{\hat{\rho}_{2(Rg)} + \hat{\theta}_{2(Rg)}}\right] + \left[\frac{\hat{\rho}_{1(Rg)}}{\hat{\rho}_{1(Rg)} + \hat{\theta}_{1(Rg)}}\right]\left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{2(Rg)}\hat{\theta}_{2(Rg)}}{\left(\hat{\rho}_{2(Rg)} + \left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{2(Rg)} + \hat{\theta}_{2(Rg)}\right)\left(\hat{\rho}_{2(Rg)} + \hat{\theta}_{2(Rg)}\right)}\right] \qquad .....(42)$$

Percentile Estimation Method (Pr):

In case of inverse Weibull distribution it is possible to use this method to obtain the estimator unknown scale parameter ρ, because of the structure of the distribution function. Since F(x) is defined in (1) :

$$F(x) = e^{-\rho x^{-\sigma}}$$

$$ln\big(F(x)\big) = -\rho x^{-\sigma}$$

$$x = \left(\frac{-ln(F(x))}{\rho}\right)^{\frac{-1}{\sigma}} \qquad .....(43)$$

since Pi ; $i = 1,2,\dots,n$ denotes the some estimate of $F\big(x_{(i)}; \sigma, \rho\big)$,then we get :

$$x_{(i)} = \left(\frac{-ln(P_i)}{\rho}\right)^{\frac{-1}{\sigma}}$$

Then the estimate of ρ can be obtained by minimizing :

$$\sum_{i=1}^{n}\left[ln\big(x_{(i)}\big) - \left(\frac{-ln(P_i)}{\rho}\right)^{\frac{-1}{\sigma}}\right]^2 = S(\rho) \qquad .....(44)$$

Deriving equation (44) with respect to ρ and equating to zero:

$$\sum_{i=1}^{n} 2\left[ln\big(x_{(i)}\big) - \rho^{\frac{1}{\sigma}}\big(-ln(P_i)\big)^{\frac{-1}{\sigma}}\right]\frac{-1}{\sigma}\rho^{\frac{1}{\sigma}-1}\left(\big(-ln(P_i)\big)^{\frac{-1}{\sigma}}\right) = 0$$

The percentile estimator of ρ ; say $\hat{\rho}_{(Pr)}$ becomes :

$$\hat{\rho}_{(Pr)} = \left[\frac{\sum_{i=1}^{n} ln\big(x_{(i)}\big)\,\big(-ln(P_i)\big)^{\frac{-1}{\sigma}}}{\sum_{i=1}^{n}\big(-ln(P_i)\big)^{\frac{-2}{\sigma}}}\right]^{\sigma} \qquad ......(45)$$

The Pr estimators of unknown scale parameters $(\rho_1, \rho_2\,, \rho_3)$ and $(\theta_1, \theta_2\,, \theta_3)$ are :

$$\hat{\rho}_{\xi(Pr)} = \left[\frac{\sum_{i_\xi=1}^{n_\xi} \ln\left(x_{\xi_{(i_\xi)}}\right)\left(-\ln(P_{i_\xi})\right)^{\frac{-1}{\sigma}}}{\sum_{i_\xi=1}^{n_\xi}\left(-\ln(P_{i_\xi})\right)^{\frac{-2}{\sigma}}}\right]^{\sigma} \; ; \xi = 1,2,3 \qquad .....(46)$$

and

$$\hat{\theta}_{\xi(Pr)} = \left[\frac{\sum_{j_\xi=1}^{m_\xi} \ln\left(y_{\xi_{(i_\xi)}}\right)\left(-\ln(P_{j_\xi})\right)^{\frac{-1}{\sigma}}}{\sum_{j_\xi=1}^{m_\xi}\left(-\ln(P_{j_\xi})\right)^{\frac{-2}{\sigma}}}\right]^{\sigma} \; ; \xi = 1,2,3 \qquad ....(47)$$

Substitution (46) and (47) in (9), the Pr estimator for reliability R ; says $\hat{R}_{(Pr)}$:

$$\hat{R}_{(Pr)} = \left[\frac{\hat{\rho}_{1(Pr)}}{\hat{\rho}_{1(Pr)} + \hat{\theta}_{1(Pr)}}\right]\left[\frac{\hat{\rho}_{2(Pr))}}{\hat{\rho}_{2(Pr)} + \hat{\theta}_{2(Pr)}}\right] + \left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{1(Pr)}\hat{\theta}_{1(Pr)}}{\left(\hat{\rho}_{1(Pr)} + \left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{1(Pr)} + \hat{\theta}_{1(Pr)}\right)\left(\hat{\rho}_{1(Pr)} + \hat{\theta}_{1(Pr)}\right)}\right]\left[\frac{\hat{\rho}_{2(Pr)}}{\hat{\rho}_{2(Pr)} + \hat{\theta}_{2(Pr)}}\right] + \left[\frac{\hat{\rho}_{1(Pr))}}{\hat{\rho}_{1(Pr)} + \hat{\theta}_{1(Pr)}}\right]\left[\frac{\left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{2(Pr)}\hat{\theta}_{2(Pr)}}{\left(\hat{\rho}_{2(Pr)} + \left(\frac{k}{m}\right)^{-\sigma}\hat{\rho}_{2(Pr)} + \hat{\theta}_{2(Pr)}\right)\left(\hat{\rho}_{2(Pr)} + \hat{\theta}_{2(Pr)}\right)}\right] \qquad .....(48)$$

Estimators Comparison:

Simulation study is introduced to estimators conduct of reliability by six methods of estimation and comparing the results by using Mean Square Error and Mean Absolute Percentage Error.

Generating Random Variables:

Assume that U be a random variable with the Uniform distribution in (0,1) ,the data for IWD can be generated by the adoption of inverse transformation for CDF where if :

$$U = F(X) \rightarrow X = F^{-1}(U) \;\text{ then } X = \left(-\frac{1}{\rho}\ln(U)\right)^{\frac{-1}{\sigma}}$$

$X_{i_\xi} = \left(-\frac{1}{\rho_{i_\xi}} ln\left(U_{i_\xi}\right)\right)^{\frac{-1}{\sigma}}; i = 1,2, \dots, n_\xi; \xi = 1,2,3$

and $Y_{j_\xi} = \left(-\frac{1}{\theta_{j_\xi}} ln(U_j)\right)^{\frac{-1}{\sigma}};$

$j = 1,2, \dots, m_\xi; \xi = 1,2,3$

Simulation Algorithm:

Simulation algorithm is written by using MATLAB program to estimate (R) and can be described out of the following steps**:**

Step1: The random samples $x_{11}, x_{12}, \dots, x_{1n_1}$, $x_{21}, x_{22}, \dots, x_{2n_2}, y_{11}, y_{12}, \dots, y_{1m_1}, y_{21}, y_{22}, \dots, y_{2m_2}$ of sizes $(n_1, n_2, m_1, m_2) =$ (15,15,15,15),(50,50,50,50),(100,100,100,100), (50,15,15,100),(15,100,50,100) and (100,50,15,50) are generated from IW.

Step2: Real parameters values are selected for 6 experiments: $(\sigma_1, \sigma_2, \rho_1, \rho_2, \theta_1, \theta_2)$ in the **Table** (1)

Table (1): The values for 6 experiments

Experiment	K	m	σ	ρ_1	ρ_2	θ_1	θ_2	
1	1.8	0.3	1.5	1.5	1.5	1.5	1.5	0.2665
2	1.8	0.3	2	1.5	1.5	1.5	1.5	0.2568
3	1.5	0.5	2	1.5	1.5	1.5	1.5	0.2763
4	1.5	0.5	2	3	3	2	2	0.3900
5	1.25	0.8	2	3	3	2	2	0.4547
6	1.25	0.8	2	2	2	3	3	0.2276

Step3: Estimating the parameters $\rho_1, \rho_2, \theta_1, \theta_2$ by (Maximum likelihood , Moment, Least Square, Weighted Least Square, Regression and Percentile) methods as in : (13), (14), (19), (20), (26), (27), (32), (33),(40),(41),(46) and (47) respectively.

Step4: Estimation of Reliability as in: (15), (21), (28), (34),(42) and (48).

Step5: Mean is calculate by: $Mean = \frac{\sum_{i=1}^{L} \hat{R}_i}{L}$

Step6 Make comparison between the six different methods of estimation by using two statistical criteria:

1.Mean Square Error, where $MSE(\hat{R}) = \frac{1}{L}\sum_{i=1}^{L}(\hat{R}_i - R)^2$

2. Mean Absolute Percentage Error , where $MAPE(\hat{R}) = \frac{1}{L}\sum_{i=1}^{L}\left|\frac{\hat{R}_i - R}{R}\right|$

Where L:Represents number of replications for any experiment.

Simulation Models

After applying the former steps of (R) for sample size (n_1,n_2,m_1,m_2) =a,b,c,d,e and f where a=(15,15,15,15),b=(50,50,50,50),c=(100,100,100,100),d=(50,15,15,100),e=(15,100,50,100) and f=(100,50,15,50) respectively **Table** values (1) in **Tables** (2), (3), (4), (5), (6), (7)

Table 2.The (Mean, MSE and MAPE) values for experiment (1)

Simple size	Criterion	ML	Mo	LS	WLS	Rg	Pr
a	Mean	0.2658	0.2650	0.2656	0.2655	0.2517	0.2523
	MSE	0.0042	0.0213	0.0050	0.0060	0.0070	0.0064
	MAPE	0.1944	0.4385	0.2116	0.2329	0.2555	0.2437
b	Mean	0.2661	0.2644	0.2658	0.2655	0.2515	0.2521
	MSE	0.0013	0.0122	0.0016	0.0027	0.0026	0.0022
	MAPE	0.1092	0.3277	0.1220	0.1573	0.1554	0.1427
c	Mean	0.2664	0.2657	0.2662	0.2656	0.2517	0.2522
	MSE	0.0007	0.0091	0.0008	0.0019	0.0015	0.0012
	MAPE	0.0772	0.2785	0.0871	0.1297	0.1167	0.1058
d	Mean	0.2680	0.2576	0.2644	0.2652	0.2493	0.2539
	MSE	0.0027	0.0161	0.0031	0.0042	0.0046	0.0043
	MAPE	0.1556	0.3786	0.1680	0.1944	0.2073	0.1978
e	Mean	0.2683	0.2487	0.2590	0.2616	0.2437	0.2549
	MSE	0.0017	0.0129	0.0021	0.0031	0.0034	0.0027
	MAPE	0.1249	0.3403	0.1390	0.1683	0.1776	0.1575
f	Mean	0.2639	0.2862	0.2770	0.2730	0.2651	0.2480
	MSE	0.0019	0.0153	0.0025	0.0034	0.0033	0.0032
	MAPE	0.1323	0.3694	0.1486	0.1747	0.1718	0.1714

Table 3. The (Mean, MSE and MAPE) values for experiment (2)

Simple size	Criterion	ML	Mo	LS	WLS	Rg	Pr
a	Mean	0.2563	0.2557	0.2562	0.2561	0.2497	0.2501
	MSE	0.0043	0.0161	0.0049	0.0058	0.0068	0.0065
	MAPE	0.2038	0.3927	0.2197	0.2389	0.2598	0.2529
b	Mean	0.2565	0.2551	0.2570	0.2573	0.2511	0.2492
	MSE	0.0013	0.0074	0.0016	0.0027	0.0024	0.0020
	MAPE	0.1114	0.2626	0.1248	0.1627	0.1548	0.1416
c	Mean	0.2567	0.2567	0.2567	0.2568	0.2504	0.2502
	MSE	0.0006	0.0048	0.0008	0.0018	0.0013	0.0011
	MAPE	0.0774	0.2078	0.0890	0.1322	0.1141	0.1009
d	Mean	0.2577	0.2557	0.2540	0.2554	0.2467	0.2526
	MSE	0.0026	0.0112	0.0030	0.0041	0.0044	0.0040
	MAPE	0.1568	0.3228	0.1705	0.1971	0.2075	0.1973
e	Mean	0.2595	0.2484	0.2494	0.2512	0.2407	0.2550
	MSE	0.0017	0.0082	0.0020	0.0029	0.0030	0.0025
	MAPE	0.1268	0.2788	0.1411	0.1695	0.1745	0.1578
f	Mean	0.2524	0.2668	0.2651	0.2616	0.2611	0.2448
	MSE	0.0019	0.0097	0.0024	0.0033	0.0034	0.0030
	MAPE	0.1352	0.3023	0.1504	0.1789	0.1795	0.1718

Table 4. The (Mean, MSE and MAPE) values for experiment (3)

Simple size	Criterion	ML	Mo	LS	WLS	Rg	Pr
a	Mean	0.2754	0.2742	0.2751	0.2749	0.2546	0.2553
	MSE	0.0044	0.0168	0.0052	0.0062	0.0073	0.0067
	MAPE	0.1907	0.3757	0.2076	0.2285	0.2525	0.2410
b	Mean	0.2759	0.2757	0.2752	0.2741	0.2537	0.2556
	MSE	0.0014	0.0085	0.0017	0.0027	0.0028	0.0025
	MAPE	0.1062	0.2592	0.1176	0.1512	0.1553	0.1476
c	Mean	0.2762	0.2754	0.2761	0.2758	0.2550	0.2549
	MSE	0.0007	0.0051	0.0009	0.0019	0.0017	0.0015
	MAPE	0.0753	0.2002	0.0847	0.1261	0.1219	0.1133
d	Mean	0.2764	0.2708	0.2729	0.2741	0.2514	0.2560
	MSE	0.0027	0.0118	0.0032	0.0043	0.0049	0.0044
	MAPE	0.1515	0.3087	0.1640	0.1893	0.2068	0.1950
e	Mean	0.2799	0.2665	0.2691	0.2705	0.2461	0.2604
	MSE	0.0018	0.0087	0.0022	0.0032	0.0038	0.0028
	MAPE	0.1228	0.2671	0.1365	0.1638	0.1824	0.1561
f	Mean	0.2737	0.2880	0.2872	0.2838	0.2687	0.2507
	MSE	0.0020	0.0103	0.0026	0.0036	0.0034	0.0035
	MAPE	0.1299	0.2896	0.1463	0.1744	0.1707	0.1745

Table 5. The (Mean, MSE and MAPE) values for experiment (4)

Simple size	Criterion	ML	Mo	LS	WLS	Rg	Pr
a	Mean	0.3852	0.3736	0.3847	0.3839	0.3594	0.3592
	MSE	0.0057	0.0213	0.0066	0.0078	0.0098	0.0093
	MAPE	0.1548	0.3016	0.1673	0.1832	0.2069	0.2011
b	Mean	0.3891	0.3820	0.3888	0.3878	0.3637	0.3634
	MSE	0.0018	0.0101	0.0022	0.0037	0.0039	0.0033
	MAPE	0.0858	0.2030	0.0967	0.1253	0.1292	0.1191
c	Mean	0.3896	0.3860	0.3894	0.3883	0.3641	0.3646
	MSE	0.0008	0.0062	0.0011	0.0024	0.0023	0.0020
	MAPE	0.0593	0.1575	0.0665	0.0998	0.0987	0.0923
d	Mean	0.3867	0.3764	0.3817	0.3816	0.3546	0.3628
	MSE	0.0036	0.0155	0.0042	0.0055	0.0069	0.0062
	MAPE	0.1223	0.2541	0.1328	0.1519	0.1722	0.1628
e	Mean	0.3914	0.3697	0.3800	0.3818	0.3528	0.3668
	MSE	0.0022	0.0110	0.0029	0.0041	0.0053	0.0040
	MAPE	0.0976	0.2143	0.1096	0.1309	0.1506	0.1308
f	Mean	0.3840	0.3935	0.3986	0.3939	0.3760	0.3567
	MSE	0.0025	0.0127	0.0031	0.0044	0.0045	0.0049
	MAPE	0.1029	0.2287	0.1137	0.1369	0.1372	0.1450

Table 6. The (Mean, MSE and MAPE) values for experiment (5)

Simple size	Criterion	ML	Mo	LS	WLS	Rg	Pr
a	Mean	0.4463	0.4270	0.4462	0.4454	0.4006	0.3982
	MSE	0.0061	0.0236	0.0071	0.0085	0.0121	0.0115
	MAPE	0.1382	0.2712	0.1497	0.1636	0.1962	0.1908
b	Mean	0.4525	0.4448	0.4515	0.4495	0.4043	0.4061
	MSE	0.0019	0.0113	0.0024	0.0039	0.0058	0.0051
	MAPE	0.0767	0.1831	0.0851	0.1095	0.1368	0.1298
c	Mean	0.4535	0.4473	0.4532	0.4516	0.4063	0.4065
	MSE	0.0009	0.0069	0.0011	0.0025	0.0040	0.0037
	MAPE	0.0534	0.1407	0.0591	0.0877	0.1164	0.1134
d	Mean	0.4506	0.4323	0.4459	0.4458	0.3984	0.4035
	MSE	0.0037	0.0169	0.0045	0.0059	0.0090	0.0078
	MAPE	0.1074	0.2248	0.1179	0.1351	0.1698	0.1584
e	Mean	0.4545	0.4323	0.4419	0.4431	0.3930	0.4095
	MSE	0.0025	0.0127	0.0034	0.0048	0.0081	0.0055
	MAPE	0.0878	0.1966	0.1020	0.1220	0.1622	0.1331
f	Mean	0.4484	0.4536	0.4632	0.4577	0.4188	0.3989
	MSE	0.0026	0.0138	0.0031	0.0044	0.0054	0.0069
	MAPE	0.0899	0.2031	0.0983	0.1172	0.1295	0.1491

Table 7. The (Mean, MSE and MAPE) values for experiment (6)

Simple size	Criterion	ML	Mo	LS	WLS	Rg	Pr
a	Mean	0.2296	0.2310	0.2296	0.2299	0.2034	0.2030
	MSE	0.0039	0.0152	0.0045	0.0054	0.0057	0.0055
	MAPE	0.2187	0.4285	0.2346	0.2565	0.2732	0.2684
b	Mean	0.2278	0.2304	0.2279	0.2283	0.2007	0.2009
	MSE	0.0011	0.0069	0.0014	0.0024	0.0025	0.0021
	MAPE	0.1169	0.2843	0.1318	0.1732	0.1796	0.1685
c	Mean	0.2281	0.2289	0.2282	0.2284	0.2008	0.2004
	MSE	0.0006	0.0045	0.0007	0.0017	0.0016	0.0015
	MAPE	0.0838	0.2249	0.0953	0.1425	0.1478	0.1420
d	Mean	0.2296	0.2260	0.2263	0.2278	0.1990	0.2026
	MSE	0.0024	0.0101	0.0027	0.0036	0.0039	0.0035
	MAPE	0.1712	0.3458	0.1826	0.2093	0.2261	0.2153
e	Mean	0.2311	0.2206	0.2220	0.2244	0.1940	0.2047
	MSE	0.0015	0.0074	0.0018	0.0027	0.0032	0.0024
	MAPE	0.1380	0.2984	0.1525	0.1860	0.2073	0.1780
f	Mean	0.2242	0.2409	0.2366	0.2335	0.2110	0.1961
	MSE	0.0016	0.0086	0.0022	0.0031	0.0028	0.0030
	MAPE	0.1423	0.3184	0.1626	0.1935	0.1883	0.1972

Conclusions:

These conclusions are according to the results of simulation:

A. We conclude from table (1) the following:

1- Reliability of model decreases with the increasing values of σ .

2- Reliability of model increases with the increasing values of ρ_1 and ρ_2 .

3- Reliability of model decreases with the increasing values of θ_1 and θ_2 .

4- Reliability of model increases with decreasing values of $(\frac{k}{m})$.

B. The best estimation method for Reliability of MSE and MAPE:

Values of parameters and samples sizes	Best method
For $(n_1, n_2, m_1, m_2) =$ $(15,15,15,15), (50,50,50,50), (100,100,100,$ $(50,15,15,100), (15,100,50,100)$ and $(100,5$ when $(\alpha, \beta_1, \beta_2, \mu_1, \mu_2) =$ $(1.5,1.5,1.5,1.5,1.5), (2,1.5,1.5,1.5,1.5)$ for $k = 1.8, m = 0.3$ $(\alpha, \beta_1, \beta_2, \mu_1, \mu_2) =$ $(2,1.5,1.5,1.5,1.5), (, 2,3,3,2,2)$ for $k = 1.5, m = 0.5$ $(\alpha, \beta_1, \beta_2, \mu_1, \mu_2) = (2,3,3,2,2), (2,2,2,3,3)$ for $k = 1.25, m = 0.8$	ML

Conflicts of Interest: None.

References:

1. Ozler B S, Gurler S. On the Reliability of k-OUT-OF-n: F system with Exchangeable components in the stress-strength setup TWMS Journal of Applied and Engineering Mathematics. 2012; 2 (1): 109-115.
2. Gogoi J, Borah M. Inference on Reliability for Cascade Model. . Journal of Informatics and Mathematical Science s. 2012; 4 (1): 77-83.
3. Sandhya K, Umamaheswari T S. Reliability of a multi component stress strength model with standby system using mixture of two exponential distribution. Journal of Reliability and Statistical Studies. 2013; 6 (2) : 105-113 .
4. Umamaheswari T S, Swathi N . Cascade reliability for generalized exponential distribution. International IJCER. 2013; 3 (1): 132-136
5. Singh D. Measuring Reliability of n-cascade system under random stress attention. IJSER . 2013; 4 (12):1400-1404.
6. Reddy D . Cascade and system Reliability for exponential distributions. DJ Journal of Engineering and Applied Mathematics. 2016 ; 2 (2) :1-8 .
7. Devi M T, Umamaheswari T S, Swathi N. Cascade system Reliability with stress and strength follow Lindley distribution. IJERT. 2016 September; 5 (9):708-714.
8. Mutkekar R R, Munoli S B. Estimation of Reliability for stress-strength cascade model. OJS. 2016; 6: 873-881
9. Karam N S, Husieen H. Frechet Cascade Stress-Strength System Reliability. Mathematical Theory and Modeling. 2017; 7 (11):12-19.
10. Karam N S, Khaleel A H. Weibull reliability estimation for (2+1) cascade model. IJAMS. 2018; 6 (1):19-23.
11. Alqallaf F, Ghitany M E, Agostinelli C. Weighted Exponential Distribution Different Methods of Estimations. AMIS. 2015; 9 (3):1167-1173.
12. Guney Y, Arslan O .Robust parameter estimation for the Marshall-Olkin extended BURR XII distribution. Communications de la Faculté des Sciences de l University Ankara Séries A1. 2017; 66 (2) :141-161.

تقدير دالة المعولية لنموذج كاسكاد (1+2)

أحمد هارون خليل[1] **ندى صباح كرم** [2]

[1] قسم الاحصاء، كلية الادارة والاقتصاد، جامعة سومر، ذي قار، العراق
[2] قسم الرياضيات، كلية التربية، الجامعة المستنصرية، بغداد، العراق

الخلاصة :

يناقش هذا البحث معولية نموذج كسكاد (1+2) لتوزيع معكوس ويبل . تم ايجاد معولية النموذج عندما تكون المتانة والاجهاد لهما توزيع معكوس ويبل حيث معلمة القياس غير معلومة ومعلمة الشكل معلومة . ست طرق تقدير (الامكان الاعظم ، العزوم ، المربعات الصغرة المربعات الصغرة المرجحة ، الانحدار و الرتب المؤية) استخدمت لإيجاد تقديرات المعولية . المقارنة بين الطرق السته المختلفة بواسطة دراسة المحاكاة بواسطة ماتلاب 2016، باستخدام معيارين احصائيين هما متوسط مربع الخطأ و معيار متوسط الخطأ النسبي المطلق ،حيث وجد ان افضل تقدير بين المقدرات الستة هو مقدر الامكان الاعظم .

الكلمات المفتاحية: عامل التوهين، المربعات الصغرة، معلمة القياس، المحاكاة، متانة - اجهاد .