Open Access **Baghdad Science Journal** **P-ISSN: 2078-8665**
Published Online First: November 2022 **2023, 20(3): 815-824** **E-ISSN: 2411-7986**

DOI: https://dx.doi.org/10.21123/bsj.2022.7007

Gogny interaction and nuclear charge distribution in 48Ca Nucleus

***Raad Falih*[1*]** ***Firas Z. Majeed*[2]**

Department of Physics, College of Science, University of Baghdad, Baghdad, Iraq.
*Corresponding author: raad.hasan1204@sc.uobaghdad.edu.iq
E-mail addresses: firas.majeed@sc.uobaghdad.edu.iq

Received 5/2/2022, Revised 12/6/2022, Accepted 14/6/2022, Published Online First 20/11/2022
Published 1/6/2023

Abstract
Charge multipole Coulomb scattering form factors in ^{48}Ca nucleus have been reproduced utilizing the theory of nuclear shell. The efficient two-body nuclear potential fpbm is considered to construct the-spin orbit term LS vectors with Harmonic Oscillator HO potential as a wave function of single particle in Fp shell. Discarded spaces (core + higher configuration) are taken into account through the Core polarization effect by model space with accurate two-body potential of Gogny to interact the LS operating particles with the discarded space pair (particle-hole) with energy of excitation equal to 2ħω. Gogny interaction has been selected as it had succeeded in nuclear shell theory. The computed results were compared with the available experimental data.

Keywords: Charge, core polarization, Electron Scattering, Form Factor, Gogney interaction.

Introduction
Electron scattering probabilities, elastic and inelastic have been studied in wide varietal models and approaches which are succeeded or failed to reproduce the experimental results. Relativistic mean field theory is one of the most important [1].
Diffraction probabilities of electrons have been studied for some low mass number isotopes, like ^{50}Ti, ^{48}Ca, ^{44}Ca, ^{42}Ca, ^{41}Ca. They had been performed in terms of the mixing configuration shell model. The off model space orbits were included in a so called core-polarization effects microscopically considering particle-hole excitations from the core orbits to the upper orbits via fp shell with 2ħω excitations. The core polarization matrix components were calculated using the theoretical Michigan sum of three range Yukawa (M3Y) interaction [2].
An account of significant neutron-proton cooperation was detected and single-particle momentum distributions for each isospins in ^{48}Ca computed from these spectral functions approve that a higher portion of high-momentum protons than neutrons are caused by neutron excess [3]. The associated neutron transfer with the role of diverse Skyrme forces effect was studied in the reactions view ^{40}Ca, ^{48}Ca+^{96}Zr. The synthesis cross-section has been computed using Wong formula and extended Wong model [4] Theoretical approaches to the quadrupole response achieve disparate findings, whereas experimental data unambiguously show the states' multipolarity and one-phonon nature. Furthermore, cross section calculations are insensitive enough to detangle between quadrupole states that are thought to be attributable to a distinct excitation mode in one of the theoretical approaches [5].
The short range of two-body potential (SRCs) was considered as most active term for the tail with great momentum of the momentum allocation of the nucleon, n(k)[6]. The two-body Skyrme type Sly5 interaction is used for the core polarization matrix elements. FPBM model space-effective interactions of Richter are adopted to generate the model space wave functions, the oscillator potential was utilized to construct the LS shell single particle basis vectors, where an analytical solution is possible. Comparisons were made with the available experimental readings for different multipolarities [7].

An algorithm of (M3Y) written in FORTRAN 90 had been modified to include realistic Gogny potential in the original code to calculates the two-

body matrix elements which are 1st order effects of core polarization.

The goal of this work is to use a realistic (NN) Gogny potential[8] as an effective residual interaction to read the effects of cast-off orbits microscopically, with a best and careful selection of LS effective two body potential which construct the LS vectors (shell model wave functions) and highly energy wave function with the same transition operations. Oscillator single particle vectors will be selected as a single basis vectors. We shall clarify the core-polarization process on the form factors of elastic and inelastic electron scattering for the low lying states of 2p1f-LS shell in ^{48}Ca isotope. Gogny interactions of H. Nakada (2002)[8] with sets P2 of Ried fitting of parameters are to be applied as a discarded space effective potential for the core polarization calculations.

General theory

Electron scattering

The diffraction probabilities of electrons from a target nucleus of atomic number Z and mass number A in the first-order Born approximation (PWBA) are provided by [9, 10]

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} f_{\text{rec}} \sum_J |F_J(q,\theta)|^2 \quad (1)$$

$\left(\frac{d\sigma}{d\Omega}\right)_{Mott}$ is the cross-section of Mott for point spinless nucleus high energy scattered electron from a, is given by:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left[\frac{Z\alpha\cos(\theta/2)}{2E_i \sin^2(\theta/2)}\right]^2 \quad (2)$$

Where $\alpha = e^2/\hbar c = (1/137)$ is the fine structure constant which is representative the interaction order, θ is the diffraction angle and E_i is the incident electron energy [2]

The nucleus recoil factor is written as[10]:

$$f_{rec} = \left[1 + \frac{2E_i}{M}\sin^2(\theta/2)\right]^{-1} \quad (3)$$

M represents the target mass [2].

$$\left|F_J(q)\right|^2 = \left(\frac{q_\mu}{q}\right)^4 \left|F_J^L(q)\right|^2 + \left[\frac{q_\mu^2}{2q^2} + \tan^2(\theta/2)\right] \left|F_J^T(q)\right|^2 \quad (4)$$

The momentum transfer components q_μ are provided by, ($\hbar$=c=1 for calculation reasons) [2]:

$$q_\mu^2 = q^2 - (E_i - E_f)^2 \quad (5)$$

with

$$q^2 = 4E_iE_f\sin^2(\theta/2) + (E_i - E_f)^2$$

$$= 4E_iE_f\sin^2(\theta/2) + \omega^2$$

Core polarization effects

Including the cast-off space microscopically as a 1st order particle-hole state perturbation (p-h), via model space to compute these effects as a residual interaction, Gogny -type hamiltonian[10]. Where a core of ^{40}Ca is expected for isotope with atomic number Z-20, mass number A>40, and neutron number N≥ 20.

The multipolarities operator $\hat{T}_\Lambda^\eta$ consists of two terms: one for matrix elements in the fp shell (model-space) and the other one for matrix elements in the off fp LS shell (core-polarization) [8]

$$\langle\Gamma_f|||\hat{T}_\Lambda^\eta|||\Gamma_i\rangle = \langle\Gamma_f|||\hat{T}_\Lambda^\eta|||\Gamma_i\rangle_{MS} + \langle\Gamma_f|||\delta\hat{T}_\Lambda^\eta|||\Gamma_i\rangle_{CP} \quad (6)$$

Where, $\langle\Gamma_f|||\hat{T}_\Lambda^\eta|||\Gamma_i\rangle_{MS}$ are the model-space matrix elements,

and, $\langle\Gamma_f|||\delta T_\Lambda^\eta|||\Gamma_i\rangle_{CP}$ are the elements of the off fp LS shell matrix.

$|\Gamma_i\rangle$ and $|\Gamma_f\rangle$ are labeled as the many body LS vectors, α and β are the initial and final single particle states [2].

The LS shell (model-space) fp-shell nuclei is definite by the ensuring configuration: (1f7/21f5/22p3/22p1/2) and,

$$\langle\Gamma_f|||\delta\hat{T}_\Lambda^\eta|||\Gamma_i\rangle_{cp} = \sum_{\alpha,\beta} OBDM(\alpha,\beta)\langle\alpha|||\delta\hat{T}_\Lambda^\eta|||\beta\rangle_{cp} \quad (7)$$

$$\langle\boldsymbol{\alpha}|\boldsymbol{\delta\hat{T}_J^\eta}|\boldsymbol{\beta}\rangle = \langle\boldsymbol{\alpha}|\boldsymbol{V_{res}}\frac{\boldsymbol{Q}}{\boldsymbol{E-H^{(0)}}}\boldsymbol{\hat{T}_J^\eta}|\boldsymbol{\beta}\rangle + \langle\boldsymbol{\alpha}|\boldsymbol{\hat{T}_J^\eta}\frac{\boldsymbol{Q}}{\boldsymbol{E-H^{(0)}}}\boldsymbol{V_{res}}|\boldsymbol{\beta}\rangle \quad (8)$$

$$\langle\alpha|\hat{T}_J^\eta\frac{Q}{E-H^{(0)}}V_{res}|\beta\rangle = \sum_{p,h}\langle h|\hat{T}_J^\eta|p\rangle \times \frac{1}{e_\beta - e_\alpha - e_p + e_h} \times \langle\alpha p|V_{res}|\beta h\rangle \quad (9)$$

$$\langle\alpha|V_{res}\frac{Q}{E-H^{(0)}}\hat{T}_J^\eta|\beta\rangle = \sum_{p,h}\langle p|\hat{T}_J^\eta|h\rangle \times \frac{1}{e_\alpha - e_\beta - e_p + e_h} \times \langle\alpha h|V_{res}|\beta p\rangle \quad (10)$$

Gogney interaction is represented by [11]:

$$\left.\begin{aligned} v_{12}^{(c)} &= \sum_n (t_n^{(SE)} P_{SE} + t_n^{TE} P_{TE} + t_n^{(SO)} P_{SO} + t_n^{(TO)} P_{TO})\, f_n^{(C)}(r_{12}) \\ v_{12}^{(LS)} &= \sum_n (t_n^{(LSE)} P_{TE} + t_n^{(LSO)} P_{TO})\, f_n^{(LS)}(r_{12}) L_{12}.(\vec{s}_1 + \vec{s}_2) \\ v_{12}^{(TN)} &= \sum_n (t_n^{(TNE)} P_{TE} + t_n^{(TNO)} P_{TO})\, f_n^{(TN)}(r_{12}) r_{12}^2 S_{12} \\ v_{12}^{(DD)} &= \{t^{(DD)} P_{SE}.[\rho(r_1)]^{a^{(SE)}} + t^{(DD)} P_{TE}.[\rho(r_1)]^{a^{(TE)}} \delta(r_{12}) \end{aligned}\right\} \quad 11$$

where $f_n^{(c)}(r_{12}) = e^{-(\mu_n r_{12})^2}$, $f_n^{(LS)} = \nabla^2\delta(r_{12})$, and μ_n : range parameter [104, 105].

Results and Discussion

Experimental data must be available to conclude that the accepted selected theoretic approach reproduces the experimental reads with an appropriate deviation or variances. The experimental readings must be sufficient modern in regard to the environments that are concerning the computation and contrast. Calculations should be sound and gently plotted to reveal the measurements perfectly. The LS effectual potential FPBM was used to generate the (1f7/2 1f5/2 2p3/2 2p1/2) ^{48}Ca shell model wave functions.

The discarded orbits were comprised to calculate the configurating contributions from model-space outside in the operation under interest.

FP LS shell is considered in this work for ^{48}Ca (shell model space having ^{40}Ca as an inert core) to all higher orbits with $2\hbar\omega$ excitation for normal transitions.

The One Body Density Matrix elements (OBDM) for all transitions considered will be referred to subsequently as each form factors are considered in turn.

The Cp effects are calculated with the Gogny N-N potential [10] as a residual interaction. The two-body matrix elements in a specific single-particle basis were deduced directly from the scattering phase shift using this technique. The HO potential is used to determine the radial component of the single-particle wave functions, with the size parameter (b) fitted to obtain the nuclei's root mean square radius (r.m.s).

The ^{48}Ca Nucleus

The nucleus ^{48}Ca is the dual behavior nucleus because it is an inert core and active nucleus with a neutron excess. It proved to be a good nuclear probing nucleus and thus provides an excellent examining base for nuclear models. Really, the isotope ^{48}Ca is more inert than ^{40}Ca,^{48}Ni and ^{56}Ni because of the filled neutron orbit $1f_{7/2}$ so that it is a useful isotope in fp LS shell nuclei. We use the single particle basis vector of the harmonic oscillator (HO) and parameter of oscillation (b= 1.988 fm).

The charge Form Factor for 0^+4 State

Charge form factors (C0) is plotted compared with the experimental readings in Fig.1, the calculated scattering probabilities were resulted by using Gogny two-body potential, readings are in an excellent agreement with the experimental data at the range of ($q \leq 1$). While for the higher (q) values the results deviate from the experimental data because of the limited actions of harmonic wave vectors.

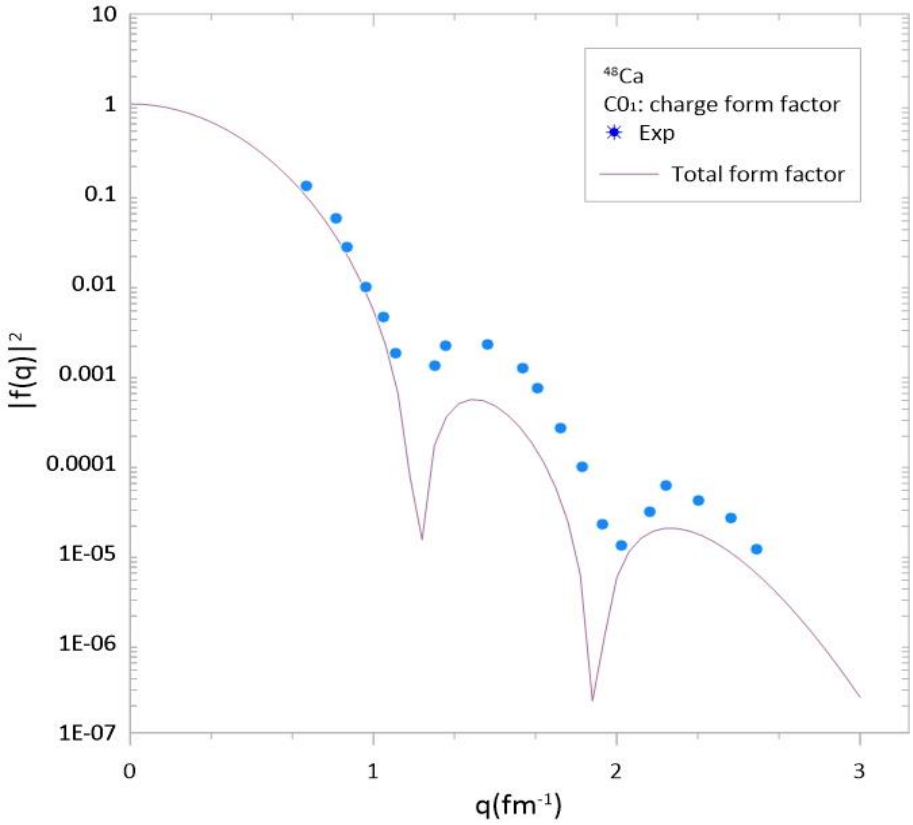

Figure 1. Coulomb C0 scattering probabilities for the first 0^+4 vector in ^{48}Ca isotope. The experimental readings are red from ref. [2]

The C2 Longitudinal Form Factor For 2^+4 state First 2^+ state at Ex=3.83MeV.

Inelastic longitudinal form factors (C2) have calculated using Gogny as shown in Fig.2 the calculated form factors using Gogny have underestimated the experimental data at all regions of (q) except at q=1, 1.8 and 2.1 fm^{-1} which have a good agreement. The theoretical result shows three peaks while the experimental data have two peaks which shifted than it.

The behaviors of results as two peaks as the experimental data but the first was underestimating the data at all (q) region which means its need more correction.

Qualitatively it has a good agreement in behavior especially at q=1.4 and q=1.5 fm^{-1} as shown in Fig.2

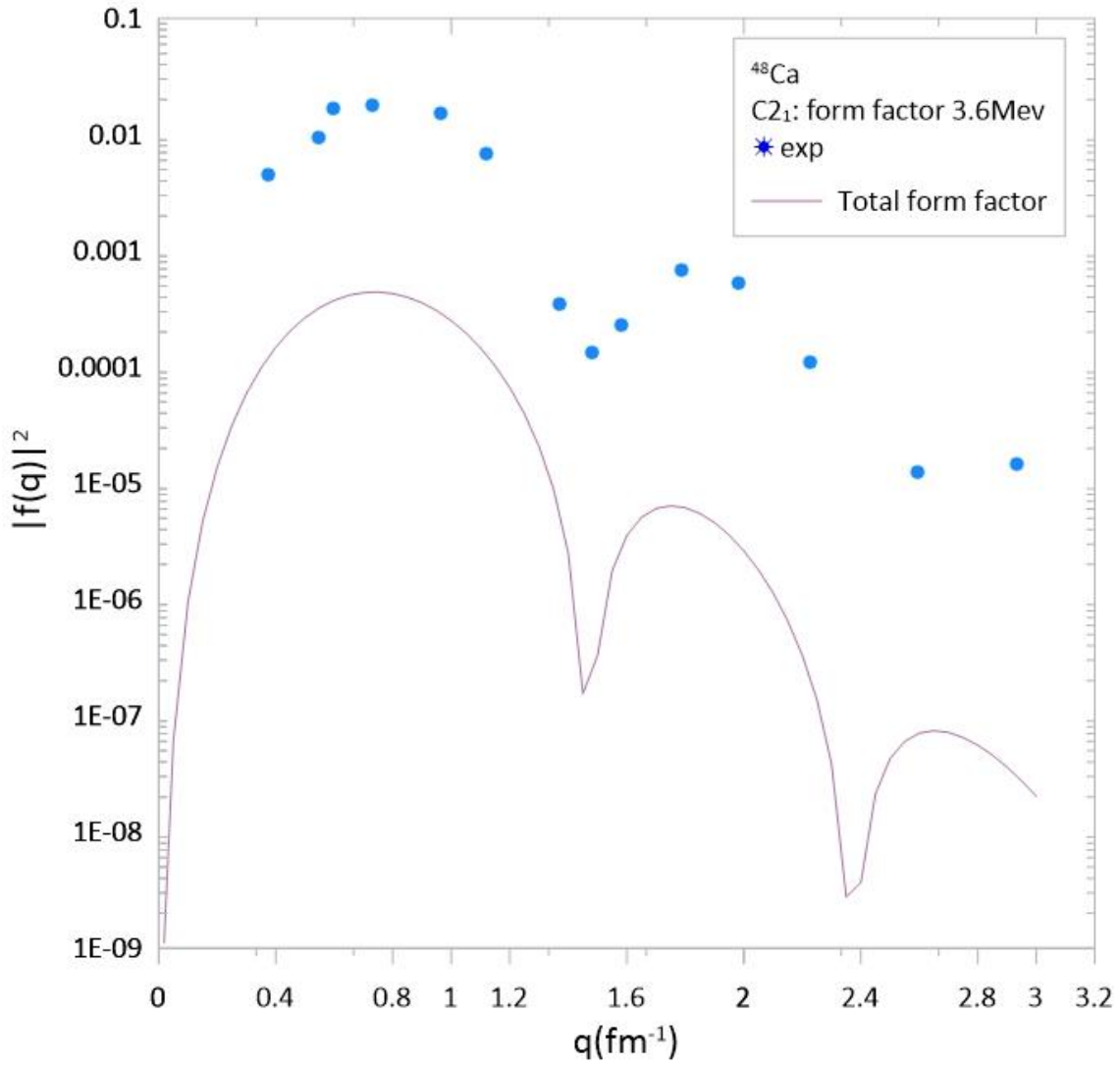

Figure 2. Coulomb C2 scattering probabilities for the first 2^+4 vector in ^{48}Ca isotope. The experimental readings are red from ref. [2]

Fourth 2^+ state at Ex=6.557MeV.

The quadruple C2 charge form factor for ^{48}Ca 1f2p shell model space is illustrated in Fig.3 using of Gogny , as a residual interaction, there is a good agreement in results with the experimental data which shifted in all region of (q), the theoretical calculations show three peaks which overestimate the data at all (q) but gave a good agreement at q=1, 1.6 and near q=2.7 fm^{-1}. This type of residual interaction cannot explain the results very well (Fig.3).

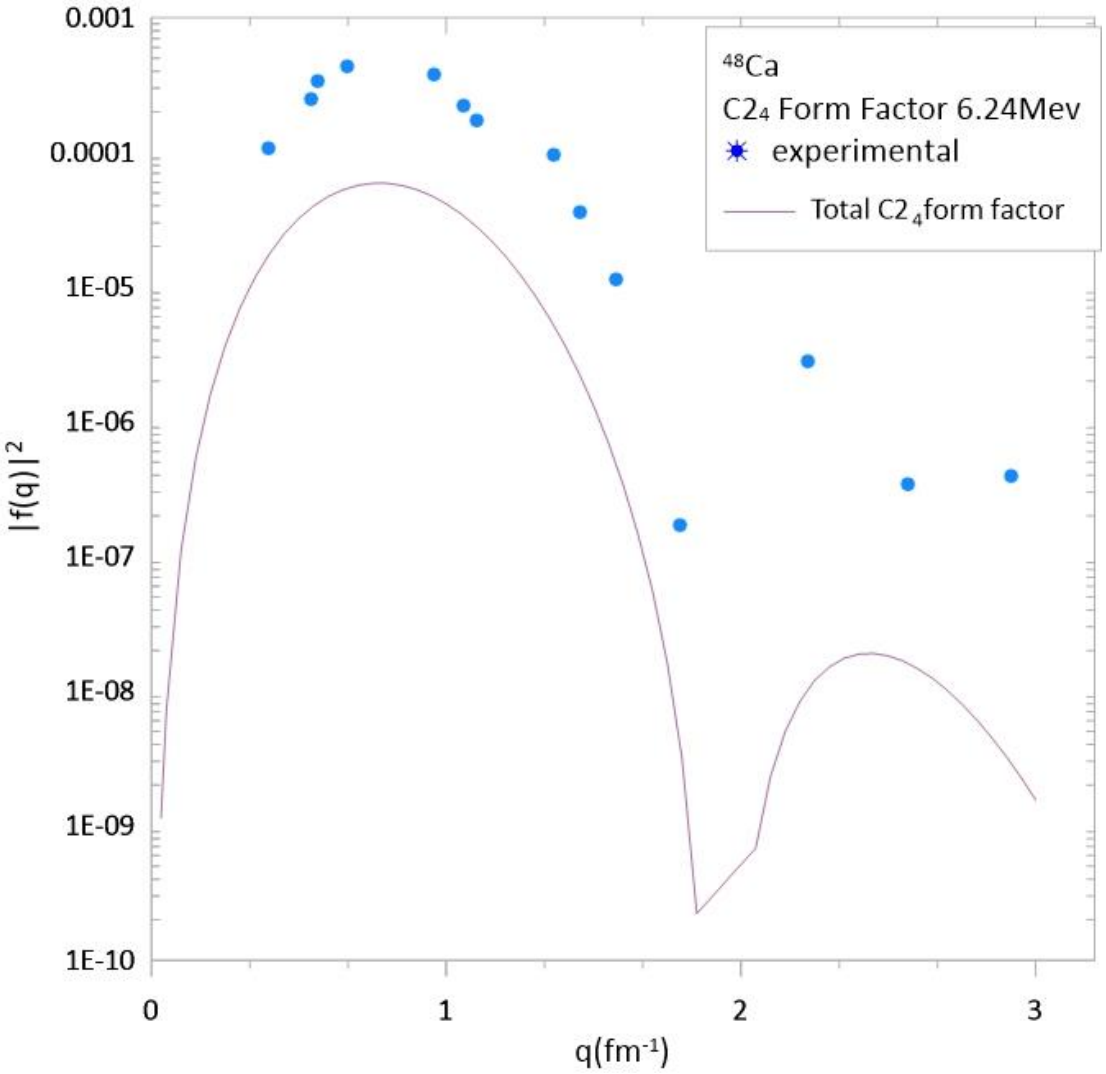

Figure 3. Coulomb C2 scattering probabilities for the second 2^+4 vector in ^{48}Ca isotope. The experimental readings are red from ref. [2]

The Inelastic Longitudinal C4 form factor (4^+4) State in ^{48}Ca

The first 4^+ state Ex=6.34 MeV

The longitudinal electron scattering C4 form factor for the first excited state with excitation energy E_x=6.34MeV has been studied in the same manner, as we have seen in the previous cases, which are used the three residual interaction Gogny results as illustrated in Fig.4 are in good agreement with experimental data, they are underestimating in the first and second lope and the diffraction minimum is deflected appreciably, the results are in harmony as a general behavior but they are underestimating for the first and second lope and the diffraction minimum are not coincided only at q=2.3 fm^{-1} have a good agreement. The results are compared with experimental data from ref. [2].

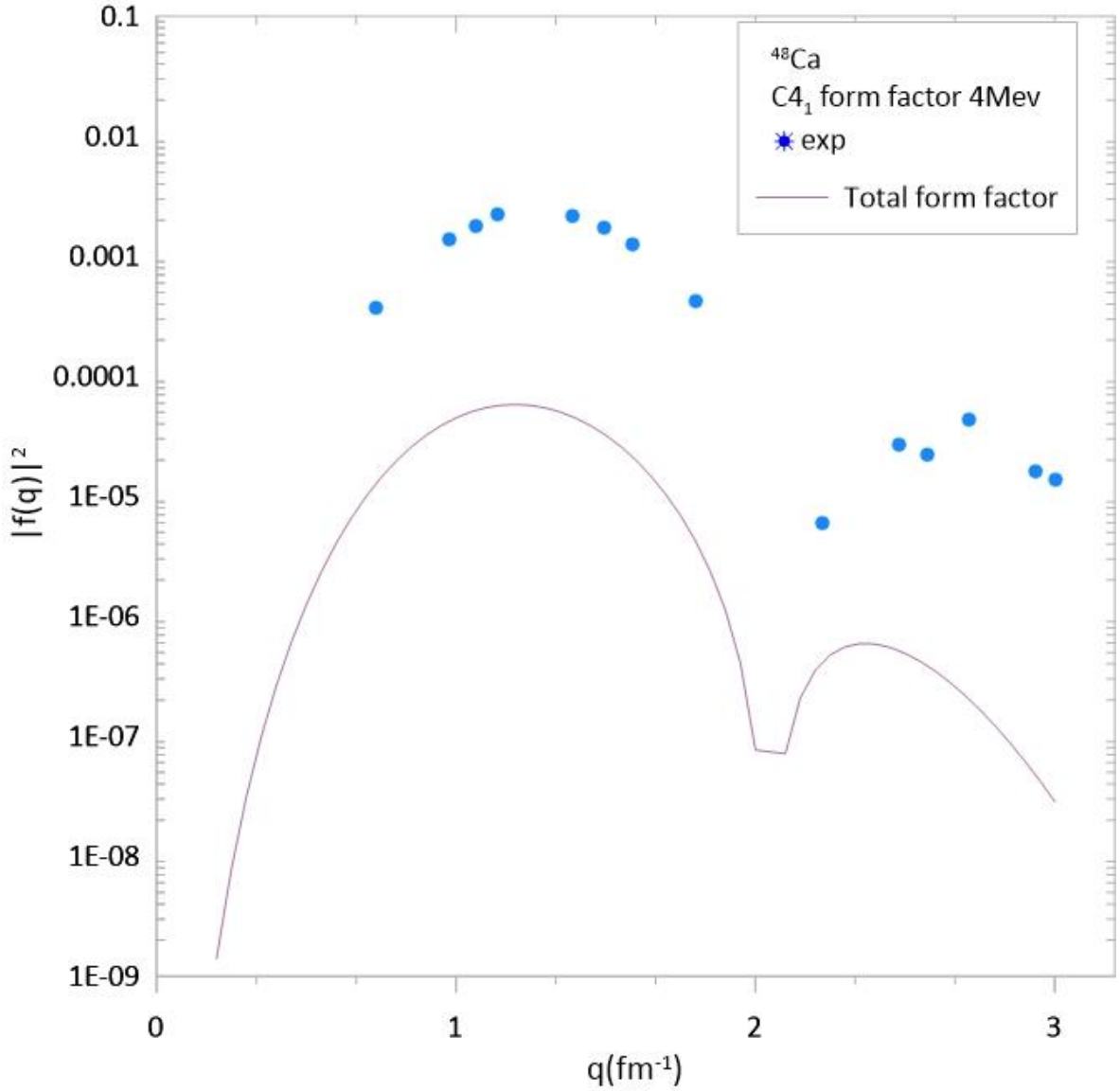

Figure 4. Coulomb C4 scattering probabilities for the first $4^{+}4$ vector in ^{48}Ca isotope. The experimental readings are red from ref. [2].

The second 4+state for ^{48}Ca at Ex=6.65 MeV.

Inelastic longitudinal (C4) form factors were calculated by using Gogny which shown in Fig.5 calculation as a function of a residual interaction which cannot explain the results very well and it shows underestimating of the data at $q \geq 1$ fm^{-1}. But there was a good agreement at q=2.4 fm^{-1}. the results go up near the experimental data while it underestimates the data at all (q) regions it shows two peaks as the experimental data behavior but with shifting. The results goes up near the data and we get a good agreement at q=1.7 till 2.6 fm^{-1} while the solid curve underestimates the data at the first region of q>0.2 till 1.7 fm^{-1}

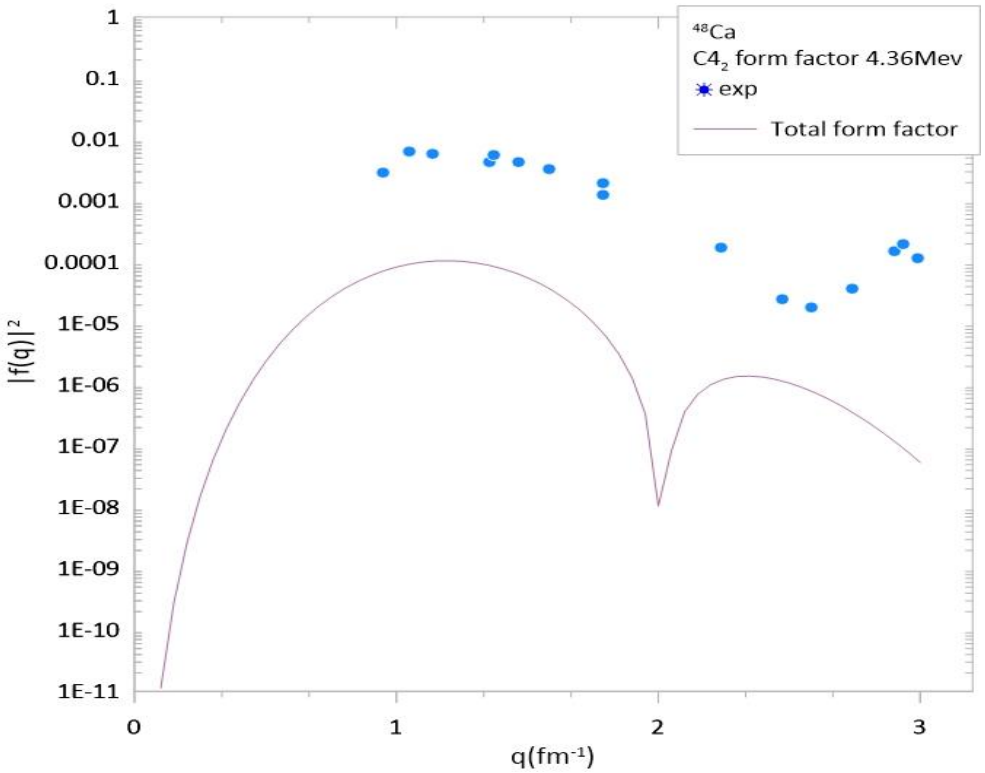

Figure 5. Coulomb C4 scattering probabilities for the second 4^+4 vector in ^{48}Ca isotope. The experimental readings are red from ref. [2].

The third 4^+ state for ^{48}Ca at Ex=7.48 MeV.

In Fig.6, the calculated results go far from the experimental readings as was expected because of the uncoincided values of excited energy matrix elements and the results are underestimate the data but with the same behavior

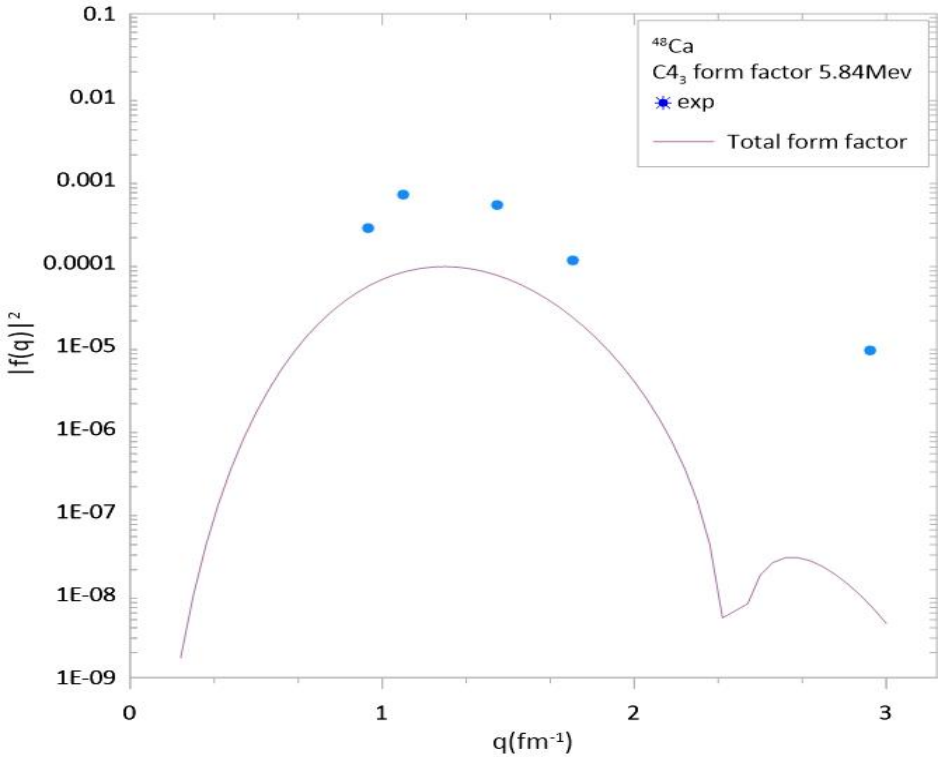

Figure 6. Coulomb C4 scattering probabilities for the third 4^+4 vector in ^{48}Ca isotope. The experimental readings are red from ref. [2]

The fourth 4^+ state for ^{48}Ca at Ex=7.79MeV

As shown in Fig.7 which represented the total C4 form factors as a function of residual interactions shows disagreement with the experimental data at all (q) regions while the results underestimate the data at all (q) regions. The experimental data are taken from ref. [2]

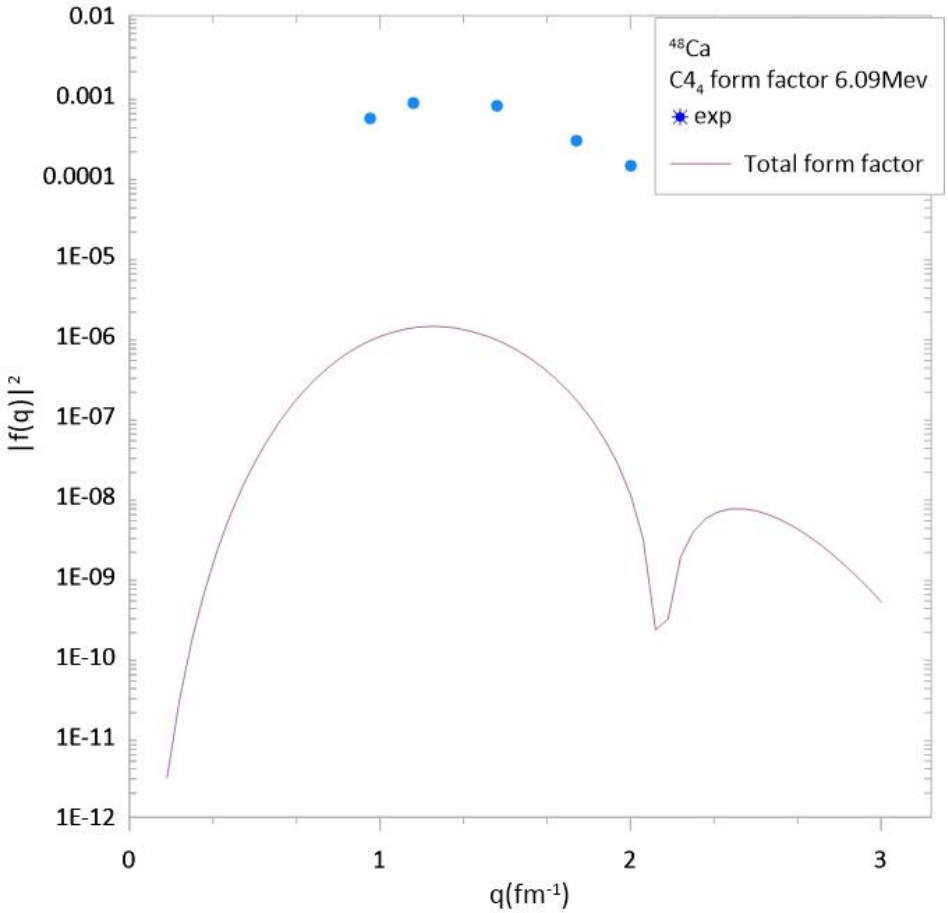

Figure 7. Coulomb C4 scattering probabilities for the fourth 4^+4 vector in ^{48}Ca isotope. The experimental readings are red from ref. [2]

Conclusions:

Harmonic oscillator wave function is still valid especially for q<2.2 fm^{-1} as it is known and the approach depends on the first order Born approximation then there are more modifications need to beused to make the reproduced values closer to measured values.

Gogny has fair action in a comparison with M3Y versions, but it might need modifications especially for fitting parameters and the use of zeroth range meson interaction is active.

Density dependence realistic versions are more useful than the independent one and the pion exchange potentials gives better as a results of charge exchange process.

Author's declaration:

- Conflicts of Interest: None.
- I hereby confirm that all the Figures and Tables in the manuscript are mine. Besides, the Figures and images, which are not mine, have been given the permission for re-publication attached with the manuscript
- Ethical Clearance: The project was approved by the local ethical committee in University of Baghdad.

Authors Contribution:

Dr. F. Z. Majeed and R. F. Hasan participated to the plan and application of the study to the results analysis and to the writing of the manuscript, the gogney interaction is the modified form of the M3Y interaction so the authors done the programming and run this subroutine in order to be used as a residual interaction to include the discarded space through the so called core polarization effect

The original copy of two codes which gotten from Dr. R. A. Radhi used for calculating form factors and residual interaction and aid in producing OBDM files from B. A. Brown's OXBASH code.

Reference:

1. Guo X, Liu J, Wang Z, Chi Z. Investigation of nonlinear isoscalar-isovector coupling in a relativistic mean field model by elastic magnetic electron scattering. Nucl Phys A. 2018; 978: 1-12
2. Majeed FZ, Mashaan SS, Inelastic longitudinal electron scattering C2 form factors in 48Ca nucleus by using sigma meson as a residual interaction. Iraqi J Sci. 2014,55(1), 151.
3. Atkinsona MC, Dickhoff WH. Investigating the link between proton reaction cross sections and the quenching of proton spectroscopic factors in 48Ca. Phys Lett B. 2019, 798: 135027.
4. Sharma I, Kumar R, Sharma MK. Relevance of different Skyrme forces in the dynamics of ^{40,48}Ca ^{96}Za reactions. Nucl Phys A. March 2019 983: 276-292.
5. Lanza E G, Pellegri L, Andres M V, Catara F, Vitturi A. Low-lying dipole and quadrupole states. Eur Phys J A. 2019; 55: 235. https://doi.org/10.1140/epja/i2019-12797-y
6. Dien N, Zhihong Y, John A, Donal D, Douglas H, The isospin dependence of Short Range Correlations through Inclusive Electron Scattering from ^{40}Ca and ^{48}Ca Et. (Dissertation) United State, University of Virginia, APS April Meeting 2019
7. Atkinson MC, Dickhoff WH. Investigating the link between proton reaction cross sections and the quenching of proton spectroscopic factors in 48Ca. Phys Lett B. 2019; 798: 135027
8. Nakada H, Sato MA. method of implementing Hartree-Fock calculations with zero- and finite-range interactions. Nucl Phys. 2003; A 714(3): 696.
9. Hameed BS. The Nuclear Structure for Exotic Neutron-Rich of $^{42, 43, 45,47}$K Nuclei. Baghdad Sci J. 2016; 13(1): 146.
10. Ali AH. investigation of the Quadrupole Moment and Form Factors of Some Ca Isotopes. Baghdad Sci J. 2020; 17(2): 0502.
11. Hilaire S, Girod M. Large-scale mean-field calculations from proton to neutron drip lines using the D1S Gogny force. Eur Phys J. 2007: A 33(2), 237.

تفاعل كوكني وتوزيع الشحنة النووية في نواة الكالسيوم 48

رعد فالح حسن د. فراس زهير مجيد

قسم الفيزياء، كلية العلوم، جامعة بغداد, بغداد, العراق.

الخلاصة:

تمت دراسة عوامل التشكل الكولومية لمتعددة الاقطاب لنواة ^{48}Ca باستخدام نموذج القشرة النووية. وقد أدخل الجهد النووي fpmb لزوج الجسيمتين للحسابات لتركيب LS. اعتمد جهد المتذبذب التوافقي Jp`t كدالة موجة للجسيم المنفرد في الفضاء Fp . وقد أضيفت الفضاءات العليا وتأثيرات القلب الخامل كتصحيح من المرتبة الاولى. تم استخدام جهد الجسيمتين الدقيق لكوكني لتفاعل الجسيمات الفعالة في الفضاء مع زوج (جسيم-فجوة) في الفضاءات المستثناة و بطاقة تهيج مقدارها $2\hbar\omega$.ان اختيار تفاعل كوكني تم على اساس نجاحه في نظرية القشرة. تم مقارنة النتائج مع القيم العملية.

الكلمات المفتاحية: الشحنة ، استثطاب القلب. استطارة الالكترون، عامل التشكل, تفاعل كوكني.