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Tadpole Domination in Graphs

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Abstract:

A new type of the connected domination parameters called tadpole domination number of a graph is introduced. Tadpole domination number for some standard graphs is determined, and some bounds for this number are obtained. Additionally, a new graph, finite, simple, undirected and connected, is introduced named weaver graph. Tadpole domination is calculated for this graph with other families of graphs.

Keywords: Dominating set, Tadpole graph, Tadpole domination number.
Mathematical subject classification: 05C69.

Introduction:

Let $G = (V, E)$ be finite, simple, connected and undirected graph where V denotes its vertices set and E its edges set. A degree of a vertex v of any graph G is defined as the number of edges incident on v. It is denoted by $deg(v)$. The minimum and maximum degrees of vertices in G are denoted by $\delta(G)$ and $\Delta(G)$, respectively. A graph is called connected if there is a path from any vertex to any other vertex in the graph. A graph which is not a connected one is called a disconnected graph, the disconnected graph can contain at least two of the connected graphs, each of these is called component. A cut-vertex of a graph G is a vertex where removing it from the graph would increase the number of components.

The girth of a graph is the length of the shortest cycle contained in a graph. A Hamiltonian graph is a graph possessing a cycle that goes through all the vertices of G. A set $D \subseteq V$ of vertices in a graph $G = (V, E)$ is called a dominating set if every vertex $v \in V$ is either an element of D or is adjacent to an element of D. The domination number of G $\gamma(G)$ is the minimum cardinality taken over all dominating sets in G (1,2). Many authors have introduced different types of domination parameters through adding conditions on the dominating set. Sampathkumar and Walikar (3) defined a connected dominating set D; a connected dominating set exists if the induced sub graph D is connected. The topic of domination parameter is associated with several studied types of dominating sets especially a connected domination number due to its importance.

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In network you need the communication between its members. Such as the internet or electricity or any social network in daily life, facilitating the business task and reducing the cost. Refer to (4,5,6,7,8,9,10,11,12) for more types of domination in graphs.

In this work, a new type of connected graph domination is introduced. If the vertices of a dominating set in a graph form a sub graph as a tadpole graph then these vertices represent a tadpole dominating set in this graph. Some results for this new domination are determined. Also, a new model of graph called weaver is initiated and its tadpole domination, is calculated.

Main results

Definition 2.1.(13) The graph obtained by joining cycle C_m to a path P_n with a bridge called Tadpole graph denoted by $T_{m,n}$. (Fig.1).

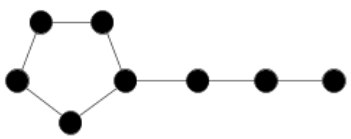


Figure 1. $T_{5,3}$ tadpole graph

Definition 2.2. A subset D of $V(G)$ is said to be a tadpole dominating set of G if D is a dominating set and the set of vertices of D forms a tadpole graph $T_{m,n}$ where $m \geq 3$, $n \geq 1$.

Definition 2.3. Let $G(V, E)$ be a graph, if D is a tadpole dominating set, then D is called a minimal tadpole dominating set if it has no proper tadpole dominating set. A minimum tadpole dominating set

is a tadpole dominating set of smallest size in a given graph.

Definition 2.4. The minimum cardinality of a minimal tadpole dominating set is called the tadpole domination number of G and is denoted by $\gamma_{TP}(G)$.

Example 2.5. Fig.2 illustrates the minimal tadpole dominating sets for different cardinality in the graph below.
$D_1 = \{v_7, v_{10}, v_4, v_5\}$, $D_2 = \{v_2, v_3, v_5, v_4, v_{10}\}$, $D_3 = \{v_8, v_9, v_{10}, v_4, v_5, v_1\}$, and $D_4 = \{v_2, v_3, v_5, v_4, v_{13}, v_{12}, v_{10}\}$.
The minimum tadpole dominating set is D_1, and the tadpole domination number is, $\gamma_{TP}(G) = 4$.

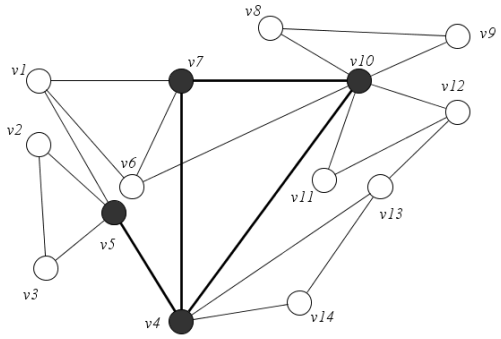


Figure 2. Minimum tadpole dominating set in G

Observation 2.6. For any graph G has a tadpole dominating set, then:

1. The order of G is $p \geq 4$.
2. $\delta(G) \geq 1, \Delta(G) \geq 3$.

Proposition 2.7. For any graph G with a minimum tadpole dominating set D if $v \in G$ is a cut vertex then, $v \in D$.
Proof. Let G has tadpole dominating set. Then G has $T_{m,n}$ as a sub graph such that each vertex of set $V - D$ is adjacent to at least one vertex in D. Let v be a cut-vertex in G. Since v is a cut vertex of G, $G - v$ is disconnected and has at least two components. From a partition of $V - \{v\}$ by letting U consist of the vertices of one of these components and W the vertices of the others. Then any two vertices $u \in U$ and $w \in W$ lie in different components of $G - v$. Therefore, every $u - w$ path in G contains v.
If $v \in V - D$, and v is dominated by some vertices say in U then the vertices of W are not dominated by any vertex in D. Therefore, $v \in D$.
Proposition 2.8. For any complete graph of order $n \geq 4$, $\gamma_{TP}(K_n) = 4$.

Proof. The girth of the complete graph is a cycle of order three. Since G is a complete graph then each vertex from these three vertices is adjacent to all other vertices of G we need exactly to one other vertex to be the vertex of the path that adjacent to this cycle to obtain a tadpole dominating set. Therefore, $\gamma_{TP}(K_n) = 4$.

Proposition 2.9. For the tadpole graph $T_{m,n}$, $m \geq 3$, $n \geq 2$, $\gamma_{TP}(T_{m,n}) = m + n - 1$.
Proof. Let G be a tadpole graph of order greater than four. Then G has only two tadpole sub graphs of orders $(m + n)$ and $(m + n - 1)$. Hence, $\gamma_{TP}(T_{m,n}) = m + n - 1$.

Observation 2.10.

1. $\gamma_{TP}(W_n) = 4$, where W_n is the wheel graph of order $n \geq 4$.
2. $\gamma_{TP}(K_{n,m}) = 5$, where $K_{n,m}$ is the complete bipartite graph of order $n \geq 2$, $m \geq 3$.
3. For the tadpole graph $T_{m,1}$, $m \geq 3$, $\gamma_{TP}(T_{m,1}) = m + 1$.

Theorem 2.11. A connected graph G has a tadpole domination if and only if:

i) There exist a maximal path P such that $V(P)$ dominates G.
ii) The maximal path P dominates a cycle in G such that there exists at most one path of order greater than 2, which is common with one vertex with this cycle.

Proof. If a graph G is connected and has a tadpole domination number, then there is a minimum tadpole dominating set say D such that it contains a cycle of order m and a path of order n such that all vertices of G are dominated by the vertices these cycle and the path. $G[D]$ contains a path forms from the vertices of the cycle and the path, where they have one vertex in common. Then D contains a path of order $m + n$, say P_1. This path is included in a maximal path say P in G. Therefore, (i) is holds. It's clear that (ii) is holds from the above proof.
Conversely, it's clear that if (i) and (ii) are satisfied, then G is connected and has a tadpole dominating set, so it has a tadpole domination number

Corollary 2.12. Let G be a graph containing at least three simple cycles joined with a common vertex with at least four vertices in every cycle then there is no tadpole dominating set for this graph.

Observations 2.13. Every tadpole dominating set D of G is a connected dominating set but not conversely.

Example 2.14. The complete graph K_n, has a connected dominating set but there is no tadpole dominating set when $n \leq 3$.

Characterizations 2.15.

1. For any graph G with $n \geq 4$ vertices, $4 \leq \gamma_{TP}(G) \leq n$.
2. Cycles and trees have no tadpole domination number.
3. Let G be a graph having a tadpole dominating set then $\gamma_{TP}(G) \geq g(G) + 1$, where, $g(G)$ is the girth of G.
4. If a spanning sub graph H of a graph G has a tadpole dominating set then G has a tadpole dominating set.
5. Let G be a graph having a tadpole dominating set for any $e = uv$ where, e is a bridge in G, if the vertices u, v are cut vertices, then $u, v \in D$.
6. If any graph G with more than one cycle has a tadpole dominating set then $\gamma_{TP}(G) \leq n - L$, where L are the pendant vertices in G.
7. If the tadpole dominating set D has pendent vertices then the number of pendent vertices in any D is exactly one.
8. For any graph G with $n \geq 4$ vertices, we have $\gamma_{TP}(G) + \Delta(G) \leq 2n - 1$.
9. Let G be a graph of order $n \geq 5$ has a tadpole dominating set D, if G is Hamilton graph then the Hamilton cycle of G do not belong to any D as the cycle of $T_{m,n}$.

Proposition 2.16. Let G and H be two graphs such that $G + H$ is not isomorphic to a cycle nor to a tree then $\gamma_{TP}(G + H) = 4,5$.

Proof. There are three cases as follows:

Case 1. If G is K_1 and H is a null graph of order m, then $G + H \cong S_{m+1}$ (S_{m+1}, is a star). Thus, in this case $G + H$ has no tadpole dominating set. Now, if G and H are two null graphs of order two, then $G + H \cong C_4$, again $G + H$ has no tadpole dominating set.

Case 2. If at least one of the two graphs G or H is not a null graph then without loss of generality say G. So, there is an edge say e in G, which forms a cycle in $G + H$ with any vertex in H. At least one of the vertices of this cycle becomes of degree greater than or equal to three. Thus, there is a path that is not contained in this cycle and joins with it by one edge. One can conclude that, this path is of order one such that a sub graph $T_{3,1}$ that contains these cycle and path which are mentioned above dominates all other vertices in $G + H$. Thus, $\gamma_{TP}(G + H) = 4$.

Case 3. If G and H are two null graphs that are not mentioned in case 1, then $G + H \cong K_{n,m}$. Thus, by Observation 2.10 (2), $\gamma_{TP}(G + H) = 5$.

Proposition 2.17. For any connected simple graph G with $n = 4$, $\gamma_{TP}(G) = 4$ if and only if $G \cong K_4$, $K_4 - \{e\}$, and $T_{3,1}$.

Proof.
Suppose that G has a tadpole domination number $\gamma_{TP}(G) = 4$. Since G is connected and is not isomorphic to P_4 nor to C_4, then one can conclude that $G \cong K_4$, $K_4 - \{e\}$, $e \in K_4$ and $T_{3,1}$.
Conversely, If $G \cong K_4$, $K_4 - \{e\}$, $e \in K_4$, and $T_{3,1}$ then $\gamma_{TP}(G) = 4$, Since each of these graphs contains spanning sub graph isomorphic to $T_{3,1}$. (Characterizations 2.15.(4)).

Theorem 2.18. For any connected graph G with $n \geq 4$, we have $\gamma(G) + \gamma_{TP}(G) + 2\ \Delta(G) + 1 \leq 3n$.

Proof. According to $\gamma(G) \leq n - \Delta(G)$ and by (Characterizations 2.15.(8)).
$\gamma_{TP}(G) + \Delta(G) \leq 2n - 1$. Thus, $\gamma_{TP}(G) + \Delta(G) \leq 2n - 1$. So, the two statements can be written as follows:
$\Delta(G) \leq n - \gamma(G)$ ... (1).
$\Delta(G) \leq 2n - 1 - \gamma_{TP}(G)$ ... (2). From (1) and (2) $\gamma(G) + \gamma_{TP}(G) + 2\ \Delta(G) + 1 \leq 3n$.

Tadpole domination in some graphs

Definition 3.1. (14) A graph of order $p = 2n$ vertices core is called sun graph. Consisting of a central complete graph K_n with an outer ring of n vertices, each of which is joined to both endpoints of the closest outer edge of the central.

Remark 3.2. For sun graph of order n, $3 \leq n \leq 6$, $\gamma_{TP}(G) = 4$. (As an example see Fig. 3).

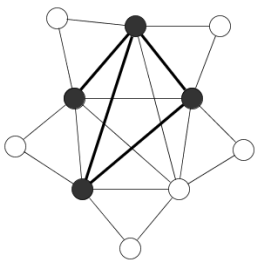


Figure 3. Sun graph with minimum tadpole dominating set, $\gamma_{TP}(G) = 4$

Theorem 3. 3. For the sun graph, $\gamma_{TP}(G) = \lceil n/2 \rceil$, where $n \geq 7$ is the order of K_n.

Proof. Every vertex in K_n is adjacent to two vertices in the outer ring. Let the vertices of K_n are $V_1 = \{v_1, v_2, v_3, \dots, v_n\}$ and the outer ring $V_2 = \{u_1, u_2, u_3, \dots, u_n\}$. Since K_n is complete, so one vertex dominates all its vertices. The girth in K_n is 3, therefore, a cycle of length 3 is chosen to belong to a dominating set. Every vertex in V_1 with odd label is adjacent to different and independent vertices from V_2. These vertices are contained in a path, choosing any three of these vertices forms a cycle, such that v_{n-1} belongs to the dominating set. So, let v_1, v_3, and v_{n-1} are the vertices belonging to a set D and the remaining vertices of set V_1 with odd labels representing a path that belongs to D as follows:
$D = \{v_{2i-1}, i = 1,2, \dots, \lceil n/2 \rceil\}$, vertices of D that dominate all the vertices of G thus, $\gamma_{TP}(G) \leq |D| \leq \lceil n/2 \rceil$. To prove the reverse inequality, the induction method on the number of vertices n is used. The results are clear if $n = 7$. Suppose that the result is true for all sun graphs of order less than n. Then $D = D_1 \cup D_2$ where D_1 is the minimum tadpole dominating set that dominates the sun graph with order less than n, when n is odd. By induction $|D_1| \geq \left\lceil \frac{n-1}{2} \right\rceil$, and hence when one vertex is added to the sun graph ($n-1$ order of the complete graph) one vertex is needed to dominate it.
Therefore, $|D| \geq \left\lceil \frac{n-1}{2} \right\rceil + 1 = \left\lceil \frac{n}{2} \right\rceil$, where $|D_2| = 1$
$|D| \geq \left\lceil \frac{n-1}{2} \right\rceil = \left\lceil \frac{n}{2} \right\rceil$ if n is even such that $|D_2| = \phi$.
Therefore, $\gamma_{TP}(G) = \left\lceil \frac{n}{2} \right\rceil$. (For example see Fig. 4) .

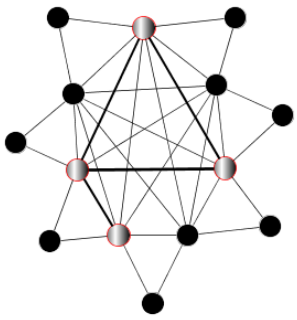


Figure 4. Sun graph with minimum tadpole dominating set, $\gamma_{TP}(G) = 4$

Definition 3.4.(14) A stacked (or generalized) prism graph $Y_{m,n}$ is a simple graph given by the Cartesian product $of\ K_n$ and C_m.
It can therefore, be formed by connecting n concentric cycle graphs C_m along spokes.
Therefore, $Y_{m,n}$ has (mn) vertices and $m(2n-1)$ edges.

Remark 3.5. For $n = 1, m \geq 3$, a stacked prism graph $Y_{m,n}$, there is no tadpole dominating set since $Y_{m,1} \cong C_m$ and for $n = 2$, $\gamma_{TP}(G) = m + 1$.

Lemma 3.6. The tadpole domination number for a stacked prism graph $Y_{3,n}$, $n \geq 3$ is $\gamma_{TP}(G) = n + 1$.

Proof. In this graph there are A copies of cycle C_3 and B copies of path P_n, $n \geq 3$. The vertices for this graph are labeled by $V(Y_{3,n}) = \{v_i^d, i = 1,2,3\ d = 1,2, \dots, n\}$. (As shown in Fig. 5). Let the set $D \subset V(Y_{3,n})$. So, it's clear that the vertices in any P_n are adjacent to all other remaining vertices of $Y_{3,n}$. Therefore, the vertices of only one path can dominate $V(Y_{3,n})$. Now to choose the shortest cycle and shortest path in $Y_{3,n}$ to get the minimum tadpole domination the cycle which vertices are $v_i^2, i = 1,2,3$ is the one. Since v_1^2 is a common vertex of C_3 and the chosen path so $n - 2 + 3$ vertices dominates the graph. Thus, $|D| = n + 1$. It is clear that D is a tadpole domination set and it is minimum.

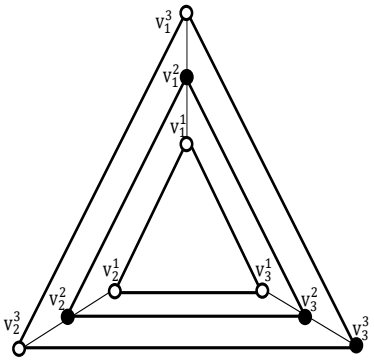


Figure 5 . A stacked prism graph, $\gamma_{TP}(Y_{3,3}) = 4$

Theorem 3.7. For a stacked prism graph $Y_{m,n}$, $m \geq 3$, $n \geq 3$, then the tadpole domination number is:

$$\gamma_{TP}(Y_{m,n}) = \begin{cases} \frac{m}{3}(n-1) + m - 1 & if\ m \equiv 0\ (mod\ 3), n \geq 3 \\ \left\lceil \frac{m}{3} \right\rceil (n-1) + m + n - 4 & if\ m = 1\ (mod\ 3), n \geq 4 \\ \left\lceil \frac{m}{3} \right\rceil (n-1) + m - 2 & if\ m = 2\ \ (mod\ 3), n \geq 5 \end{cases}$$

Proof. A stacked prism graph $Y_{m,n}$, $m \geq 3$, $n \geq 3$ is formed from A copies of cycle C_m and B copies of path P_n. Let the vertices of this graph be labeled by $V(Y_{m,n}) = \{v_i^d, i = 1,2,3, \dots m\ d = 1,2, \dots, n\}$. Three cases are obtained as follows:

Case 1.
i) When $m = 3$ the proof is in Lemma 3.6.

ii) When $m > 3$: If $m \equiv 0\ (mod\ 3)$then, let $D = \left[\{v_i^2, i = 1,2,\dots,m\} \cup \{v_1^d\ d = 2,\dots,n\} \cup \{v_{3d-1}^n, v_{3d}^n, \forall\, d = 1,2,\dots,\frac{m}{3}-1\} \cup \{v_{3j-2}^d, d = 4\dots n, \forall\, j = 2\dots\frac{m}{3}-1\}\right]$.

For the same reason in Lemma (3.6), the graph is dominated by vertices of C_m^2.
$V(C_m^2) = \{v_i^2, i = 1,2,\dots,m\}$, and for the path P_1^d, $V(P_1^d) = \{v_1^d,\ d = 2,\dots,n\}$. Since each P_n^d is adjacent to two different paths, therefore, $\frac{m}{3}$ paths are adjacent to two different paths by jumping two adjacent paths. To complete the chosen path, the set $\{v_{3d-1}^n, v_{3d}^n, \forall\, d = 1,2,\dots,\frac{m}{3}-1\}$ must be taken to get these paths whose vertices are in the following set: $\{v_{3j-2}^d, d = 4\dots n, \forall\, j = 2,\dots,\frac{m}{3}\}$. The number of vertices in every path is equal to $n\frac{m}{3}$. There are $\frac{m}{3}$ common vertices between C_m^2 and the chosen paths. The path $\{v_1^d, d = 2,..,n\}$ is chosen to be of order $n-1$. Such that $|D| = n\frac{m}{3} + m - \frac{m}{3} - 1$. Hence, the vertices in D represent the minimum tadpole dominating set. (See Fig. 6 *a*).

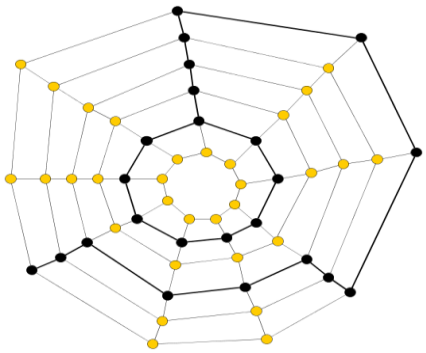
Case 2. If $m \equiv 1\ (mod\ 3)$, then as in case. 1 the graph is dominated by the vertices of set D. There is only one path of vertices v_{m-2}^d which is not dominated by any vertices in D. This path of order $n-3$, will be taken to dominate its vertices and add these vertices to the set D in case 1. Therefore, $D = \left[\{v_i^2, i = 1,2,\dots,m\} \cup \{v_1^d\ d = 2,\dots,n\} \cup \{v_{3d-1}^n, v_{3d}^n, \forall\, d = 1,2,\dots,\left\lceil\frac{m}{3}\right\rceil - 1 \cup \{v_{3j-2}^d, d = 4\dots n, \forall\, j = 2\dots\left\lceil\frac{m}{3}\right\rceil\} \cup \{v_{m-2}^d\ d = 4,\dots,n\}\right]$. Hence, $|D| = n\left\lceil\frac{m}{3}\right\rceil + m - \left\lceil\frac{m}{3}\right\rceil + n - 4$.
The vertices in D represent the minimum tadpole dominating set. (As an example see Fig. 6 b).

Case 3. If $m \equiv 2\ (mod\ 3)$, as in case. 2 the graph is dominated by the vertices in D.
In this case there are two adjacent paths of vertices v_{m-1}^d and v_{m-2}^d, they are not dominated by any other vertices in D. So we will take the vertices v_{m-2}^d to dominate the vertices v_{m-1}^d. But to continue forming the path from the path with the vertices v_{m-4}^d to the path with vertices v_{m-2}^d we pass through the vertex v_{m-3}^4, so we have the following set:

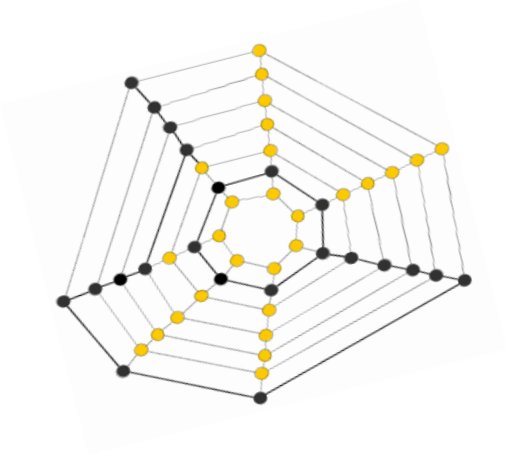
$D = \left[\{v_i^2, i = 1,2,\dots,m\} \cup \{v_1^d\ d = 2,\dots,n\} \cup \{v_{3d-1}^n, v_{3d}^n, \forall\, d = 1,2,\dots,\left\lceil\frac{m}{3}\right\rceil - 2 \cup \{v_{3j-3}^d, d = 4\dots n, \forall\, j = 2\dots\left\lceil\frac{m}{3}\right\rceil\} \cup \{v_{m-3}^4\}\right]$.

Therefore, $|D| = n\left\lceil\frac{m}{3}\right\rceil + m - \left\lceil\frac{m}{3}\right\rceil - 2$.
Hence, the vertices in D represent the minimum tadpole dominating set. (As an example see Fig. 6 c).

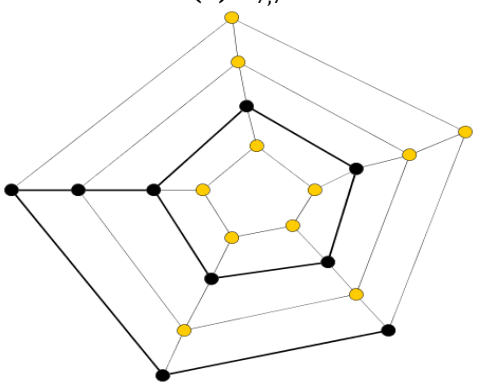
Example 3.8. For the graph $Y_{m,n}$, $\gamma_{TP}(Y_{9,6}) = 23$, $\gamma_{TP}(Y_{7,7}) = 22$, $\gamma_{TP}(Y_{5,4}) = 9$.



(a): $Y_{9,6}$



(b): $Y_{7,7}$



(c): $Y_{5,4}$

Figure 6. Minimum tadpole dominating set in stacked prism graphs

In the following definition, a new graph is introduced named "weaver graph".

Definition 3.9. The "weaver graph" is a simple graph denoted by $S_{m,h}$. It is formed by warping the path P_n of the tadpole graph $T_{m,h}$ around itself (m is the order of cycle in $S_{m,h}$ and h is the order of path in each copies of the path). $S_{m,h}$ has $(mh + 1)$ vertices and $m(2h - 2)$ edges and one pendent vertex.(As an example see Fig. 7)**.**

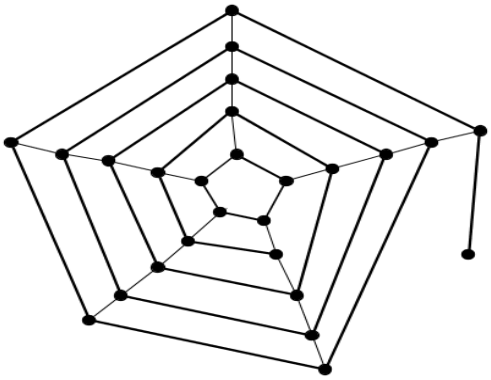


Figure 7 : The Weaver graph $S_{5,5}$

Proposition 3.10. For the weaver graph $S_{m,h}$, the tadpole domination number is

$$\gamma_{TP}(S_{m,h}) = \begin{cases} \frac{m}{3}(h-1)+m & if\ m \equiv 0\ (mod\ 3), h \geq 3 \\ \left\lceil \frac{m}{3} \right\rceil (h-1)+m+h-3 & if\ m = 1\ (mod\ 3), h \geq 4 \\ \left\lceil \frac{m}{3} \right\rceil (h-1)+m-3 & if\ m = 2\ (mod\ 3), h \geq 5 \end{cases}$$

Proof.

The proof is in the same manner that is used in Theorem 3.7.

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Conflicts of Interest: None.

هيمنة الشرغوف في البيانات

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الخلاصة:

في هذا البحث، تم استحداث نوع جديد من انواع الهيمنة المتصلة تسمى هيمنة الشرغوف. تم تحديد رقم الهيمنة الشرغوفية لبعض الرسوم البيانية القياسية، وتم الحصول على بعض الحدود الدنيا والعليا لهذا الرقم. وأخيراً ، تم تقديم رسم بياني جديد ، منتهي، وبسيط، وغير موجه، ومتصل، اسميناه البيان النساج، ويرمز إليه بـ $S_{m,h}$ ، تم حساب الهيمنة الشرغوفية له مع عائلات أخرى من البيانات .

الكلمات المفتاحية: المجموعة المهيمنة، البيان الشرغوفي، رقم الهيمنة الشرغوفية.