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An Application of the Banzhaf Values for Cooperating Among Producers of Waste Processing in the Al-Mahmudiya Factory

Huda Hadi 

Jabbar Abbas* 

Department of Applied Sciences, University of Technology, Baghdad, Iraq

*Corresponding author: jabbar.a.ghafil@uotechnology.edu.iq

E-mail addresses: as.18.98@grad.uotechnology.edu.iq

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Abstract:

The game theory has been applied to all situations where agents' (people or companies) actions are utility-maximizing, and the collaborative offshoot of game theory has proven to be a robust tool for creating effective collaboration strategies in a broad range of applications. In this paper first, we employ the Banzhaf values to show the potential cost to waste producers in the case of a cooperation and to reduce the overall costs of processing non-recyclable waste during cooperation between producers. Secondly, we propose an application of the methodology to study a case for five waste producers' waste management in the Al-Mahmudiya factory with the aim of displaying the potential cost to waste producers in case of cooperation. Lastly, the obtained results of the proposed framework will strongly help professionals to formulate and improve well-organized strategies for the waste management system of the future.

Keywords: Banzhaf value, Cooperative game, Decision-making process, Game theory, Waste management.

Introduction:

The waste issue is one of the thorny issues that hinder development plans in many countries, as it is necessary to highlight and build an integrated system to achieve the maximum possible benefit from it. Therefore, most developed countries have had to establish a modern manufacturing system based on decreasing the participation of fresh input factors and recycling of waste.

As not all waste is recycled in waste recycling plants, there must be cooperation between waste producers to reduce their overall costs of treating non-recyclable waste.

Many researchers have published several papers on this issue. The following is some of the previous research in the context of waste management: Cheng et al. ¹ integrated multi-criteria decision analysis and linear programming to optimize the selection of a landfill site and the waste flows costs considered in the waste management problem; Karmperis et al. ² reviewed multi-criteria decision analysis model and introduced the waste management bargaining game to support decision-makers; He et al. ³ proposed an agent-based waste treatment model as a complex adaptive system, and private operators set gate fees to vie for waste in

low-information competition; Nguyen-Trong et al. ⁴ modeled the optimization of collection services on transportation of municipal solid waste; Feyzi et al. ⁵ provided a new MCDM framework and evaluated sustainability-based criteria to select the most appropriate site for solid waste incineration power plants.

The collaborative offshoot of game theory has proven to be a robust tool for creating effective collaboration strategies in a broad range of applications. The coalitional game framework to waste producers' dispute has been introduced in ⁶⁻⁷, and three application-oriented types of cooperative games in the waste management problems and its comparison have been implemented in ⁸. The Banzhaf value ⁹ for cooperative games can be used as a solution concept and to evaluate the cooperation existing among players. To name a few, the application of the Banzhaf value to communication situations has been given by Alonso-Meijide and Fiestras-Janeiro ¹⁰, Gallego et al. ¹¹ to Compute the power of the political groups in the European Parliament, and Fragnelli and Pusillo ¹² applied the Banzhaf value for detecting abnormally expressed genes.

In recent literature, the generalization of Banzhaf value and its possible applications have been discussed by various researchers¹³⁻¹⁶. The interaction phenomena among a set of players, which can be seen as an extension of the notion of value, has been applied to multi-criteria decision-making in the framework of aggregation by the Choquet integral¹⁷⁻²⁰.

The aim of this research is to apply a mathematical model based on cooperative game theory to model cooperation between producers in waste management. First, the Banzhaf value is used to show the potential cost to waste producers in the case of cooperation and to reduce the overall costs of processing non-recyclable waste during a cooperation between producers. Then, the methodology is used to study a case for the processing of waste management in Baghdad. The importance of the study is the application of cooperative games in waste management problems enabling analysis and prediction of waste producers' behavior which will strongly help professionals to formulate and improve well-organized strategies for the waste management system of the future.

The rest of the paper is organized as follows. Section 2 provides an overview of the Banzhaf value. A mathematical model for decision-making in waste management is described in Section 3. Section 4 introduces a case study for the process of waste management in Baghdad. Finally, Section 5 concludes the paper with some conclusions.

The Banzhaf Value:

The Banzhaf value presents a sharing structure that poises cooperation power in the cooperative game problems. This section is for explicit analytical definitions of Banzhaf value⁴, and the one-point solution is usually called power index.

Let $N = \{1, \dots, n\}$ be a finite set of players and $G(N)$ be a set of the power set of N . Each non-empty subset of N is called a coalition. The set N is referred to as the grand coalition. For each coalition $S \subseteq N$, a worth function $v(S)$ gives the payoffs that any coalition of players can get. This function $v(S)$ is called the characteristic function of the game. For the empty \emptyset coalition, the characteristic function $v(\emptyset) = 0$. A cooperative game is generally defined by a pair (N, v) , where N is a set of players and v is a coalition value function.

Definition 1:⁹ For the player $i \in N$, the Banzhaf value $\varphi_i(N, v)$, is a value on cooperative game (N, v) defined by

$$\varphi_i(N, v) = \sum_{S \subseteq N \setminus i} \frac{1}{2^{n-1}} (v(S \cup i) - v(S)), \quad 1$$

Note that, when $i \notin N$, implies $\varphi_i(N, v) := 0$.

Example 1: Consider a four-person game with $N = \{1, 2, 3, 4\}$ where the characteristic functions are given as:

$$\begin{aligned} v(\emptyset) &= v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\{4\}) = 0, \\ v(\{1, 2\}) &= 4, \quad v(\{1, 3\}) = 4, \\ v(\{1, 4\}) &= 4, \quad v(\{2, 3\}) = 6, \quad v(\{2, 4\}) = 6, \\ v(\{3, 4\}) &= 4, \quad v(\{1, 2, 3\}) = 6, \\ v(\{1, 2, 4\}) &= 8, \quad v(\{1, 3, 4\}) = 10, \\ v(\{2, 3, 4\}) &= 10, \\ v(\{1, 2, 3, 4\}) &= 12. \end{aligned}$$

The Banzhaf value for player 1 can be computed as follows:

$$\begin{aligned} \varphi_1(N, v) &= \sum_{S \subseteq N \setminus 1} \frac{1}{2^{n-1}} (v(\{S \cup 1\}) - v(\{S\})), \\ \varphi_1(N, v) &= \frac{1}{2^{4-1}} (v(\{1\}) - v(\{\emptyset\})) \\ &\quad + \frac{1}{2^{4-1}} (v(\{1,2\}) - v(\{2\})) \\ &\quad + \frac{1}{2^{4-1}} (v(\{1,3\}) - v(\{3\})) \\ &\quad + \frac{1}{2^{4-1}} (v(\{1,4\}) - v(\{4\})) \\ &\quad + \frac{1}{2^{4-1}} (v(\{1,2,3\}) - v(\{2,3\})) \\ &\quad + \frac{1}{2^{4-1}} (v(\{1,2,4\}) - v(\{2,4\})) \\ &\quad + \frac{1}{2^{4-1}} (v(\{1,3,4\}) - v(\{3,4\})) \\ &\quad + \frac{1}{2^{4-1}} (v(\{1,2,3,4\}) - v(\{2,3,4\})) = \frac{22}{8}. \end{aligned}$$

Also, the Banzhaf values for the other players (2, 3, and 4), are as follows.

$$\varphi_2(N, v) = \frac{30}{8}, \quad \varphi_3(N, v) = \frac{32}{8}, \quad \varphi_4(N, v) = \frac{34}{8},$$

Therefore, the Banzhaf vector $\varphi_B(N, v)$ for players have been found as

$$\varphi_B(N, v) = (\varphi_1, \varphi_2, \varphi_3, \varphi_4) = \left(\frac{22}{8}, \frac{30}{8}, \frac{32}{8}, \frac{34}{8} \right).$$

A Mathematical Model for Decision-Making in Waste Management

This section describes a Mathematical model for decision-making in waste management by using game theory. According to the definition of

cooperative game theory, this theory concentrates on mathematical models of interaction indices among rational participants (the players) of the modeled dispute (the game) ²¹.

In waste management, the common dispute of waste producers can be formalized as a game with the characteristic function $v(S)$ given by the following. Let $M = \{1, \dots, m\}$ be a set of waste recycle plants; w_1^c, \dots, w_m^c denote their capacities and c_1^g, \dots, c_m^g denote their gate fees. The set of producers is $N = \{1, \dots, n\}$, and their waste productions are w_1^p, \dots, w_n^p . The matrix specifies the transportation costs $[c_{i,j}^t]$, where $c_{i,j}^t$ indicates the cost of transportation from the point of origin to the point of destination (i.e. from the producer $i \in N$ to the plant $j \in M$). The quantity of garbage sent by the manufacturer $i \in N$ to the waste recycle plant $j \in M$ in tonnes is denoted by $x_{i,j}$.

The distinguishing feature of the characteristic function $v(S)$, $S \subseteq N$ is given by the following model as an optimization problem.

Model of the optimization problem

$$v(S) = \text{Min}_{x_{i,j}: i \in S, j \in M} \sum_{j \in M} \sum_{i \in S} (c_{i,j}^t + c_j^g) x_{i,j}$$

Subject to

$$\sum_{i \in S} x_{i,j} \leq w_j^c - \sum_{i \in N \setminus S} x'_{i,j}, \quad \forall j \in M,$$

$$\sum_{j \in M} x_{i,j} = w_i^p, \quad \forall i \in S,$$

$$x_{i,j} \geq 0, \quad \forall i \in S,$$

$$\forall j \in M,$$

Where,

$$\{x'_{i,j}: i \in N \setminus S, \forall j \in M\} =$$

$$\text{Min}_{x_{i,j}: i \in N \setminus S, j \in M} \sum_{j \in M} \sum_{i \in N \setminus S} (c_{i,j}^t + c_j^g) x_{i,j} \text{ Subject to}$$

$$\sum_{i \in N \setminus S} x_{i,j} \leq w_j^c, \quad \forall j \in M,$$

$$\sum_{j \in M} x_{i,j} = w_i^p, \quad \forall i \in N \setminus S,$$

$$x_{i,j} \geq 0, \quad \forall i \in N \setminus S, \forall j \in M.$$

Note that, the characteristic function for the empty coalition equals zero by definition. Waste handling costs are presented as the sum of transportation costs and gate fees multiplied by the amount of treated waste. The central assumption of

the entire model is that all waste generated can be treated by waste recycling plants. The value of a characteristic function $v(S)$ corresponds to the minimum of the total costs of coalition members who have made the correct decision after a coalition of all outsiders has reduced the total cost of the operation. It presents the worst-case scenario cost estimation in a setting with landfills and reflects general principles of decision-maker cooperation in waste producers' disputes. As a result, the above-mentioned characteristic function will serve as the foundation for all game classes evaluated.

In order to fair allocation for the cooperative game problem for the waste producers, the resulting costs are compared using the Banzhaf values with a writing algorithm and computer program (MatLab program) as a working procedure. The following algorithm is the way to compute the values of the characteristic function $v(S)$.

Algorithm 1: Computations $v(S)$

Step1: Let $N = \{1, \dots, n\}$, $M = \{1, \dots, m\}$,

Input: Gate fees (c_1^g, \dots, c_m^g) , and transportation costs $(c_{1,1}^t, \dots, c_{n,m}^t)$,

Step2:

for $i = 1$ to number of players n , $i \in S$,

for all $j = 1$ to m do

Input: $x(i, j)$;

Step3: if

$$x'_{i,j} = \text{Min}_{x(i,j)} \left(\sum_{i \in N} \sum_{j \in M} c_{i,j}^t + c_{i,j}^g \right) x(i, j);$$

end if

end i

end j

Step4: for $i \in S$,

Calculate the characteristic function $v(S)$ using the model of the optimization problem;

end for

Step5: Return to Step2

Case study: The process of waste management in Baghdad

This section applies the mathematical model described in Section 3 to study the waste

management process in Baghdad. Then, Banzhaf values for this study are calculated with the aim of displaying the potential cost to waste producers in case of cooperation and formulating well-structured strategies for the waste management system in the future.

The waste sorting and recycling plant in Al-Mahmudiya (Al-Yusufiyah district) is the only factory in Baghdad governorate that sorts and recycles waste with a design and an actual production capacity of 200 tons /day (8 hours) , with 8 working hours.

In this waste sorting and recycling plant (Al-Mahmudiya factory), there are five the waste producers: The first is AL Central Bank of Iraq piston (CBI), the second is Al-Rawi Water Company piston (AWC), the third is the Directorate of Water and Sewage piston (DWS), the fourth is Asia dyes piston (AD), and the fifth is Dora piston (DP). Therefore, the set of producers is $N = \{1, 2, 3, 4, 5\}$, and there is only one waste recycling plant 1 (i.e. $M = \{1\}$).

Table 1. The data of waste producers in waste sorting and recycling AL-Mahmudiya plant.

W_i^P	c_{ij}^t	c_j^g	w_j^c
$W_1^P = 500kt$	$c_{11}^t = 30$	$c_1^g = 65$	$w_1^c = 500$
$W_2^P = 700kt$	$c_{21}^t = 35$	$c_1^g = 65$	$w_1^c = 500$
$W_3^P = 750kt$	$c_{31}^t = 40$	$c_1^g = 65$	$w_1^c = 500$
$W_4^P = 750kt$	$c_{41}^t = 50$	$c_1^g = 65$	$w_1^c = 500$
$W_5^P = 800kt$	$c_{51}^t = 55$	$c_1^g = 65$	$w_1^c = 500$

The data, with respect to the capacity of the waste sorting and recycling plant, gate fees, waste production, and transportation cost from the point of

origin to the intended destination, were collected from the AL-Mahmudiya factory, which is shown in Table 1, and, as illustrated in Fig. 1.

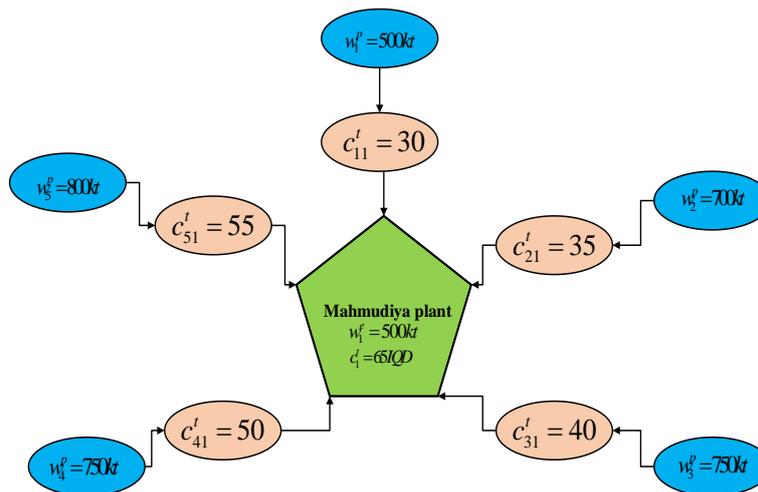


Figure 1. A situation illustrates the waste sorting and recycling plant in AL-Mahmudiya factory.

The characteristic functions for all coalitions of waste producers $v(S)$, $S \subseteq N = \{1,2,3,4,5\}$ are computed by the model of the optimization problem

as mentioned above through the algorithm presented in section 3, and as shown in Table 2.

Table 2. The characteristic function values

Coalitional set of producers	{∅}	{1}	{2}	{3}	{4}	{5}	{1,2}	{1,3}
(N, v)	0	47.5	87.5	86.25	75	84	135	133.75
Coalitional set of producers	{1,4}	{2,3}	{2,4}	{2,5}	{3,4}	{3,5}	{4,5}	{1,5}
(N, v)	122.5	173.75	162.5	171.5	161.25	170.25	159	131.5
Coalitional set of producers	{1,2,3}	{1,2,4}	{1,2,5}	{1,3,5}	{1,3,4}	{1,4,5}	{2,3,4}	{2,4,5}
(N, v)	221.25	210	219	217.75	208.75	206.5	248.75	246.5
Coalitional set of producers	{2,3,5}	{3,4,5}	{1,2,3,4}	{1,2,4,5}	{1,3,4,5}	{1,2,3,5}	{2,3,4,5}	{1,2,3,4,5}
(N, v)	257.75	245.25	296.25	294	292.75	305.25	332.75	380.25

Using Eq. 1, the Banzhaf values for each producer are presented in Table 3. For example, to find Banzhaf value of the waste producer 1 (i. e. $\varphi_1(N, v)$):

$$\varphi_1(N, v) = \sum_{s \subseteq N \setminus 1} \frac{1}{2^{n-1}} (v(s \cup 1) - v(s)),$$

$$\begin{aligned} \varphi_1(N, v) &= \frac{1}{2^{5-1}} (v(\{1\}) - v(\emptyset)) \\ &+ \frac{1}{2^{5-1}} (v(\{1,2\}) - v(\{2\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,3\}) - v(\{3\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,4\}) - v(\{4\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,5\}) - v(\{5\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,2,3\}) - v(\{2,3\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,3,4\}) - v(\{3,4\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,4,5\}) - v(\{4,5\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,2,5\}) - v(\{2,5\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,2,4\}) - v(\{2,4\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,3,5\}) - v(\{3,5\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,2,3,4\}) - v(\{2,3,4\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,2,3,5\}) - v(\{2,3,5\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,2,4,5\}) - v(\{2,4,5\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,3,4,5\}) - v(\{3,4,5\})) \\ &+ \frac{1}{2^{5-1}} (v(\{1,2,3,4,5\}) - v(\{2,3,4,5\})) \\ &= 47.36 \end{aligned}$$

Table 3. The Banzhaf values

Waste producer	i=1	i=2	i=3	i=4	i=5
$\varphi_i(N, v)$	47.36	69.95	78.7	86.24	96

Table 3 shows the individually each waste producer. Thus, the Banzhaf vector $\varphi_B(N, v)$ for waste producers is (47.36, 69.95, 78.7, 86.24, 96). Evidently, the highest value is the AL Dora piston (DP)= 96, therefore waste producer 5 has the maximum capacity to influence the outcome of the game, and it will be a greater contribution to the disposal or treatment of non-recyclable waste. That is, it will be a greater contribution than the rest of the waste producers. On the other hand waste producer 1 (CBI) has the minimum capacity to influence the outcome of the game, and it will be the least contribution than the rest of the waste producers.

Conclusions:

This paper employed a simple mathematical model to represent the application of cooperative game theory to waste management. The proposed results in this paper are as follows.

- The first result introduces an employ of the Banzhaf values to show the potential cost to waste producers in the case of cooperation and to reduce the overall costs of processing non-recyclable waste during a cooperation between producers.
- The second result proposes an application of the methodology to study a case for five the waste producers' waste management in the Al-Mahmudiya factory with the aim of displaying the potential cost to waste producers in case of cooperation.
- The results of the proposed framework will strongly help professionals to formulate and

improve well-organized strategies for the waste management system of the future.

Although the inference of the results from this study is enough to formulate well-organized strategies for the waste management system of the future, there is scope for improvement of the proposed framework. Also, we can employ other values different from the Banzhaf interaction value to show the potential cost to waste producers, and study a case for more than five the waste producers in waste management with the aim of displaying the potential cost to waste producers in case of cooperation, which are open questions for future researches. For a large-scale problem, cooperation might become beneficial when the capacity of the local waste processor (Al-Mahmoudiya factory) is insufficient and the producers are forced to send their waste to more distant ones. Therefore, we recommend establishing more waste processors in Baghdad and other administrative units of Iraq.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Technology.

Authors' contributions statement:

Both authors participated in the work in an objective and supportive manner. The first author (H. H.) contributed to the data collection and analysis. The second author (J. A.) contributed by employing a simple mathematical model to represent the application of cooperative game theory to waste management in Baghdad.

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تطبيق قيم بانزاف للتعاون بين منتجي معالجة النفايات في مصنع المحمودية

جبار عباس

هدى هادي

قسم العلوم التطبيقية، الجامعة التكنولوجية، بغداد، العراق.

الخلاصة:

تعد قضية النفايات من القضايا الشائكة التي تعرقل أهداف النمو في العديد من البلدان، حيث من المهم التأكيد على نظام متكامل وإنشاء نظام متكامل لتحقيق أقصى استفادة منه. نتيجة لذلك، كان على معظم البلدان الصناعية إنشاء نظام تصنيع حديث يعتمد على تقليل استخدام عناصر المدخلات الجديدة وإعادة تدوير النفايات. نظرًا لأنه لا يتم إعادة تدوير جميع القمامة في مصانع إعادة تدوير النفايات، لهذا يجب على منتجي النفايات العمل معًا لخفض التكاليف الإجمالية للتعامل مع النفايات غير القابلة لإعادة التدوير. الهدف من هذا البحث هو تطبيق إطار رياضي قائم على نظرية الألعاب التعاونية لنمذجة التعاون بين المنتجين في إدارة النفايات. في هذه الورقة أولاً، نستخدم قيم Banzhaf لإظهار التكلفة المحتملة لمنتجي النفايات في حالة التعاون ولتقليل التكاليف الإجمالية لمعالجة النفايات غير القابلة لإعادة التدوير أثناء التعاون بين المنتجين. ثانيًا، نقترح تطبيقًا للمنهجية لدراسة حالة لإدارة نفايات خمسة منتجين في مصنع المحمودية بهدف عرض التكلفة المحتملة لمنتجي النفايات في حالة التعاون. أخيرًا، ستساعد النتائج التي تم الحصول عليها من إطار العمل المقترح المهنيين بقوة على صياغة وتحسين استراتيجيات جيدة التنظيم لنظام إدارة النفايات في المستقبل.

الكلمات المفتاحية: قيمة بانزاف، اللعبة التعاونية، نظرية الألعاب، عملية اتخاذ القرار، إدارة النفايات