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The Effective M3Y Residual Interaction In ^{41}Ca As a Nuclear Diffraction Grating of Electrons

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Abstract:

The total and individual multipole moments of magnetic electron scattering form factors in ^{41}Ca have been investigated using a widely successful model which is the nuclear shell model configurations keeping in mind of $1f_{7/2}$ subshell as an L-S shell and Millinar, Baymann, Zamick as L-S shell (F7MBZ) to give the model space wave vector. Also, harmonic oscillator wave functions have been used as wave function of a single particle in $1f_{7/2}$ shell. Nucleus ^{40}Ca as core closed and Core polarization effects have been used as a corrective with first order correction concept to basic computation of L-S shell and the excitement energy has been implemented with 2ħω. The core polarizability effect has been utilized to incorporate the rejected space (core + higher arrangement) via L-S shell with a realism interaction of effective M3Y P2 interaction to connect the model space particles in motion with the spouse (p-h). The two body M3Y interactions have been utilized as an interaction residue to calculate the core polarizability matrix elements. Finally, the theoretical result of the form factor has been compared with the experimental results.

Keywords: Electron Scattering, Form Factor, ^{41}Ca, residual interaction.

Introduction:

Electron scattering is a useful technique for the purpose of study the characteristics and structure of target nucleus, elastic electron scattering is the operation in which the nucleus remain in it ground state before and after the scattering and the process is changing the direction of the nucleus spatially, the target stays in its original state just absorbing the recoil momentum and therefore alteration its kinetic energy. The elastic electron scattering is good as it gives details about other aspects such as projectile, target nuclei potential which is important to perform exact theoretical calculations for non-elastic operations and provides a powerful experimental tool for the determination of the charge distributions, radii and transition probabilities [1].

The magnetic electron scattering has supplied information on nuclear current distribution and single particle characteristics of nuclear wave function. The magnetic electron scattering has given specifics on both nucleons, in contrast to dispersion of charges that also is most delicate to proton [2]. ^{41}Ca is the touchstone nucleus to measure the nuclear radius for $1f_{7/2}$ orbit and it is the most important nuclear system because it provides a starting point for microscopic descriptions of nuclei [2]. It is also important to investigate the nuclear spatial distribution of magnetic dipole moments in isotopes more complicated like ^{48}Ca [3].

Shell-model calculations have been used to study the nuclear structure of ^{23}Na, ^{25}Mg, ^{27}AL, ^{41}Ca nuclei (energy level - elastic and inelastic electron-nucleus scattering - probability of the transition). The Wildenthal interaction of sd shell in the proton-neutron formalism, which are universal sd shell potential (A), universal sd shell potential (B), GXFP1 interaction for the fp shell had been utilized with the N-N actual interaction M3Y as a two body potential for Core polarizability analysis. The radial part of the wave vectors had been deduced with oscillator and Woods-Saxon forces. Stats from electron scattering Form Factors construct had been exhibited the Core polarizability effects were required to get an acceptable describe of the experimental measurement with no changeable parameters [4].

Using shell model with effective W0 interaction, elastic magnetic electron scattering form factors and magnetic dipole moments for exotic nuclei of potassium isotopes K (A= 42, 43, 45, 47) have been determined. The single particle wave vectors of harmonic oscillator (HO) potential have been utilizing with oscillator parameters b, based on interaction the valence nucleons were assumed to move in the d3f7 model space [5].

The sensibility of single valence neutron characteristics and elastic magnetic form factors to three different relativisms mean-field (RMF) parameter sets. The wave function of the external most neutron subshell, nuclear density, valence neutron density, magnetic form factors are calculated numerically. With nuclear masses and isospin asymmetry larger than, the separation has been become more visible. The spectroscopic factors for ^{17}O , ^{41}Ca have been measured by fitting the experimental data [6].

It has been shown that a deformed figuration improve concord with experiment in deformed nucleus. While generating spherical nucleus by appropriately considering the spherical limit of the deformed model and the Nucleon-Nucleon interactions effect. The ability of the model to explain magnetic form factor has been deduced [7].

Electron-Nucleus Scattering Form Factors of selected Nucleus of Medium Mass revealed that the commitments of the quadrupole frame components FC2(q) in 93Nb and 115 in cores, which were characterized by the undeformed $3s-1g$ shell demonstrate, were vital for getting a great understanding between the hypothetical and exploratory frame components [8].

Shell model was taken into consideration to determine the electric quadrupole moments for Calcium isotopes ^{20}Ca (N = 21, 23, 25, and 27) in the fp shell. By using the set of charges effective the core excitation effect was obtained utilizing microscopic theory and best results for quadrupole moments are obtained utilizing Bohr-Mottelson (B-M) charges effective. Bohr-Mottelson (B-M) charges effective have been used to study the behavior of Calcium isotopes form factors. Bohr-Mottelson (B-M) charges effective have been used to study the behavior of Calcium isotopes form factors. To obtain the wave vector by utilizing effective potential $fpd6$ and fp space model of the two bodies [9].

Elastic Magnetic form factor from odd-A nucleus has been provided. Plane wave Born approximation has been used in the calculation, The characteristics of the one body have been generated in a deformed self-consistent mean field determined according to a Skyrme HF+BCS theory. Also approximation of the cranked took in to account collective effects and the results from multiple stable nucleuses have been compared to the experimental data [10].

The aim of the present work is to include a new form of M3Y interaction as a residual potential and measure its suitability to reproduce the results under interest.

Theory:

The reduced matrix elements with single particle of the electron scattering measurable $\hat{T}_{\Lambda}^{\eta}$ is informed as the sum of the one body density matrix elements (OBDM) times the single particle transition matrix elements and summed over the production which is given by [9]:

$$\langle \Gamma_f ||| \hat{T}_{\Lambda}^{\eta} ||| \Gamma_i \rangle = \sum_{\alpha,\beta} OBDM\,(\Gamma_i, \Gamma_f, \alpha, \beta)\, \langle \alpha\ ||| \hat{T}_{\Lambda}^{\eta} |||\ \beta \rangle \qquad 1$$

Where β and α represent the initial and final single particle states, correspondingly (isospin is included). The states $|\Gamma_i\rangle$ and $|\Gamma_f\rangle$ are the initial and final states of the nucleus and $\Lambda = JT$ is the multi-polarity. The reduced matrix element with many particle of the electron scattering operator $\hat{T}_{\Lambda}^{\eta}$ combined from two parts, the first is the "zero ħω" matrix element and the other is the "2ħω" matrix element [9]:

$$\langle \Gamma_f ||| \hat{T}_{\Lambda}^{\eta} ||| \Gamma_i \rangle = \langle \Gamma_f ||| \hat{T}_{\Lambda}^{\eta} ||| \Gamma_i \rangle_{MS} + \langle \Gamma_f ||| \delta\hat{T}_{\Lambda}^{\eta} ||| \Gamma_i \rangle_{CP} \qquad 2$$

Where

$\langle \Gamma_f ||| \hat{T}_{\Lambda}^{\eta} ||| \Gamma_i \rangle_{MS}$ is the model space reduced matrix element (zero $\hbar\omega$) .

$\langle \Gamma_f ||| \delta\hat{T}_{\Lambda}^{\eta} ||| \Gamma_i \rangle_{CP}$ is the discarded L − S shell reduced matrix element .

$|\Gamma_i\rangle$ and $|\Gamma_f\rangle$ are the first and second states of the nucleus , correspondingly.

The theory of perturbation with first order shows that the matrix elements of single particle for the higher excited state can be written as [9]:

$$\langle \alpha | \delta\hat{T}_J^{\eta} | \beta \rangle = \left\langle \alpha \left| V_{res} \frac{Q}{E - H^{(0)}} \hat{T}_J^{\eta} \right| \beta \right\rangle + \left\langle \alpha \left| \hat{T}_J^{\eta} \frac{Q}{E - H^{(0)}} V_{res} \right| \beta \right\rangle \qquad 3$$

The single particle energies are calculated according to [9] :

$$e_{nlj} = \left(2n + l - \frac{1}{2}\right)\hbar\omega + \begin{cases} -\frac{1}{2}(l+1)\langle f(r)\rangle_{nl} & for\ j = l - \frac{1}{2} \\ \frac{1}{2} l \ \langle f(r)\rangle_{nl} & for\ j = l + \frac{1}{2} \end{cases} \quad 4$$

With:

$$\langle f(r)\rangle_{nl} \approx -20A^{-2/3}\ MeV \quad 5$$

$$\hbar\omega = (45/A^{1/3} - 25/A^{2/3})\ MeV \quad 6$$

The matrix elements of single particle that have been reduced in spin and isospin are written in terms of the matrix elements of single particle that have been reduced in spin [9]:

$$\langle\alpha \| |\hat{T}^{\eta}_{\Lambda}| \| \beta\rangle = \sqrt{\frac{2T+1}{2}} \sum_{t_z} I_T(t_z) \langle j_2 \| |\hat{T}^{\eta}_{\Lambda}| \| j_1\rangle \quad 7$$

With

$$\sum_{t_z} I_T(t_z) = \begin{cases} 1 & for\ \ T = 0 \\ (-1)^{\frac{1}{2}-t_z} & for\ \ T = 1 \end{cases} \quad 8$$

Where $t_z = \frac{1}{2}$ for proton and $t_z = -\frac{1}{2}$ for neutron.

For the matrix elements of two bodies of the residual interaction $\langle\alpha\alpha_1|V_{res}|\beta\alpha_2\rangle_\Gamma$ and $\langle\alpha\alpha_2|V_{res}|\beta\alpha_1\rangle_\Gamma$ which are given in the Eq. (3). For the core polarization two body potential, the M3Y interaction [11] is adopted. scattering form factors which includes total spin(J) and (q) between first and second excited states of total angular momentum $J_{i,f}$ and charge space spin $T_{i,f}$ respectively are [2] :

$$|F^{\eta}_J(q)|^2 = \frac{4\pi}{Z^2(2J_i+1)} \times \left| \sum_{T=0,1} (-1)^{T_f - T_{Z_f}} \begin{pmatrix} T_f & T & T_i \\ -T_{Z_f} & M_T & T_{Z_i} \end{pmatrix} \langle J_f T_f \| |\hat{T}^{\eta}_{JT}| \| J_i T_i\rangle \right|^2 \times |F_{c.m}(q)|^2 |F_{f.s}(q)|^2 \quad 9$$

Where $|F_{c.m}(q)|^2$ *and* $|F_{f.s}(q)|^2$ are the center of mass correction and finite size correction respectively.

The M3Y functional N-N force, yielded from a fit of the G-matrix elements produced on Reid and Elliott soft-core N-N potential, in an oscillator waves, is the sum of three Yukawa's potential with spans 0.25 fm for a medium span attractive part, 0.4 fm for a short range repulsive part and 1.414 fm to ensure a long range tail of the one-pion exchange potential (OPEP). The widely utilized form of the M3Y effective interaction $v_{eff}(r)$ is given by [11]:

$$v_{eff}(r) = 7999\frac{e^{-4r}}{4r} - 2134\frac{e^{-2.5r}}{2.5r} \quad 10$$

The Eq. (10) reflects the spin and isospin independent terms of the central component of the effective N-N potential, and that the One Pion Exchange Potential objectives are absent here.

On the basis of the σ and ω meson fields of the Relativistic Mean Field Lagrangian, as:

$$v_{eff}(r) = \frac{g^2_\omega}{4\pi} B_\omega \frac{e^{-m_\omega r}}{r} - \frac{g^2_\sigma}{4\pi} A_\sigma \frac{e^{-m_\sigma r}}{r} \quad 11$$

The Equation (11) could be related to the phenomenological M3Y effective N-N potential in Equation (10). In Equation (11), $m_\omega, m_\sigma, g_\omega, g_\sigma$ are, correspondingly the masses and correlation constants of the ω and σ mesons. The parameters $B_\omega = \left(1 + \frac{1}{2}\left(\frac{m_\omega}{M}\right)^2\right)$ and $A_\sigma = \left(1 - \frac{1}{4}\left(\frac{m_\sigma}{M}\right)^2\right)$ are dependent on relativistic corrections $\left(\frac{m_i}{M}\right)^2$; $i = (\omega, \sigma)$, which is useful for the field energies m_i becoming equivalent to nucleon mass M [11].

Result and Discussion:

Elastic magnetic electron scattering form factors for ^{41}Ca with ^{40}Ca as inert core, F7MBZ has been utilized to construct model space factor taking into account single particle potential (Harmonic oscillator) and all the theory are analyzed through nuclear shell theory. The discarded space inclusion which is core excitation with modern effective M3Y N-N potential to couple p-h pair via model space and the generated wave function for $1f_{7/2}$ model space and it OBDM are calculated using OXBASH code . From Fig.1 which represents M1 scattering pattern for model space (blue), core polarization (red), total M1 form factor (green) it is clear that the three contributions are somehow in face specially at low momentum transfer q < 1.5 fm^{-1} and the dominant is the core contribution over the model space and from the value of total form factor they are (core and model space) interfer distractively and the diffraction minima between three contributions deviated as momentum transfer increase.

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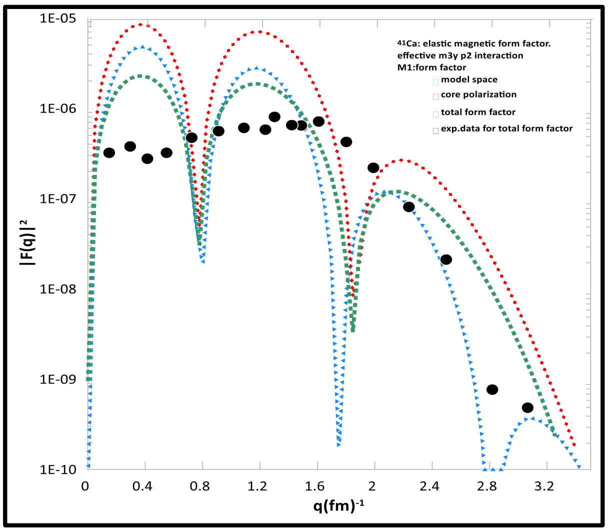

Figure1. Elastic magnetic form factor M1 in ^{41}Ca with model space contribution and core polarization effect .The experimental results are from [4].

In an another picture the behaviour of M3 form factor is nearly as same as that of M1 but for the second lobe in Fig.2 the contributions of the two spaces are rippled were the first lobe reveals for the first lobe the dominate is model space over the core part and the surprizing that the total form factor is resulted destructively for the first lobe and constructively for the second lobe. The model space contribution is very weak in the region for a momentum transfer (q) less than 2.5 fm^{-1} .

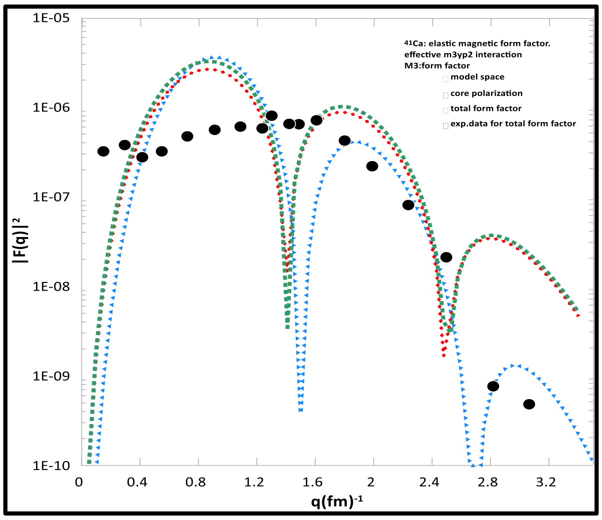

Figure 2. Elastic magnetic form factor M3 in ^{41}Ca with model space contribution and core polarization effect .The experimental results are from [4].

In Fig.3 M5 form factor is shown, the model space is constructively with core part so that the total form factor of M5 is larger than the two contributions starting from q = 0 fm^{-1} to q = 3.5 $fm^{-1}$and the model space part is too weak with respect to core part in the region q = 2.2 fm^{-1}.

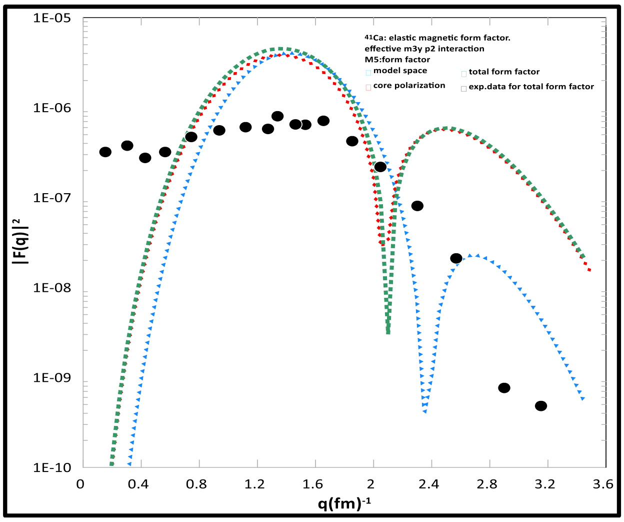

Figure 3. Elastic magnetic form factor M5 in ^{41}Ca with model space contribution and core polarization effect. The experimental results are from [4].

Finally, the form factor of M7 is shown in Fig.4 where two contributions interfere constructively and they are in face. From the Fig.4 the values of $F(q)^2$ are between 10^{-10} and 10^{-5} they are (M1, M3, M5, M7) they are different in three diffraction patterns.

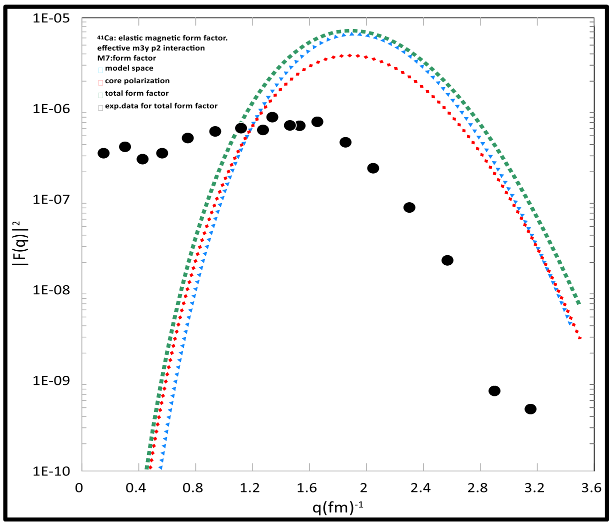

Figure 4. Elastic magnetic form factor M7 in ^{41}Ca with model space contribution and core polarization effect. The experimental results are from [4].

For Fig.5 which represents the total elastic magnetic electron scattering form factor in ^{41}Ca and it is clear that the solid curve shifted

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quantitatively but qualitatively they are coherent and in some extent the coincidences are decreased with the increase of q and that is not surprising if we remember that we choose single orbit rather than fp shell model space and one particle- one hole excitation instead of higher excitation energy (4,6,8 $\hbar\omega$) and the conclusion is that effective M3Y interaction had a fitting parameters need to be readjusted, customarily the use of different interaction might produce a clear picture about the effectiveness of a residual interactions and their impact on core polarization contribution.

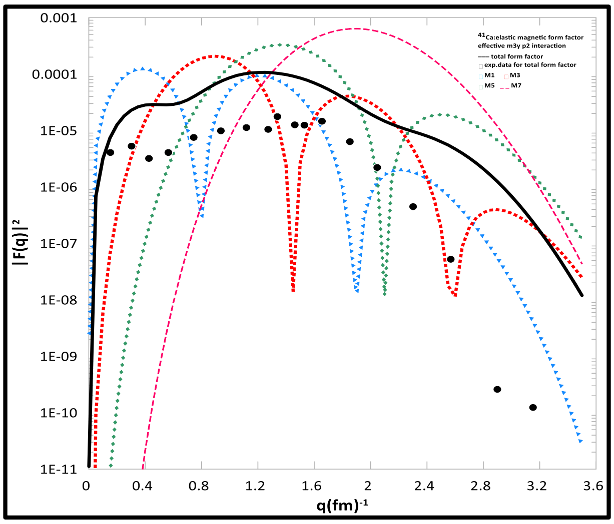

Figure5. Magnetic form factors of ^{41}Ca disassembled into M1, M3, M5 and M7 multipole components.
The experimental results are from [4].

Conclusions:

For all magnetic form factors , they are overestimating and they do not well reproduce the experimental data in comparison with the results published in references (2, 3, 4, 9) and these results prove that the original version of M3Y is more suitable than that reproduced in reference (11) if we remember that the effective M3Y used in this paper eliminates the spin orbit and tensor parts which are very sensitive to magnetic component of electromagnetic interaction between scattered electron and ground state deformed nucleus.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Baghdad.

Authors' contributions statement:

F. Z. M. and R. M. H. contributed to the design and implementation of the research to the analysis of the results and to the writing of the manuscript.

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تفاعل المتبقي المؤثر M3Y في ^{41}Ca كمحزز حيود نووي للإلكترونات

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الخلاصة:

تمت تحقيق عوامل التشكل للاستطارة الالكترونية المغناطيسية المرنة متعددة الاقطاب الكلية والمنفردة لنظير ^{41}Ca باستخدام أنموذج القشرة النووية الناجح و الواسع التطبيق واعتبار الغلاف $1f_{7/2}$ كأنموذج فضاء واعتماد التفاعل F7MBZ كتفاعل مؤثر لتشكيل دوال موجة انموذج الفضاء. اعتمدت دالة المتذبذب التوافقي كدالة جسيم منفرد , كما تم اعتماد نواة ^{40}Ca كقلب خامل حيث يتم استقطابه واشراكه في حسابات عوامل التشكل المغناطسية المرنة من خلال عملية استقطاب القلب حيث يتم خلع نيوكليون من مدارات القلب الخامل وتهييجه الى مستويات عليا وبفرق طاقة 2ħω وكما تم الاعتماد على التفاعل الواقعي (effective M3Y P2) في حسابات استقطاب القلب والذي يفاعل الزوج(جسيم –فجوة) ان هذا التفاعل هو نموذج مطور حديثا للتفاعل الاصلي واخيرا تم مقارنة النتائج النظرية مع ما هو متوفر من نتائج عملية.

الكلمات المفتاحية: استطارة الالكترون ,عامل التشكل, ^{41}Ca ,تفاعل المتبقي .