Development Of Six-Degree Of Freedom Strapdown Terrestrial INS Algorithm

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Abstract

Many of accurate inertial guided missile systems need to use more complex mathematical calculations and require a high speed processing to ensure the real-time operation. This will give rise to the need of developing an efficient high-speed computing system with modern inertial guided algorithms. In this work, a flexible computer simulation program for the terrestrial strapdown INS algorithm was developed and implemented for six-degree of freedom missile flight simulator. The evaluation of the algorithm is based on the accuracy of the proposed algorithm with real data.

1. Introduction

The celestial strapdown inertial navigation system is mechanized in inertial frame. This frame is widely used for spacecraft applications in which geographical information in not required [1]. But, for terrestrial navigation, the inherent time-varying relationship between the inertial and geographic frames complicates the space-stable system design [2].

Thus, the inertial frame implementation results in the most straight-forward navigation-state differential equations, which is not commonly used. The reasons for this lack of use are the difficulty in calculating gravitational forces, and the terrestrial navigation has the same coordinate used by the GPS system, for GPS aiding navigation system [3, 4].

In this work the terrestrial algorithm will be derived and implemented for six-degree of freedom. This algorithm represents a developed version of the algorithm described in [5].

The navigation equations are expressed on the basis of navigation frames, which are used in the navigation society. So, the descriptions and assumptions of the coordinate systems transformation, will introduced at Appendix A.

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2. Terrestrial Strapdown System Dynamic Equation

The differential equation of the relative quaternion between body coordinate and geographic coordinate given by [2, 6]:

$$\dot{u} = \tfrac{1}{2}\Omega_{ib}^{b} \cdot u - \tfrac{1}{2}\Omega_{in}^{b} \cdot u \quad \ldots(1)$$

Where, the angular velocity skew-symmetric matrix Ω_{in}^{i} and Ω_{bn}^{b} are given by [5]:

$$\Omega_{in}^{b} = \begin{bmatrix} 0 & -w_D & w_E & w_N \\ w_D & 0 & -w_N & w_E \\ -w_E & w_N & 0 & w_D \\ -w_N & -w_E & -w_D & 0 \end{bmatrix} \ldots\ldots$$

......(2)

$$\Omega_{ib}^{b} = \begin{bmatrix} 0 & w_Y & -w_P & w_R \\ -w_Y & 0 & w_R & w_P \\ w_P & -w_R & 0 & w_Y \\ -w_R & -w_P & -w_Y & 0 \end{bmatrix} \ldots\ldots$$

......(3)

and

$$\begin{bmatrix} w_N \\ w_E \\ w_D \end{bmatrix} = \begin{bmatrix} \left(|w_{ie}| + \dot{l}\right)\cos L \\ -\dot{L} \\ -\left(|w_{ie}| + \dot{l}\right)\sin L \end{bmatrix} \ldots\ldots$$

.......(4)

where

$[L,\ l,\ h]$: are geodetic positions (latitude, longitude, and height)

w_R, w_P, w_Y are the body angular velocities in the body coordinate (roll, pitch, and yaw), respectively.

Body fixed coordinate to navigation coordinate (C_b^n) can be described in terms of the quaternion parameters [6]

$C_b^n =$

$$\begin{bmatrix} u_0^2 + u_1^2 - u_2^2 - u_3^2 & 2(u_1u_2 - u_0u_3) & 2(u_1u_3 + u_0u_2) \\ 2(u_0u_3 + u_1u_2) & u_0^2 - u_1^2 + u_2^2 - u_3^2 & 2(u_2u_3 - u_0u_1) \\ 2(u_1u_3 - u_0u_2) & 2(u_0u_1 + u_2u_3) & u_0^2 - u_1^2 - u_2^2 + u_3^2 \end{bmatrix}$$

........(5)

The differential equations of the vehicle position in terms of latitude, longitude, and heading can be arranged in matrix form instead of set of equations as described in [6, 7]:

$$\begin{bmatrix} \dot{L} \\ \dot{l} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} 1/(R_N + h) & 0 & 0 \\ 0 & 1/\left((R_N + h)\cos L\right) & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}$$

........(6)

where

$[V_N\ V_E\ V_D] = V^n$: geodetic velocity vector (north, east, and down)

R_N and R_E: are the radii of curvature in the north and east direction and given by [8]:

$$R_N = \frac{r_e}{\left(1 - e^2 \sin^2(L)\right)^{1.5}} \quad \ldots..(7)$$

$$R_E = \frac{r_e}{\sqrt{\left(1 - e^2 \sin^2(L)\right)}} \quad \ldots\ldots.(8)$$

and e : eccentricity (= 0.0818)

The differential equations relating the second derivative of the geodetic position and velocities can be derived as [5]:

$$\begin{bmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{bmatrix} = C_b^n \cdot f^b + \begin{bmatrix} -\left[\dfrac{V_E}{(R_E + h)\cos L} + 2w_e\right] V_E \sin L + \dfrac{V_N V_D}{(R_N + h)} \\ \left[\dfrac{V_E}{(R_E + h)\cos L} + 2w_e\right] V_N \sin L + \dfrac{V_E V_D}{(R_E + h)} + 2w_e V_D \cos L \\ -\dfrac{V_E^2}{(R_E + h)} - \dfrac{V_N^2}{(R_N + h)} - 2w_e V_E \cos L + g_e \end{bmatrix}$$

.........(9)

where

f^b : Specific force outputs in the body coordinate = $[f_x\ f_y\ f_z]^T$

g_e : Gravity force applied on down direction

Gravity force (g_e) can be found from initial gravity g_0 [3, 9]:

$$g_0 = 9.780327\begin{bmatrix} 1 + 0.0053024\sin^2(L) - \\ 0.0000058\sin^2(2L) \end{bmatrix} \cdots$$

.......(10)

and

$$g_e = g_0 - \left[3.0877\times10^{-6} - 0.0044\times10^{-6}\sin^2(L)\right]h + 0.072\times10^{-12}h^2$$

......(11)

Equations (1, 6, and 9), represent the mechanization equation for the terrestrial navigation system. The following sections will give the details of the developed algorithm to solve the above equations.

3. Initialization

Initial velocities and positions in ECEF coordinate system are loaded to missile computer before missile launch, where the data should be loaded to the missile processor through launcher computer.

Initial Position and velocity

Missile initial position in ECEF coordinate system is computed from [8]:

$$\left.\begin{aligned} X_e(0) &= \left(R_E + H_0\right)\cos l_0 \cos L_0 \\ Y_e(0) &= \left(R_E + H_0\right)\sin l_0 \cos L_0 \\ Z_e(0) &= \left[R_E\left(1 - e^2\right) + H_0\right]\sin L_0 \end{aligned}\right\}$$

.......(12)

while initial velocity are zero in ECEF frame.

Initial quaternion parameters

By applying the transformation matrix [8]

$$C_b^n = \begin{bmatrix} 0 & \sin\chi & \cos\chi \\ 0 & \cos\chi & -\sin\chi \\ -1 & 0 & 0 \end{bmatrix} \ldots\ldots(13)$$

Euler angles directly obtained by applying the equations (A11-A13).

And initial quaternion can computed from [8]:

$$\left.\begin{aligned} u_0 &= \cos(\Psi/2)\cos(\Theta/2)\sin(\Phi/2) + \sin(\Psi/2)\sin(\Theta/2)\cos(\Phi/2) \\ u_1 &= \cos(\Psi/2)\sin(\Theta/2)\cos(\Phi/2) - \sin(\Psi/2)\cos(\Theta/2)\sin(\Phi/2) \\ u_2 &= \cos(\Psi/2)\sin(\Theta/2)\sin(\Phi/2) + \sin(\Psi/2)\cos(\Theta/2)\cos(\Phi/2) \\ u_3 &= \cos(\Psi/2)\cos(\Theta/2)\cos(\Phi/2) - \sin(\Psi/2)\sin(\Theta/2)\sin(\Phi/2) \end{aligned}\right\}$$

.........(14)

Initial gravity

Initial gravity, which effect on the Z_e-axis in ECEF frame is computed by executing Eq (10).

4. *Updating Algorithm*

Update quaternion parameters

Gyros outputs (w_R, w_P, and w_Y) and equations (2, 3, and 4) are used to solve the Eq.(1).

Compute transformation matrices (C_b^n)

Using the above computed quaternion parameters, and by applying the algorithm described in [5], the transformation between the body and the navigation coordinate can be found. This algorithm have more accurate results and more reliable for real time applications then matrix described in Eq.(5).

Compute missile velocity and position

Missile velocity in navigation frame (V_N, V_E, and V_D) is determined by integrating the Eq.(9), where gravity vector computed by applying equations.(10 and 11).

Missile position in ECEF coordinate is computed using Eq.(6).

Updating gravity force and other parameters

Missile position (latitude, longitude, and altitude) used to find the curvature in the north and east direction using equations (7) and (8), and gravity force from the equations (10) and (11).

The above steps are summarized as a flow chart described in figure (1).

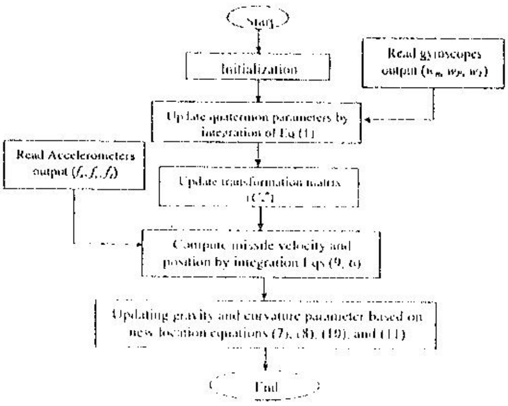


Figure (1): Flow diagram of terrestrial navigation algorithm

5 Algorithms Evaluations:

The data of figures (2) and (3), which represent the accelerometers and gyros outputs for a six-degree missile simulator, are used as inputs to navigation algorithm described in previous sections. The initial position is taken at point (33.310° latitude, 44.4038° longitude, and 0 m altitude). These values used in the terrestrial navigators to predict position time elapsed. The comparison between a true velocity and position of typical trajectory and the terrestrial INS without error velocity and position output are shown in figure (4 a and b). While figure (5 a, b, and c) show the comparison of velocity in X-axis, Y-axis, and Z-axis between the true and celestial INS output without error. Figure (6 a, b, and c) show the comparison of position in X-axis, Y-axis, and Z-axis between the true and calculated output without error.

Figure (7 a and b)) shows the comparison between the true velocity and position of typical trajectory and the algorithm output with error velocity and position output.

From these figures, we note that the developed algorithm is perfect without any errors. While we know that there exist errors in any INS sensors system and that errors (which depend on type of INS) will be grows as time increase. The above figures are based on several source errors, as shown in table (1).

6. Conclusions

The following points summarize the main conclusions in this work:

1. A pure INS implementation can suffer from unbounded growth in the position and the velocity error due to the integration of inertial measurements that will contain various forms of error.
2. The accuracy of the inertial strapdown guided missile system is sensitive for the gyro drift rate, therefore when the gyro drift rate is zero the corresponding error in the computation of the actual range will be also zero. Also the accuracy of the inertial strapdown guided missile system is sensitive for the bias error in the accelerometers.
3. At the end of this paper, we must stress that the strapdown navigation computer should be fast enough to do all the strapdown calculations in a few milliseconds during the flight, making this system to operate within a real-time mode; therefore the speed of the processor, the access time of the RAM with its capacity, and the type of the programming language which will be used in the building of the software, all these features must be taken carefully into account when we are building this real-time system.

5. Future Works

Future work may be directed in the following directions:

1. Further algorithms can be developed to make synthesized

practical strapdown system. Such algorithms include startup, self-test. sensor calibration... etc.

2. The software can be improved to achieve all the requirements of the modern guidance theories. which need more flexible programming language. to get the high performance and accuracy for the guided missile systems.
3. The pipeline technique for the navigation computer can be used to reduce the operating time foe the guided missile systems with very high degree of accuracy.
4. Increase the accuracy by using the aided INS system, such as Global Position System (GPS). And to lead the slowest possible growth of navigation error in between. GPS updates using any estimation technique, such as Kalman f lter.

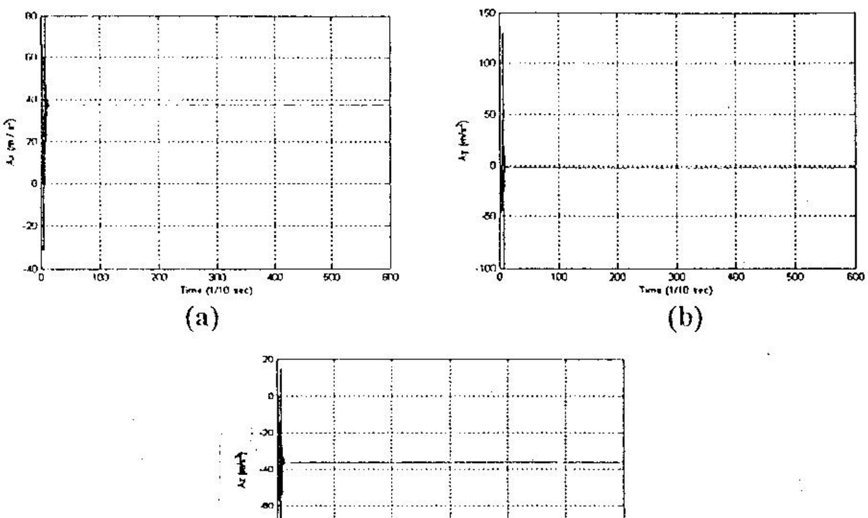


Figure (2): Accelerometers outputs (a) Ax, (b) Ay, and (c) Az

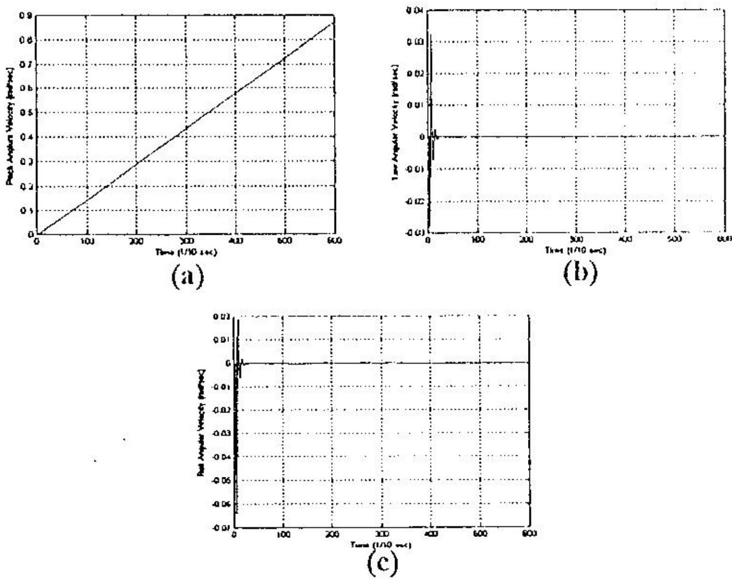


Figure (3): Gyros outputs (a) Pitch, (b) Yaw, and (c) Roll

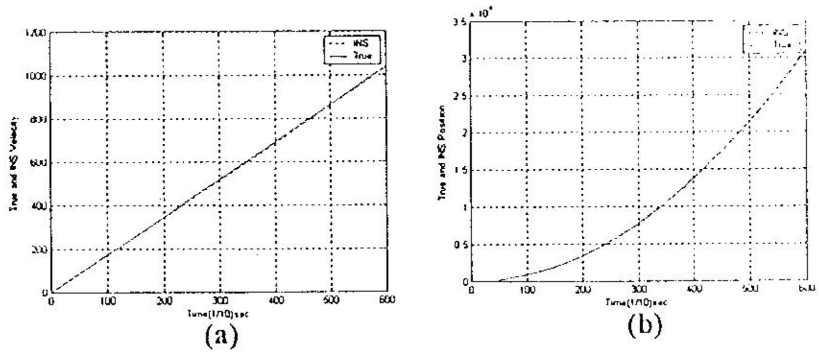
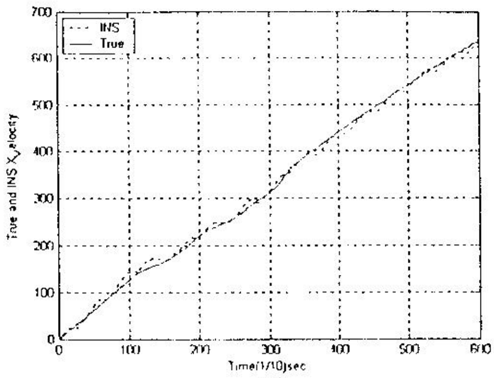
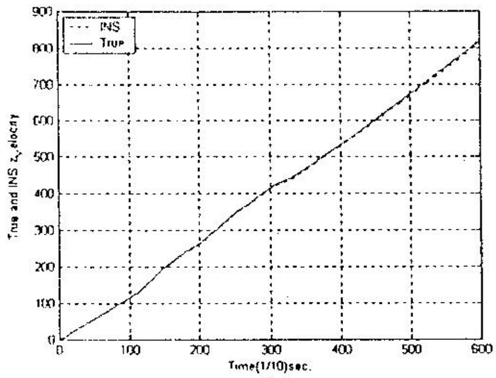


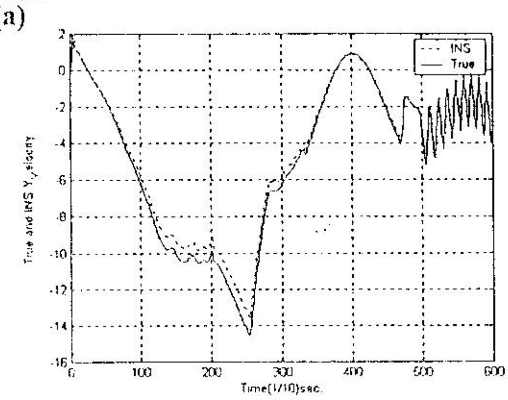
Figure (4): Comparison between True and Terrestrial INS without error (a) Velocity (b) Position



(a)

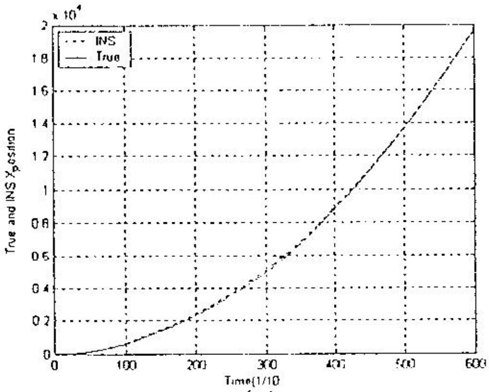


(b)

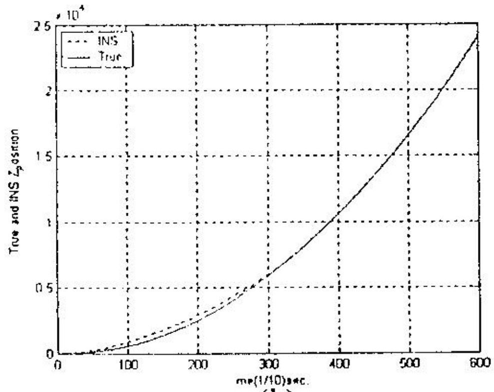


(c)

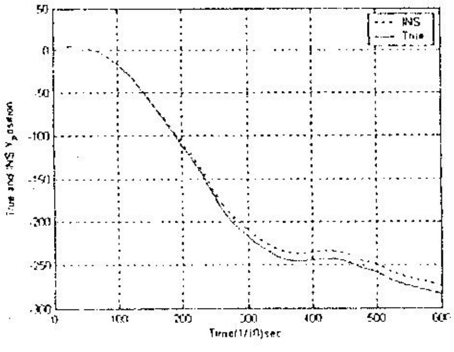
Figure (5): Comparison between True and Terrestrial INS without error (a) Velocity in X-axis (b) Velocity in Z-axis (c) Velocity in Y-axis



(a)

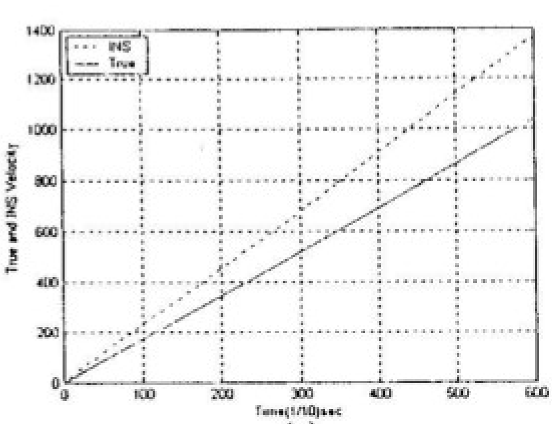


(b)

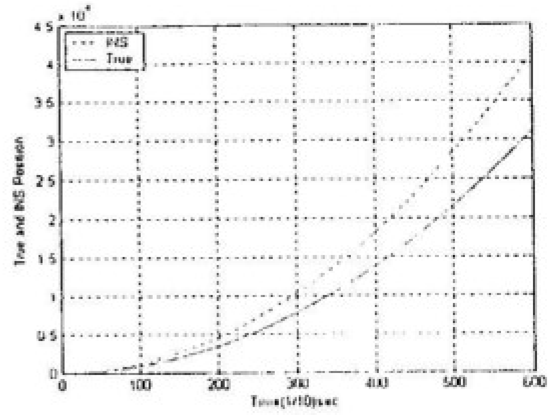


(c)

Figure (6): Comparison between True and Terrestrial INS without error (a) Position in X-axis (b) Position in Z-axis (c) Position in Y-axis



(a)



(b)

Figure (7): Comparison between True and Terrestrial INS with error (a) Velocity (b) Position

Gyro Errors	*Accelerometer Errors*
Constant = $12^0/h$ **g dependent** = $2^0/h/g$ **Random** = $10^0/h$	**Bias** = $0.001\times g$ **Scale factor** = 5% **Random** = $2g\times10^{-5}$

Table (1): Gyros and Accelerometers errors

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Appendix A : Reference Frame and its Transformation

There are so many frames needed in navigation according to the requirements f he pproach roblem. These frames are orthogonal (right-handed coordinate frames). A brief description of these frames and its transformations are given below [1, 3].

A.1 Inertial Frame (i-frame: x_i, y_i, z_i)

A frame that has its origin in the mass center of the earth and is non-rotating relative to the stars can be considered as an inertial frame for the measurements made in the vicinity of the earth, where:

0 = At the center of the earth

x_i = Initially through intersection of equator and Greenwich meridian in the direction of sky at time Go-inertial (close to launch-time).

y_i = Completes right-hand triad.

z_i = Coincident with the earth axis in the direction of north.

A.2 Earth-Centered-Earth-Fixed Frame (e-frame; x_e, y_e, z_e)

It is also called ECEF (Earth Center Earth Fixed) frame. This frame is similar to the inertial frame but it is fixed in the earth and rotate with it, where:-

0 = At the center of the earth

x_e = Through the intersection of the equator and Greenwich meridian in the direction of sky.

y_e = Completes right-hand triad.

z_e = Coincident with the earth axis in the direction of north.

Inertial to earth-fixed and reverse transformation matrices (C_i^e) are defined by earth rotation angle [3]:

$$\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = C_i^e \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \text{(A-1)}$$

$$C_e^i = \begin{bmatrix} \cos(w_e t) & \sin(w_e t) & 0 \\ \sin(w_e t) & \cos(w_e t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \ldots \text{(A-2)}$$

$$C_i^e = (C_e^i)^T \quad \ldots \text{(A-3)}$$

Where

w_e = Earth angular velocity (7.2921×10^{-5} rad / sec).

C_i^e= Transformation matrix from the inertial to Earth frame.

C_e^i= Transformation matrix from the Earth to inertial frame.

A.3 Navigation Frame (n-frame; x_n, y_n, z_n)

Is a local navigation frame which has its origin conceding with that of the sensor fiame and it has the north, east, and down direction. The north direction is pointing to the North pole of the earth, the down direction is normal line to the local point on the reference ellipsoid, the east direction completes the right-hand orthogonal set; where:

0 : At the vehicle center of mass projection to the earth surface

x_n = N : North direction

y_n = E : East direction

z_n = D : Directed to the center of the earth

The transformation matrix which converts from the *e*-frame to the *n*-frame (C_e^n) can be expressed by geographic position angles [4]:

$$\begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} = C_e^n \begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} \ldots \text{(A-4)}$$

$$C_n^e = \begin{bmatrix} -\sin l \cos V & -\sin V & -\cos l \cos V \\ -\sin l \sin V & \cos V & -\cos l \sin V \\ \cos l & 0 & -\sin l \end{bmatrix} \dots (A\text{-}5)$$

$$C_e^n = (C_n^e)^T \quad \dots (A\text{-}6)$$

Where

$\Delta l = l - l_0$

$l_0 =$ The initial geographical longitude.

$l =$ The geographical longitude.

$L =$ The geographical latitude.

And can be calculated from [1]

$$L = \tan^{-1} \frac{Y_e}{X_e} \dots (A-7)$$

$$l = \tan^{-1} \frac{Z_e}{\sqrt{X_e^2 + Y_e^2}} \dots (A-8)$$

A.4 Body Frame (b-frame; R, P, Y):

It is a frame that is fixed to the vehicle. Therefore it is the same angular rotations of the vehicle. i.e., roll, pitch, and yaw. The roll axis is pointing forward, the pitch axis is pointing out right-hand side and yaw axis is pointing downward (all with respect to the vehicle). as shown in figure (A-1).

The transformation matrix relationship from body to navigation frames (C_b^n) can be obtained from Euler angles. [1, 3]:

$$C_n^b = \begin{bmatrix} \cos\Psi\cos\Theta & \sin\Psi\cos\Theta & -\sin\Theta \\ -\sin\Psi\cos\Phi + \cos\Psi\sin\Theta\sin\Phi & \cos\Psi\cos\Phi + \sin\Psi\sin\Theta\sin\Phi & \cos\Theta\sin\Phi \\ \sin\Psi\sin\Phi + \cos\Psi\sin\Theta\cos\Phi & -\cos\Psi\sin\Phi + \sin\Psi\sin\Theta\cos\Phi & \cos\Theta\cos\Phi \end{bmatrix}$$

... .(A-9)

Where

Φ, Θ, and ψ = Euler angles (roll, pitch, and yaw angles, respectively)

also,

$C_b^n = (C_n^b)^{-1}$ (A-10)

The Euler angles can also be determined from the DCM by the following equation [7]:

$$\Theta = -\tan^{-1}\left(\frac{c_{31}}{\sqrt{1 - c_{31}^2}}\right) \dots\dots (A-11)$$

$$\Phi = atan2\left(c_{32}, c_{33}\right) \dots\dots (A-12)$$

$$\Psi = atan2\left(c_{21}, c_{11}\right) \dots\dots (A-13)$$

Where

c_{ij}'s, $1 \le i, j \le 3$ -- are the (i, j)[th] elements of DCM (C_b^n).

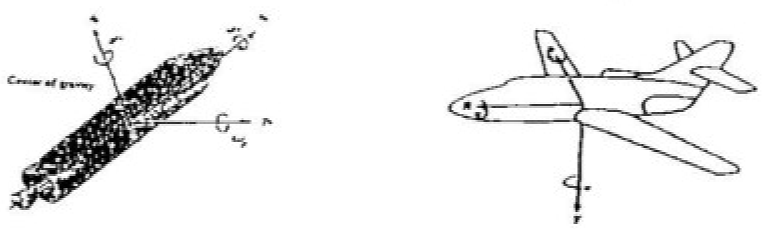
atan2 = is the four quadrant inverse tangent funct on.

Euler angles directly related with angular velocity of the missile (gyros outputs) by [3]:

$$\begin{bmatrix} \dot\Theta \\ \dot\Phi \\ \dot\Psi \end{bmatrix} = \begin{bmatrix} 1 & \sin\Phi\tan\Theta & \cos\Phi\tan\Theta \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \dfrac{\sin\Phi}{\cos\Theta} & \dfrac{\cos\Phi}{\cos\Theta} \end{bmatrix} \begin{bmatrix} w_R \\ w_P \\ w_Y \end{bmatrix}$$

.... (A-14)

Where w_R, w_P, and w_Y = roll rate, pitch rate, and yaw rate angles, respectively of the missile body.



تطوير خوارزمية Strapdown من الدرجة السادسة من الحرية لمنظومة INS من النوع الارضي

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الخلاصة

تحتاجُ العديد منْ أنظمةِ الصاروخ الموجّهِ الدقيقة لإستعمال الحسابات الرياضيةِ الأكثر تعقيدا وتتطلبُ معالجة بسرعةٍ عالية لضمان العمليةِ الفورية في تحديد الموقع. من هنا جائت الحاجة الماسة لتطوير نظامَ حاسبات سريع وكفوء ويعمل بالزمن الحقيقي مَع خوارزمياتَ تحديد الموقع الحديثة. في هذا العمل، طور برنامج مرن لمحاكاة خوارزميةَ الملاحة الأرضية. طوّرَ وطبّق للدرجةِ السادسة منْ محاكي طيران قذيفة. إنّ تقييمَ الخوارزمية مستند على دقةِ الخوارزمية المُقترحةِ بالبيانات الحقيقية.