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Comparison of Some of Estimation methods of Stress-Strength Model: $R = P(Y < X < Z)$

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Abstract:

In this study, the stress-strength model $R = P(Y < X < Z)$ is discussed as an important parts of reliability system by assuming that the random variables follow Invers Rayleigh Distribution. Some traditional estimation methods are used to estimate the parameters namely; Maximum Likelihood, Moment method, and Uniformly Minimum Variance Unbiased estimator and Shrinkage estimator using three types of shrinkage weight factors. As well as, Monte Carlo simulation are used to compare the estimation methods based on mean squared error criteria.

Key words: Stress-Strength Model, Inverse Rayleigh Distribution, Least Square Estimator, Maximum Likelihood Method Estimator, Moment method, Uniformly Minimum Variance Unbiased Method, Shrinkage Methods.

Introduction:

The many technological and mechanical malfunction problems that emerge due to the constant developments of life might not be the only reason behind the study of reliability and stress-strength models as a parts of reliability system since these models have several applications in different sciences areas, like reliability design of systems, quality control, physics, engineering and medicine. According to Rao 'the reliability can be defined as 'the probability of a device performing its function over a specified period of time and under specified operating conditions'(1). The probability equation: $R = P(Y < X)$ of "stress-strength" reliability shows that if the component which has a random strength represented by X is greater than the stress represented by the random variables Y where both of X and Y are independent then the system works statistically. Statistical studies of stress-strength reliability system were introduced for the first time in 1956 by Birnbaum (2). This paper is interested in the model R= $P(Y < X < Z)$ that discusses the case where the strength X should not only be greater than stress Y but also be smaller than stress Z , for example person's blood pressure has two limits systolic and diastolic and his/her blood pressure should lie within these limits (3). Singh in 1980, constructed maximum likelihood (ML), the minimum variance unbiased (MVU), and empirical estimators of stress-strength model $R = P(Y < X < Z)$ by assuming that strength of a component lies in an interval where Y, X and Z are independent random variable based on normal distribution (4). After that in 2013 Wang *et al.* made statistical inference for R using nonparametric normal- approximations and the jackknife empirical likelihood under the assumption that the three samples were, independent, without ties among them (5). The Maximum likelihood estimator, moment estimator (ME) and mixture estimator (Mix) for the estimation of R=$(Y < X < Z)$ and the stresses Y and Z and the strength X have Weibull distribution with common known shape and scale parameters and X, Y and Z are independent constructed in 2013 by Amal *et al.*(6). In 2016 Patowary *et al.* discussed the technique of Reliability estimation for P$(Y < X < Z)$ of n-standby system (n=1, 2), through Monte-Carlo simulation (MCS) (7). In this work, the estimation of the system reliability P(Y < X < Z) based on the Inverse Rayleigh Distribution is considered. In addition, Monte Carlo simulation is performed for comparing several methods.

Model Description

Inverse Rayleigh Distribution (IRD) is first suggested by Trayer in 1964 see (8) and it has many applications in the area of reliability studies. The probability density function (pdf) and the corresponding cumulative distribution function (CDF) of one parameter Inverse Rayleigh Distribution are respectively defined as:

$$f(x;\gamma)=\frac{2\gamma}{x^3}\exp\left(\frac{-\gamma}{x^2}\right)\quad ;\ x>0\ \ ,\ \gamma>0$$
…(1)

$$F(x;\gamma)=\exp\left(-\frac{\gamma}{x^2}\right)\quad ;x>0\ \ ,\ \gamma>0$$
... (2)

where γ is the scale parameter for IRD see (9). The Reliability and Hazard Rate functions of Inverse Rayleigh distribution are given, respectively as follows

$$R(t;\gamma)=1-F(x;\gamma)=1-\exp\left(-\frac{\gamma}{t^2}\right)$$

$$H(t;\gamma)=-Ln\,R(t)=-Ln\left(1-\exp\left(-\frac{\gamma}{t^2}\right)\right)$$

For more details about IRD see [10,11,12,13 and 14]

This article discusses the estimation of reliability stress-strength model $R=P(Y<X<Z)$ in case of IRD. Let the random variables X,Y and Z be independent and follow IRD with the scale parameters $\gamma_1,\gamma_2,\gamma_3$ respectively. the p.d.f of the strength X is

$$f_x(x;\gamma_1)=\frac{2\gamma_1}{x^3}e^{\left(\frac{-\gamma_1}{x^2}\right)}\quad ;\ x>0\ ,\ \gamma_1>0$$
…(3)

Consequently, the pdf. of the stresses $Y and Z$ are given respectively by

$$f_y(y;\ \gamma_2)=\frac{2\gamma_2}{y^3}e^{\left(\frac{-\gamma_2}{y^2}\right)}\quad ;\ y>0\ \ ,\gamma_2>0$$
… (4)

$$f_z(z;\ \gamma_3)=\frac{2\gamma_3}{z^3}e^{\left(\frac{-\gamma_3}{z^2}\right)}\quad ;\ z>0\ \ ,\gamma_3>0$$
…(5)

the reliability system of the model $P(Y<X<Z)$ (where Y,Z are random stress and X represent the system strength)is given by

$$R=P(Y<X<Z)$$

$$=\int_0^\infty P(Y<X\,,X<Z)f(x)\,dx$$

$$=\int_0^\infty F_y(x)\,\bar{F}_z(x)\,f(x)dx=\int_0^\infty Fy(x)[1-Fz(x)]f(x)\,dx$$

$$=\int_0^\infty F_y(x)f(x)\,dx-\int_0^\infty F_y(x)\,F_z(x)f(x)dx$$

$$=\int_0^\infty e^{\left(-\frac{\gamma_2}{x^2}\right)}\ \frac{2\gamma_1}{x^3}.e^{\left(\frac{-\gamma_1}{x^2}\right)}dx-\int_0^\infty e^{\left(-\frac{\gamma_2}{x^2}\right)}e^{\left(-\frac{\gamma_3}{x^2}\right)}.\ \frac{2\gamma_1}{x^3}e^{\left(\frac{-\gamma_1}{x^2}\right)}dx$$

$$=\frac{\gamma_1}{\gamma_1+\gamma_2}-\frac{\gamma_1}{\gamma_1+\gamma_2+\gamma_3}$$

So that :

$$R=\frac{\gamma_1\gamma_3}{(\gamma_1+\gamma_2)(\gamma_1+\gamma_2+\gamma_3)}$$
…(6)

Maximum Likelihood Estimation of *R*

The Maximum likelihood method is an important and commonly, since it contained properties for good estimate and has invariant property . In this section, the MLE obtained for $R=P(Y<X<Z)$ throw the derivation of scale parameters $\gamma_1,\gamma_2,\gamma_3$,of the r.v_s. X,Y,Z as following: Let $x_1,x_2,\ldots\ldots x_n$ be a random strength sample of size n with pdf. as in (eq.3) then let $y_1,y_2,\ldots\ldots y_{m_1}$ and $z_1,z_2,\ldots\ldots z_{m_2}$ be the random stresses with pdf. as in eq.4 and eq.5 respectively, the Maximum Likelihood function of the observed sample is :

$$L(\gamma_1,\gamma_2,\gamma_3|data)=\prod_{i=1}^n f(x_i).\prod_{j=1}^{m_1} f(y_j).\prod_{k=1}^{m_2} f(z_j)$$

$$=\prod_{i=1}^n\frac{2\gamma_1}{x_i^3}e^{\left(\frac{-\gamma_1}{x_i^2}\right)}.\prod_{j=1}^{m_1}\frac{2\gamma_2}{y_j^3}e^{\left(\frac{-\gamma_2}{y_j^2}\right)}.\prod_{k=1}^{m_2}\frac{2}{z_k^3}e^{\left(\frac{-\gamma_3}{z_k^2}\right)}$$

$$=(2\gamma_1)^n+\prod_{i=1}^n\frac{1}{x_i^3}e^{\left(-\gamma_1\sum_{i=1}^n\frac{1}{x_i^2}\right)}.(2\gamma_2)^{m_1}\prod_{j=1}^{m_1}\frac{1}{y_j^3}e^{\left(-\gamma_2\sum_{j=1}^{m_1}\frac{1}{y_j^2}\right)}$$

$$.\ (2\gamma_3)^{m_2}\prod_{k=1}^{m_2}\frac{1}{z_k^3}\ e^{\left(-\gamma_1\sum_{k=1}^{m_2}\frac{1}{z_k^2}\right)}$$
... (7)

Taking the logarithm of likelihood function eq.7 then differentiating the result partially with respect to $\gamma_1,\gamma_2,\gamma_3$ and equalizing to zero respectively to get the estimated parameters $\hat{\gamma}_1,\hat{\gamma}_2,\hat{\gamma}_3$ as follows:

$$\frac{\partial lnL}{\gamma_1}=\frac{n}{\gamma_1}-\sum_{i=1}^n\frac{1}{x_i^2}=0$$

$$\frac{\partial lnL}{\gamma_2}=\frac{m_1}{\gamma_2}-\sum_{j=1}^{m_1}\frac{1}{y_j^2}=0$$

$$\frac{\partial lnL}{\gamma_3}=\frac{m_2}{\gamma_3}-\sum_{K=1}^{m_2}\frac{1}{z_k^2}=0$$

thus

$\hat{\gamma}_{1MLE} = \frac{n}{T_1}$, where $T_1 = \sum_{i=1}^{n} \frac{1}{x_i^2}$ …(8)

$\hat{\gamma}_{2MLE} = \frac{m_1}{T_2}$, where $T_2 = \sum_{j=1}^{m_1} \frac{1}{y_j^2}$ …(9)

$\hat{\gamma}_{3MLE} = \frac{m_2}{T_3}$, where $T_3 = \sum_{k=1}^{m_2} \frac{1}{z_k^2}$ … (10)

$$\hat{R}_{MLE} = \frac{\hat{\gamma}_{1MLE}\hat{\gamma}_{3MLE}}{(\hat{\gamma}_{1MLE}+\hat{\gamma}_{2MLE})(\hat{\gamma}_{1MLE}+\hat{\gamma}_{2MLE}+\hat{\gamma}_{3MLE})}$$

When substituting the equations 8, 9 and 10 in $\hat{R}_{MLE}$ this leads to the estimation of the stress-strength model $R = P(Y < X < Z)$ using Maximum likelihood Estimator as bellow

$$\hat{R}_{MLE} = \frac{\frac{n}{T_1} * \frac{m_1}{T_2}}{\left(\frac{n}{T_1}+\frac{m_1}{T_2}\right)\left(\frac{n}{T_1}+\frac{m_1}{T_2}+\frac{m_2}{T_3}\right)} \quad \text{… (11)}$$

Moments Method Estimator

This method is one of conventional methods because its ease. The basic idea of this method is to equate certain sample characteristics, such as the mean, to the corresponding population expected values. Then solving these equations for unknown parameter values yields the estimators. The sample mean of the r.v$_s$. X, Y, Z of IRD with unknown scale parameters γ_1, γ_2andγ_3 respectively are given by

$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$\bar{Y} = \frac{1}{m_1}\sum_{j=1}^{m_1} y_j$

$\bar{Z} = \frac{1}{m_2}\sum_{k=1}^{m_2} z_k$

While r^{th} moments will be:

$E(x^r) = \Gamma\left(1-\frac{r}{2}\right)\gamma_1^r \qquad r = 1,2,\dots\dots$

$E(y^r) = \Gamma\left(1-\frac{r}{2}\right)\gamma_2^r \qquad r = 1,2,\dots\dots$

and

$E(z^r) = \Gamma\left(1-\frac{r}{2}\right)\gamma_3^r \qquad k = 1,2,\dots\dots$

Equalize the sample mean with the first moment of X , Y and Z the estimates of γ_1, γ_2 and γ_3 become:

$\hat{\gamma}_{1MOM} = (\frac{1}{n}\sum_{i=1}^{n} x_i)/\Gamma\left(\frac{1}{2}\right)$ …(12)

$\hat{\gamma}_{2MOM} = (\frac{1}{m_1}\sum_{j=1}^{m_1} y_i)/\Gamma\left(\frac{1}{2}\right)$ …(13)

$\hat{\gamma}_{3MOM} = (\frac{1}{m_2}\sum_{k=1}^{m_2} z_k)/\Gamma\left(\frac{1}{2}\right)$ …(14)

Where $\Gamma\left(\frac{1}{2}\right) = \sqrt[2]{\pi}$

Substituted eq.12, 13and eq.14 in eq.6 to obtain the approximate moment estimator $\hat{R}_{MOM}$ for stress-strength reliability (R) as follows:

$$\hat{R}_{MOM} = \frac{\hat{\gamma}_{1MOM}\hat{\gamma}_{3MOM}}{(\hat{\gamma}_{1MOM}+\hat{\gamma}_{2MOM})(\hat{\gamma}_{1MOM}+\hat{\gamma}_{2MOM}+\hat{\gamma}_{3MOM})} \quad \text{…(15)}$$

Uniformly Minimum Variance Unbiased Estimators.

An unbiased estimator $\hat{\gamma}$ of γ is called the uniformly minimum variance unbiased estimator (UMVUE) if and only if Var($\hat{\gamma}$) ≤ Var($\hat{\gamma}_{ub}$) for any $x \in X$ and any other unbiased estimator of α (15). The Uniformly Minimum Variance Unbiased Estimator (UMVUE) of the scale parameters $\gamma_1, \gamma_2$and γ_3 of the random variables X ,Y and Z respectively of IRD will be discussed in this section. The IRD belongs to the exponential family with densities of the form

$f(x;\gamma) = a(\gamma)b(x)e^{(\sum_{j=1}^{m}\rho_j(\gamma)k_j(x))}$,

where $a(\gamma), b(x) > o$ $, \gamma < x < \beta$ and

$\gamma = \gamma_1\gamma_2,\dots,\gamma_k$ with $\alpha_j < \gamma_j < \delta_j$ and each of γ, β, α_j *and* δ_j are constants

Put $a(\gamma) = 2\gamma$; $b(x) = 1/x^3$; $\rho_j(\gamma) = -\gamma$; $k_j(x) = \frac{1}{x^2}$

then T_i is a complete sufficient statistic for (γ_i) for i=1, 2,3..

Where $T_1 = \sum_{i=1}^{n}\frac{1}{x_i^2}$, $T_2 = \sum_{j=1}^{m_1}\frac{1}{y_j^2}$ and $T_3 = \sum_{k=1}^{m_2}\frac{1}{z_k^2}$

the distribution of T_i can be found by following supposition:

$q_1 = \frac{1}{X^2}$, $q_2 = \frac{1}{y^2}$ and $q_3 = \frac{1}{z^2}$

consequently

$\xi(q_1) = f_x\left(X = \frac{1}{\sqrt[2]{q_1}}\right).\left|\frac{dx}{dq_1}\right|$ …(16)

$\xi(q_2) = f_y\left(Y = \frac{1}{\sqrt[2]{q_2}}\right).\left|\frac{dy}{dq_2}\right|$ …(17)

$\xi(q_3) = f_z\left(Z = \frac{1}{\sqrt[2]{q_3}}\right).\left|\frac{dz}{dq_3}\right|$ …(18)

Substitute eq.3 in eq.16 , eq.4 in eq.17 and eq.5in eq.18 to get

$\xi(q_1) = \gamma_1 e^{(-\gamma_1 q_1)}$

$\xi(q_2) = \gamma_2 e^{(-\gamma_2 q_2)}$

$\xi(q_3) = \gamma_2 e^{(-\gamma_3 q_3)}$

So $q_1 \sim Exp(\gamma_1)$, $q_2 \sim Exp(\gamma_2)$ *and* $q_3 \sim Exp(\gamma_3)$ and $T_1 \sim \Gamma(n,\gamma_1)$, $T_2 \sim \Gamma(m_1,\gamma_2)$ and $T_3 \sim \Gamma(m_2,\gamma_3)$

the density function of T_1 is

$U(t_1) = \frac{\gamma_1^n}{\Gamma(n)} t^{n-1}. e^{(-\gamma_1 t_1)}$; $t_1 > 0, \gamma_1 > 0, n > 0$

Thus $\left(\frac{1}{T_1}\right) = \frac{\gamma_1}{n-1}$, and $\left(\frac{n-1}{T_1}\right)$ is an unbiased estimator of (γ_1), then by Lehmann-Scheffe theorem the (UMVUE) of (γ_1) is giving by

$$\hat{\gamma}_{1(UMVU)} = \frac{n-1}{T_1} \quad \dots(19)$$

Consequently obtain (UMVUE) of γ_2, γ_3 as follows

$$\hat{\gamma}_{2(UMVU)} = \frac{m_1-1}{T_2} \quad \dots (20)$$

$$\hat{\gamma}_{2(UMVU)} = \frac{m_2-1}{T_3} \quad \dots(21)$$

Substituted (eq.19,20and 21) in(eq.6) to obtain the approximate UMVU($\hat{R}_{UMVU}$)estimator for stress-strength reliability R as follows:

$$\hat{R}_{UMVU} = \frac{\hat{\gamma}_{1UMVU}\hat{\gamma}_{3UMVU}}{(\hat{\gamma}_{1UMVU} + \hat{\gamma}_{2UMVU})(\hat{\gamma}_{1UMVU} + \hat{\gamma}_{2UMVU} + \hat{\gamma}_{3UMVU})} \quad \dots(22)$$

Least Square Estimator (LS)

The least squares method estimators depending on the idea of minimizing the sum of square error between the value and it's expected value. Suppose $x_1, x_2, \dots\dots x_n$ a random strength sample of size n follows IRD$(x; \gamma_1)$ and $y_1, y_2, \dots\dots y_{m_1}$and $z_1, z_2, \dots\dots z_{m_2}$are random stresses of size m_1 and m_2 respectively and both have pdf of IRD as in eq.4,and eq.5 then let :

$$Q_1 = \sum_{i=1}^{n} \left[F(x_i, \gamma_1) - E\big(F(x_i, \gamma_1)\big)\right]^2 = \rho_i \quad \dots(23)$$

$$Q_2 = \sum_{i=1}^{m_1} \left[F(y_j) - E\left(F(y_j)\right)\right]^2 = \rho_{1j} \quad \dots(24)$$

$$Q_3 = \sum_{i=1}^{m_2} \left[F(z_j) - E\left(F(z_j)\right)\right]^2 = \rho_{2j} \quad \dots(25)$$

Where ρ_i, ρ_{1j}andρ_{2j} are the plotting position such that:

$$\rho_i = \frac{i}{n+1} = E\big(F(x_i, \gamma_1)\big) \;\; ; \; i = 1,2, \dots, n$$

$$\rho_{1j} = \frac{j}{m_1+1} = E\left(F(y_j)\right) \; ; j = 1,2, \dots m_1$$

And

$$\rho_{3j} = \frac{j}{m_2+1} = E\left(F(z_j)\right) \;\; ; \; j = 1,2, \dots m_1$$

Hence according to CDF formula in(eq. 2)

$$F_x(x_i, \gamma_1) = \mathrm{e}^{-\left(\frac{\gamma_1}{x_i^2}\right)} \text{ then } \big(F_x(x_i, \gamma_1)\big)^{-1} = \mathrm{e}^{\frac{\gamma_1}{x_i^2}}$$

$$F_y(y_i, \gamma_2) = \mathrm{e}^{-\left(\frac{\gamma_2}{y_i^2}\right)} \rightarrow \left(F_y(y_i, \gamma_2)\right)^{-1} = e^{\frac{\gamma_2}{y_i^2}}$$

$$F_z(z_i, \gamma_3) = e^{-\left(\frac{\gamma_3}{z_j^2}\right)} \rightarrow \big(F_z(z_i, \gamma_3)\big)^{-1} = e^{\frac{\gamma_3}{z_j^2}}$$

After simplification and replacing $F_x(x_i, \gamma_1)$, $F_y(y_i, \gamma_2)$ and $F_z(z_i, \gamma_3)$ by plotting position ρ_i, $\rho_{1j}$and ρ_{2j} then equalized to zero, the following equations were obtained:

$$ln(\rho_i)^{-1} - \frac{\gamma_1}{x_i^2} = 0 \quad \dots(26)$$

$$ln\big(\rho_{1j}\big)^{-1} - \frac{\gamma_2}{y_i^2} = 0 \quad \dots(27)$$

$$ln\big(\rho_{2j}\big)^{-1} - \frac{\gamma_3}{z_j^2} = 0 \quad \dots(28)$$

Substitution eq.26 in eq.23 to get:

$$\sum_{i=1}^{n} \left[ln(\rho_i)^{-1} - \frac{\gamma_1}{x_i^2}\right]^2 = \sum_{i=1}^{n} \left[(ln(\rho_i)^{-1})^2 - 2\frac{\gamma_1}{x_i^2}(ln(\rho_i)^{-1}) + \left(\frac{\gamma_1}{x_i^2}\right)^2\right]$$

taking the first derivative with respect to γ_1 and equating the result to zero, to get:

$$\hat{\gamma}_{1LS} = \frac{-(\sum_{i=1}^{n} \ln \rho_i / x_i^2)}{\sum_{i=1}^{n} 1/x_i^2} \quad \dots (29)$$

By the same way Substitution eq.27 in eq.24 and eq.28 in eq.25 for getting

$$\hat{\gamma}_{2LS} = \frac{-(\sum_{j=1}^{m} \ln \rho_i / y_j^2)}{\sum_{j=1}^{m} 1/y_j^2} \quad \dots (30)$$

$$\hat{\gamma}_{3LS} = \frac{-\sum_{j=1}^{m_2} p2j/z_j^2}{\sum_{j=1}^{m_2} 1/z_j^2} \quad \dots(31)$$

Substitute eq.29, 30 and 31 in eq.6 to obtain the approximate LS estimator ($\hat{R}_{LS}$) for stress-strength reliability R as follows:

$$\hat{R}_{LS} = \frac{\hat{\gamma}_{1LS}\hat{\gamma}_{3LS}}{(\hat{\gamma}_{1LS}+\hat{\gamma}_{2LS})(\hat{\gamma}_{1LS}+\hat{\gamma}_{2LS}+\hat{\gamma}_{3LS})} \quad \dots(32)$$

Shrinkage Estimation Methods (Sh)

Thompson in 1968, suggested a new idea for estimate the unknown parameter by shrinking the classical estimator($\hat{\gamma}$) to a prior information(prior estimate) α_0 due the past studies or previous experiences using shrinkage weight factor $\Theta(\hat{\gamma})$ as below(16):

$$\hat{\gamma}_{sh} = \Theta(\hat{\gamma})\hat{\gamma}_{ub} + (1 - \Theta(\hat{\gamma}))\gamma_0 \;\; ; \;\; 0 \le \Theta(\hat{\gamma}) \le 1$$

Where $\hat{\gamma}_{sh}$ is the shrinkage estimator of γ and $\hat{\gamma}_{ub}$ is unbiased estimator of γ as it is defined previously where $\hat{\gamma}_{ub} = \hat{\gamma}_{(UMVU)}$. The factor $\Theta(\hat{\gamma})$ $(0 \leq \Theta(\hat{\gamma}) \leq 1)$, represent a shrinkage weight factor which can be obtained by minimizing the mean squared error of $\hat{\gamma}_{sh}$ or it can be considered as a constant , a function of sample size or a function of $\hat{\gamma}_{ub}$ (ad hoc basis) . Noted $\Theta(\hat{\gamma})$ refers to the believe of unbiased estimator $\hat{\gamma}_{ub}$, and $(1-\Theta(\hat{\gamma}))$ represents to believe of γ_0. Although the shrinkage estimator is biased, it is well known that it has minimum quadratic risk compared to classical estimators (mostly the maximum likelihood estimator or unbiased estimator); (17). In this section shrinkage estimator obtained according to Thompson's shrinkage technique also for more details see (18,19 and 20) .

Noted that, $E\left(\hat{\gamma}_{iub} = \frac{v-1}{T_i}\right) = \gamma$ and $var\left(\hat{\gamma}_{iub} = \frac{v-1}{T_i}\right) = \frac{\gamma_i^2}{v-2}$

Where, i=1, 2 ,3 and v refer to n , m_1 $m_2$or respectively depends on i.

Thus, the shrinkage estimator of the scale parameters $\gamma_1, \gamma_2, \gamma_3$ of the random variables X,Y, Z that follows IRD will be as follows:

$$\hat{\gamma}_{ish} = \Theta_i(\hat{\gamma}_i)\gamma_{ub} + (1-\Theta_i(\hat{\gamma}_i))\gamma_{i0}, \quad i=1,2,3 \quad \ldots(33)$$

Constant Shrinkage Weight Function(Sh1)

Here assume that the constant shrinkage weight factor is equal to the following:

$\Theta_i(\hat{\gamma}_i) = 0.03$; i=1,2,3

Then applying in eq. 32 to obtain the following shrinkage estimators

$$\hat{\gamma}_{1sh_1} = \Theta_1(\hat{\gamma}_1)\hat{\gamma}_{1ub} + (1-\Theta_1(\hat{\gamma}_1))\gamma_{1_0} \quad \ldots(34)$$

$$\hat{\gamma}_{2sh_1} = \Theta_2(\hat{\gamma}_2)\hat{\gamma}_{2ub} + (1-\Theta_2(\hat{\gamma}_2))\gamma_{2_0} \quad \ldots(35)$$

$$\hat{\gamma}_{3sh_1} = \Theta_3(\hat{\gamma}_3)\hat{\gamma}_{3ub} + (1-\Theta_3(\hat{\gamma}_3))\gamma_{3_0} \quad \ldots(36)$$

where, γ_{i_0} (i=1,2,3) are prior information of γ_i.

Substitute equations 34, 35 and 34 in equation 6 to get the constant shrinkage estimator $\hat{R}_{sh_1}$ of S-S reliability (R) using shrinkage estimator $\hat{R}_{sh1}$ as

$$\hat{R}_{sh_1} = \frac{\hat{\gamma}_{1sh_1}\hat{\gamma}_{3sh_1}}{(\hat{\gamma}_{1sh_1}+\hat{\gamma}_{2sh_1})(\hat{\gamma}_{1sh_1}+\hat{\gamma}_{2sh_1}+\hat{\gamma}_{3sh_1})} \quad \ldots(37)$$

Shrinkage weight function (Sh_{wf})

In this subsection, the shrinkage weight factor has been suggested to be a function of n, $m_1$and m_2 respectively in eq.33 as bellow

$\Theta_1(\hat{\gamma}_1) = e^{-n}/n$

$\Theta_2(\hat{\gamma}_2) = e^{m_1}/m_1$

And $\Theta_3(\hat{\gamma}_3) = e^{-m_2}/m_2$

So

$$\hat{\gamma}_{1sh_2} = e^{-n}/n\,\gamma_{1ub} + (1-e^{-n}/n)\gamma_{1_0} \quad \ldots(38)$$

$$\hat{\gamma}_{2sh_2} = e^{m_1}/m_1\,\gamma_{2ub} + (1-e^{m_1}/m_1)\gamma_{2_0} \quad \ldots(39)$$

$$\hat{\gamma}_{3sh_2} = e^{-m_2}/m_2\,\gamma_{3ub} + (1-e^{-m_2}/m_2)\gamma_{3_0} \quad \ldots(40)$$

Substitute equations 38,39and 40 in equation 6 to get the estimation of reliability models($\hat{R}_{sh2}$) using shrinkage function estimator as following :

$$\hat{R}_{sh_2} = \frac{\hat{\gamma}_{1_{sh2}}\hat{\gamma}_{3_{sh2}}}{(\hat{\gamma}_{1_{sh2}}+\hat{\gamma}_{2_{sh2}})(\hat{\gamma}_{1_{sh2}}+\hat{\gamma}_{2_{sh2}}+\hat{\gamma}_{3_{sh2}})} \quad \ldots(41)$$

Modified Thompson Type Shrinkage Weight Function (MTShwf)

In this subsection, the modification to the shrinkage weight factor of Thompson type estimator has been suggested as the following equation :

$$\psi(\hat{\gamma}_{iub}) = \frac{(\hat{\gamma}_{iub}-\hat{\gamma}_{i_0})^2}{(\hat{\gamma}_{iub}-\hat{\gamma}_{i_0})+var(\hat{\gamma}_{iub})}(0.03) \quad for\ i = 1,2,3$$

Therefore, the modified Thompson type shrinkage estimator will be

$$\hat{\gamma}_{iTH} = \psi(\hat{\gamma}_i)\hat{\gamma}_{iub} + (1-\psi(\hat{\gamma}_i))\gamma_{i_0} for\ i = 1,2,3 \quad \ldots(42)$$

Substitute eq.40 in the eq.5 when i=1,2,3 then the modified Thompson type shrinkage estimation of the (S-S) reliability is as below

$$\hat{R}_{TH} = \frac{\hat{\gamma}_{1_{TH}}\hat{\gamma}_{3_{TH}}}{(\hat{\gamma}_{1_{TH}}+\hat{\gamma}_{2_{TH}})(\hat{\gamma}_{1_{TH}}+\hat{\gamma}_{2_{TH}}+\hat{\gamma}_{3_{TH}})} \quad \ldots(43)$$

Simulation

Monte Carlo simulation is used to compare the estimators obtained in this study. samples of different sizes, n = 10, 35, and 75 has been generated from one parameter inverse Rayleigh distribution based on MSE criteria, with 1000 replicates. The steps of Simulation for Mote Carlo as follows;

Step1: Generate random samples as $u_{i1}, u_{i2}, \ldots, u_{in_i}$, $v_1, v_2, \ldots, v_{m_1}$, $and\ w_1, w_2, \ldots, w_m,$ $for\ all$ $i = 1,2,\ldots,k.$ Respectively which follow the continuous uniform distribution defined on the interval (0,1).

Step2: Transform the above uniform random m samples to random samples follows IRD using the cumulative distribution function (CDF) as follow;

$F(x) = e^{\frac{-\alpha}{x^2}}$:

$ui = e^{\frac{-\alpha}{x^2}}$:

$x_i = [\alpha_1 / -\ln(ui)]^{\frac{1}{2}}$

And, by the same method, the following equations have been gotten:

$y_j = [\alpha_2 / -\ln(vj)]^{\frac{1}{2}}$

$z_r = [\alpha_2 / -\ln(wr)]^{\frac{1}{2}}$

Step3: Compute the R from equation (6).
Step4: Recall the R of the maximum likelihood estimator using equation (11).
Step4*:* Find Moment estimate for reliability by using equation (16).
Step4*:* Find Least Square Estimator estimate for reliability by using equation (32).
Step5: find the uniformly minimum variance unbiased method of R using equation (22).
Step6: Apply Shrinkage estimators of reliability using equations (37,41,and 43).

Step7: Based on L=1000 replicate. Calculate the MSE as follows:

$\text{MSE} = \frac{1}{L}\sum_{i=1}^{L}(\hat{R}_i - R)^2$

Results of Simulation

In this section, the simulation results used to determine the best outcome of the proposed estimation methods (MLE, MOM, UMM, Sh_1, Sh_2, and Sh_3) for the reliability R of stress -strength $P(X < Y < Z)$ based on IRD. The Mote Carlo simulation was coded using Matlab b 2016.
Tables (1, 3,5 and 7) present the simulation results for estimation value of the different reliability values R at different values of scale parameters $\gamma_1, \gamma_2, \gamma_3$
Tables (2,4,6 and 8) present the simulation results for MSE of all the proposed estimation methods. Based on the results, the shrinkage estimator ($\hat{R}_{\text{Sh2}}$) using Shrinkage weight function as shown in these tables was the best one and had less MSE after $\hat{R}_{Th}$ in all cases for the R= $P(Y_1 < X < Y_2)$. Therefore, Under the current results, it is observed that the results of $\hat{R}_{\text{Sh2}}$ are better than the results of other approaches .

Table 1. Estimation value of $\mathbf{R = 0.098039}$, when $\boldsymbol{\gamma_1 = 2}$, $\boldsymbol{\gamma_2 = 4}$ and $\boldsymbol{\gamma_3 = 2.5}$

(n, m_1, m_2)	$R\hat{}\_MLE$	$R\hat{}\_Ub$	$R\hat{}\_MOM$	$R\hat{}\_LS$	$R\hat{}\_Sh1$	$R\hat{}\_sh2$	$R\hat{}\_Th$
(20,20,20)	7.6133e-02	7.8542e-02	1.8203e-01	8.2483e-02	9.4314e-02	9.8018e-02	9.7815e-02
(20,50,20)	7.6402e-02	7.6605e-02	9.9133e-02	7.5856e-02	9.4043e-02	9.8018e-02	9.7726e-02
(20,50,50)	7.3768e-02	7.5943e-02	1.8407e-01	7.7321e-02	9.4103e-02	9.8018e-02	9.7758e-02
(50,50,20)	7.5741e-02	7.5117e-02	9.0120e-02	7.5774e-02	9.4270e-02	9.8018e-02	9.7824e-02
(50,50, 50)	7.4522e-02	7.5310e-02	1.7631e-01	7.6474e-02	9.4102e-02	9.8018e-02	9.7756e-02
(50,20, 20)	7.5264e-02	7.6768e-02	1.3754e-01	8.1799e-02	9.4039e-02	9.8018e-02	9.7714e-02
(75,20,20)	7.5215e-02	7.7226e-02	1.1268e-01	8.2633e-02	9.4571e-02	9.8018e-02	9.7815e-02
(75,75,75)	7.4726e-02	7.5393e-02	1.7868e-01	7.6178e-02	9.4271e-02	9.8018e-02	9.7645e-02
(75,50,20)	7.4369e-02	7.3330e-02	7.8528e-02	7.4950e-02	9.3882e-02	9.8018e-02	9.7709e-02
(75,50,50)	7.6306e-02	7.6914e-02	1.6578e-01	7.8573e-02	9.4457e-02	9.8018e-02	9.7807e-02

Table 2. MSE value of $\mathbf{R = 0.098039}$, $\boldsymbol{\gamma_1 = 2}$, $\boldsymbol{\gamma_2 = 4}$ and $\boldsymbol{\gamma_3 = 2.5}$

(n, m_1, m_2)	$R\hat{}\_MLE$	$R\hat{}\_Ub$	$R\hat{}\_MOM$	$R\hat{}\_LS$	$R\hat{}\_Sh1$	$R\hat{}\_sh2$	$R\hat{}\_Th$
(20,20,20)	$1.8161e-03$	$1.9312e-03$	$1.1319e-02$	$1.1873e-03$	$1.1590e-04$	$4.3931e-10$	$8.4065e-07$
(20,50,20)	$1.8063e-03$	$1.9362e-03$	$9.2717e-04$	$1.7864e-03$	$1.3343e-04$	$4.3931e-10$	$3.8167e-06$
(20,50,50)	$1.8996e-03$	$2.0146e-03$	$1.0516e-02$	$1.6048e-03$	$1.2000e-04$	$4.3931e-10$	$3.6060e-06$
(50,50,20)	$1.8294e-03$	$1.8736e-03$	$1.3455e-03$	$1.7913e-03$	$1.0565e-04$	$4.3931e-10$	$7.1668e-07$
(50,50, 50)	$1.9836e-03$	$2.0071e-03$	$1.0610e-02$	$1.7348e-03$	$1.1245e-04$	$4.3931e-10$	$4.1862e-06$
(50,20, 20)	$1.9153e-03$	$1.9189e-03$	$6.8979e-03$	$1.2592e-03$	$1.3262e-04$	$4.3932e-10$	$9.1571e-06$
(75,20,20)	$1.8698e-03$	$1.8396e-03$	$4.8031e-03$	$1.1456e-03$	$9.8420e-05$	$4.3931e-10$	$1.6262e-06$
(75,75,75)	$1.8742e-03$	$1.8962e-03$	$1.0533e-02$	$1.7283e-03$	$1.1886e-04$	$4.3931e-10$	$1.5975e-05$
(75,50,20)	$1.9071e-03$	$1.9418e-03$	$1.8031e-03$	$1.8496e-03$	$1.3719e-04$	$4.3931e-10$	$4.4574e-06$
(75,50,50)	$1.8500e-03$	$1.8438e-03$	$9.5939e-03$	$1.5495e-03$	$1.0109e-04$	$4.3931e-10$	$2.4365e-06$

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Table 3. Estimation value of $R = 0.19780$, when $\gamma_1 = 1.5$, $\gamma_2 = 2$ and $\gamma_3 = 3$

(n, m_1, m_2)	R^_MLE	R^_Ub	R^_MOM	R^_LS	R^_Sh1	R^_sh2	R^_Th
$(20, 20, 20)$	$1.50E-01$	$1.55E-01$	$9.89E-02$	$1.98E-01$	$1.97E-01$	$1.64E-01$	$1.91E-01$
$(50, 20, 20)$	$1.48E-01$	$1.51E-01$	$7.77E-02$	$1.98E-01$	$1.97E-01$	$1.65E-01$	$1.91E-01$
$(75, 20, 20)$	$1.48E-01$	$1.51E-01$	$6.50E-02$	$1.98E-01$	$1.97E-01$	$1.65E-01$	$1.91E-01$
$(20, 50, 20)$	$1.55E-01$	$1.56E-01$	$4.61E-02$	$1.98E-01$	$1.97E-01$	$1.57E-01$	$1.91E-01$
$(20, 50, 50)$	$1.53E-01$	$1.57E-01$	$9.78E-02$	$1.98E-01$	$1.97E-01$	$1.60E-01$	$1.91E-01$
$(50, 75, 75)$	$1.49E-01$	$1.51E-01$	$9.21E-02$	$1.98E-01$	$1.97E-01$	$1.53E-01$	$1.90E-01$
$(50, 50, 50)$	$1.51E-01$	$1.53E-01$	$9.93E-02$	$1.98E-01$	$1.97E-01$	$1.59E-01$	$1.91E-01$
$(75, 50, 50)$	$1.52E-01$	$1.53E-01$	$9.27E-02$	$1.98E-01$	$1.97E-01$	$1.60E-01$	$1.91E-01$
$(75, 75, 2075)$	$1.49E-01$	$1.49E-01$	$9.64E-02$	$1.98E-01$	$1.97E-01$	$1.52E-01$	$1.90E-01$
$(75, 20, 75)$	$1.49E-01$	$1.55E-01$	$1.34E-01$	$1.98E-01$	$1.98E-01$	$1.76E-01$	$1.92E-01$

Table 4. MSE value of $R = 0.19780$ when $\gamma_1 = 1.5$, $\gamma_2 = 2$ and $\gamma_3 = 3$

(n, m_1, m_2)	R^_MLE	R^_Ub	R^_MOM	R^_LS	R^_Sh1	R^_sh2	R^_Th
$(20, 20, 20)$	$7.07E-03$	$7.27E-03$	$1.10E-02$	$5.26E-03$	$9.54E-06$	$4.251E-11$	$4.52E-04$
$(50, 20, 20)$	$7.21E-03$	$7.12E-03$	$1.60E-02$	$5.12E-03$	$1.46E-05$	$4.295E-11$	$3.90E-04$
$(75, 20, 20)$	$7.15E-03$	$6.96E-03$	$1.91E-02$	$5.10E-03$	$9.17E-06$	$4.250E-11$	$4.10E-04$
$(20, 50, 20)$	$6.18E-03$	$6.54E-03$	$2.32E-02$	$6.54E-03$	$4.98E-06$	$4.250E-11$	$3.95E-04$
$(20, 50, 50)$	$6.45E-03$	$6.87E-03$	$1.10E-02$	$6.00E-03$	$1.02E-05$	$4.259E-11$	$3.90E-04$
$(50, 75, 75)$	$6.91E-03$	$7.37E-03$	$1.23E-02$	$7.25E-03$	$1.51E-05$	$4.250E-11$	$5.27E-04$
$(50, 50, 50)$	$6.66E-03$	$6.68E-03$	$1.08E-02$	$5.98E-03$	$4.31E-05$	$4.252E-11$	$4.35E-04$
$(75, 50, 50)$	$6.74E-03$	$6.78E-03$	$1.24E-02$	$6.04E-03$	$1.34E-05$	$4.205E-11$	$4.76E-04$
$(75, 75, 207$	$7.32E-03$	$7.35E-03$	$1.15E-02$	$7.45E-03$	$4.14E-05$	$4.215E-11$	$5.67E-04$
$(75, 20, 75)$	$6.79E-03$	$6.45E-03$	$9.04E-03$	$3.72E-03$	$1.20E-06$	$4.205E-11$	$2.51E-04$

Table 5. Estimation value of $R = 0.1684$, when $\gamma_1 = 2.2$, $\gamma_2 = 3.3$ and $\gamma_3 = 4$

(n, m_1, m_2)	R^_MLE	R^_Ub	R^_MOM	R^_LS	R^_Sh1	R^_sh2	R^_Th
$(20, 20, 20)$	$1.30E-01$	$1.33E-01$	$1.33E-01$	$1.43E-01$	$1.62E-01$	$1.68E-01$	$1.68E-01$
$(50, 20, 20)$	$1.26E-01$	$1.29E-01$	$1.29E-01$	$1.40E-01$	$1.62E-01$	$1.68E-01$	$1.68E-01$
$(75, 20, 20)$	$3.12E-02$	$1.34E-01$	$1.34E-01$	$1.44E-01$	$1.63E-01$	$1.68E-01$	$1.68E-01$
$(20, 50, 20)$	$1.33E-01$	$1.34E-01$	$1.34E-01$	$1.35E-01$	$1.63E-01$	$1.68E-01$	$1.68E-01$
$(20, 50, 50)$	$1.29E-01$	$1.31E-01$	$1.31E-01$	$1.35E-01$	$1.62E-01$	$1.68E-01$	$1.68E-01$
$(50, 75, 75)$	$1.26E-01$	$1.27E-01$	$1.27E-01$	$1.28E-01$	$1.62E-01$	$1.68E-01$	$1.68E-01$
$(50, 50, 50)$	$1.28E-01$	$1.29E-01$	$1.29E-01$	$1.34E-01$	$1.62E-01$	$1.68E-01$	$1.68E-01$
$(75, 50, 50)$	$1.28E-01$	$1.29E-01$	$1.29E-01$	$1.35E-01$	$1.62E-01$	$1.68E-01$	$1.68E-01$
$(75, 75, 2075)$	$1.31E-01$	$1.32E-01$	$1.32E-01$	$1.34E-01$	$1.63E-01$	$1.68E-01$	$1.68E-01$
$(75, 20, 75)$	$1.29E-01$	$1.34E-01$	$1.34E-01$	$1.54E-01$	$1.63E-01$	$1.68E-01$	$1.68E-01$

Table 6. MSE value of R = 0.1684 when $\gamma_1 = 2.2$, $\gamma_2 = 3.3$ and $\gamma_3 = 4$

(n, m_1, m_2)	$R\hat{}\_MLE$	$R\hat{}\_Ub$	$R\hat{}\_MOM$	$R\hat{}\_LS$	$R\hat{}\_Sh1$	$R\hat{}\_sh2$	$R\hat{}\_Th$
(20, 20, 20)	4.99E − 03	5.17E − 03	5.17E − 03	3.32E − 03	3.80E − 04	1.79E − 11	2.52E − 05
(50, 20, 20)	5.43E − 03	5.38E − 03	5.38E − 03	3.74E − 03	2.94E − 04	1.79E − 11	6.44E − 06
(75, 20, 20)	4.88E − 03	4.79E − 03	4.79E − 03	3.17E − 03	3.39E − 04	1.79E − 11	2.06E − 05
(20, 50, 20)	4.46E − 03	4.73E − 03	4.73E − 03	4.35E − 03	3.19E − 04	1.79E − 11	7.82E − 06
(20, 50, 50)	5.12E − 03	5.46E − 03	5.46E − 03	4.56E − 03	3.51E − 04	1.79E − 11	2.96E − 06
(50, 75, 75)	5.49E − 03	5.55E − 03	5.55E − 03	5.54E − 03	3.69E − 04	1.79E − 11	7.27E − 06
(50, 50, 50)	5.21E − 03	5.30E − 03	5.30E − 03	4.71E − 03	4.37E − 04	1.79E − 11	1.31E − 05
(75, 50, 50)	5.18E − 03	5.19E − 03	5.19E − 03	4.62E − 03	2.99E − 04	1.79E − 11	1.71E − 05
(75, 75, 2075)	4.78E − 03	4.81E − 03	4.81E − 03	4.69E − 03	2.81E − 04	1.79E − 11	3.90E − 06
(75, 20, 75)	5.07E − 03	4.95E − 03	4.95E − 03	2.82E − 03	3.07E − 04	1.79E − 11	1.62E − 05

Table 7. Estimation value of R = 0.175000, when $\gamma_1 = 1.5$, $\gamma_2 = 2.5$ and $\gamma_3 = 3.5$

(n, m_1, m_2)	$R\hat{}\_MLE$	$R\hat{}\_Ub$	$R\hat{}\_MOM$	$R\hat{}\_LS$	$R\hat{}\_Sh1$	$R\hat{}\_sh2$	$R\hat{}\_Th$
(20, 20, 20)	1.32E − 01	1.36E − 01	1.36E − 01	1.75E − 01	1.75E − 01	1.45E − 01	1.68E − 01
(50, 20, 20)	1.35E − 01	1.38E − 01	1.38E − 01	1.75E − 01	1.75E − 01	1.49E − 01	1.69E − 01
(75, 20, 20)	1.31E − 01	1.34E − 01	1.34E − 01	1.75E − 01	1.74E − 01	1.44E − 01	1.68E − 01
(20, 50, 20)	1.35E − 01	1.36E − 01	1.36E − 01	1.75E − 01	1.75E − 01	1.35E − 01	1.68E − 01
(20, 50, 50)	1.35E − 01	1.37E − 01	1.37E − 01	1.75E − 01	1.75E − 01	1.40E − 01	1.69E − 01
(50, 75, 75)	1.35E − 01	1.36E − 01	1.36E − 01	1.75E − 01	1.75E − 01	1.37E − 01	1.68E − 01
(50, 50, 50)	1.32E − 01	1.34E − 01	1.34E − 01	1.75E − 01	1.74E − 01	1.38E − 01	1.68E − 01
(75, 50, 50)	1.34E − 01	1.35E − 01	1.35E − 01	1.75E − 01	1.75E − 01	1.40E − 01	1.69E − 01
(75, 75, 2075)	1.35E − 01	1.36E − 01	1.36E − 01	1.75E − 01	1.75E − 01	1.36E − 01	1.68E − 01
(75, 20, 75)	1.34E − 01	1.39E − 01	1.39E − 01	1.75E − 01	1.75E − 01	1.57E − 01	1.69E − 01

Table 8. MSE value of R = 0.175000, $\gamma_1 = 1.5$, $\gamma_2 = 2.5$ and $\gamma_3 = 3.5$

(n, m_1, m_2)	$R^{\wedge}_{MLE}$	$R\hat{}\_Ub$	$R\hat{}\_MOM$	$R\hat{}\_LS$	$R\hat{}\_Sh1$	$R\hat{}\_sh2$	$R\hat{}\_Th$
(20, 20, 20)	6.16E − 03	6.33E − 03	6.33E − 03	3.90E − 03	1.36E − 05	8.420E − 11	3.51E − 04
(50, 20, 20)	5.77E − 03	5.73E − 03	5.73E − 03	3.52E − 03	1.53E − 06	8.428E − 11	2.77E − 04
(75, 20, 20)	6.37E − 03	6.27E − 03	6.27E − 03	4.25E − 03	4.92E − 05	8.402E − 11	4.72E − 04
(20, 50, 20)	5.82E − 03	6.07E − 03	6.07E − 03	5.70E − 03	1.05E − 05	8.452E − 11	3.73E − 04
(20, 50, 50)	5.70E − 03	5.99E − 03	5.99E − 03	4.67E − 03	4.10E − 06	8.042E − 11	2.91E − 04
(50, 75, 75)	5.88E − 03	5.96E − 03	5.96E − 03	5.31E − 03	5.93E − 06	8.942E − 11	4.03E − 04
(50, 50, 50)	6.23E − 03	6.28E − 03	6.28E − 03	5.09E − 03	3.13E − 05	8.492E − 11	4.58E − 04
(75, 50, 50)	5.92E − 03	5.88E − 03	5.88E − 03	4.84E − 03	1.13E − 05	8.402E − 11	3.50E − 04
(75, 75, 2075)	6.03E − 03	6.08E − 03	6.08E − 03	5.69E − 03	4.48E − 06	8.402E − 11	4.13E − 04
(75, 20, 75)	5.96E − 03	5.92E − 03	5.92E − 03	3.10E − 03	2.27E − 06	8.422E − 11	2.95E − 04

Conclusion:

The estimation of S-S reliability for Inverse Rayleigh distribution was introduced in this paper using different methods as; Maximum Likelihood, Moment method, Uniformly Minimum Variance Unbiased, Constant Shrinkage Estimation Method, Shrinkage Function Estimator, Modified Thompson Type Shrinkage Estimator. Monte Simulation was exhibited. Based on the results, the performance of shrinkage weight function ($\hat{R}_{Sh2}$) was appropriate behavior and it is efficient estimator than the others in the sense of MSE based on four parameters (β_1, β_2, β_3). While $\hat{R}_{Th}$ had the second rank.

Authors' declaration:

- Conflicts of Interest: None.

- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Garmian.

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مقارنة بعض طرائق التقدير لنموذج الأجهاد-المتانة $R = P(Y < X < Z)$

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الخلاصة:

كجزء مهم من نظام المعولية بفرض أن R= P(Y<X<Z) في هذا البحث تمت مناقشة نموذج الاجهاد –المتانة المتغيرات العشوائية تتبع توزيع معكوس رالي . بعض طرق التقدير التقليدية استخدمت لتخمين المعلمات وهي: طريقة الامكان الاعظم ، طريقة العزوم ،طريقة المقدر غير المتحيز ذي اقل تباين وثلاث طرائق من عوامل الاوزان المقلصة . بالاضافة الى ذلك تم استخدام محاكاة مونت كارلو للمقارنة بين طرق التخمين المستخدمة بناءً على متوسط مربعات الخطأ.

الكلمات المفتاحية: نموذج الاجهاد – المتانة ، :توزيع معكوس رالي ، مقدر المربعات الصغرى ،طريقة الامكان الاعظم، طريقة الحد الأدنى من التباين غير المتحيز ،طريقة التقلص.