Steady and Unsteady flow of non-newtonian fluid in curved pipe with triangular cross-section

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Abstract

A numerical study is presented for steady and unsteady slow flow of a viscous fluid of second order in a region bounded by a right-angled isosceles triangle. The particular flow considered is the secondary flow generated in the plane of the cross-section by the primary axial flow, under action of the pressure gradient, through a slightly curved pipe of triangular cross-section. Two cases are considered; the first one is the steady case in which it is found that the motion equations, which are describing the fluid motion, are controlled by two parameters namely; Dean number and the non-Newtonian parameter. In the second case it is found that the motion equations are controlled, in addition to the parameters mentioned above, by third parameter namely; the frequency parameter. Solutions, of the first case, are expanded in terms of Dean number. While in the second case the solutions are firstly expanded in terms of Dean number and secondly in terms of the frequency parameter. Perturbations equations are solved by Galerkin method after eliminating the dependency on time .In both cases, the effect of the parameters mentioned above on the secondary flow and the axial velocity is studied.

1-Introduction

The flow of Newtonian and non-Newtonian fluids has been the subject of extensive theoretical studies till date. Dean, [4], was the first researcher who worked in flow analysis of Newtonian fluids in curved pipes. He introduces a toroidal coordinate system to show that the relation between pressure gradient and the rate of flow through a curved pipe with circular cross-section of incompressible Newtonian fluid is dependent on the curvature. In that paper he could not show this dependence but he did in his second paper, [5]. He modified his analysis by including the higher order terms and was able to show that the rate of flow is slightly reduced by curvature.

Collins and Dennis [3], in their paper consider flow of Newtonian fluid in curved pipe with triangular cross-section (right angle). They shown that the first appearance of the vortices was in the secondary flow near the corner of 45^0 and give detailed study of all corner regions, which was made by refining the grid size of the numerical scheme.

The present paper investigates the steady and unsteady flow of non-Newtonian fluid in curved pipe with triangular cross-section. An orthogonal coordinates system has been framed to describe the fluid motion and it is found that the motion equations, in the case of steady flow, are controlled by two parameters namely; Dean number

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and the non-Newtonian parameter. Solution for the secondary flow and the axial velocity are derived as perturbations over straight pipe appearing through the Dean number. Galerkin method of analyses has been employed to derive each perturbation solution; these solutions have been developed in Cartesian coordinates for harmonic and biharmonic equations. This case is ended with studying the effect of the non-dimensional parameters mentioned above on each of the secondary motion and the axial velocity. The second case deals with the unsteady state flow of non-Newtonian fluid in a curved pipe. In this case we use different approach to write the

continuity and motion equations for the sake of simplicity. Here it is found that the equations, which are describing the fluid motion, are controlled by three parameters namely; Dean number, the non-Newtonian parameter and the frequency parameter. Solutions of the flow are firstly expanded in terms of Dean number and secondly in terms of the frequency parameter. Perturbations equations are solved by Galerkin method after eliminating the dependency on time. In the last two sections of this paper, the effect of the parameters mentioned above on the secondary flow and the axial velocity is studied. A bout 200 case has been tested to analysis this flow, and we choose (45) case to clarify this analysis. Up to author knowledge this problem is not considered yet.

2-Mathematical formulation

A typical triangular cross-section of a curved pipe is shown in figure 1, where C is the center of the circle in which the pipe lies. The angle at vertex O is 90^0 and the cross-section is symmetrical about the axis CN, with CO=L and ON=a. It is assumed that the ratio a/L<< 1 (i.e. The dimensions

of the cross-section are small in comparison with the radius L) and the velocity components are independent of ϕ but P is not, where P is the pressure, which is varing linearly with ϕ, where ϕ is the angle which the plane of given cross-section makes with a fixed cross-section, as shown in figure 1.The problem can be described in terms of the coordinate system (x_1, y_1, ϕ), where the origin is taken at the vertex O. The assumption on the pressure depends on the case under moderation whether it is steady or unsteady and it will mention letter on.
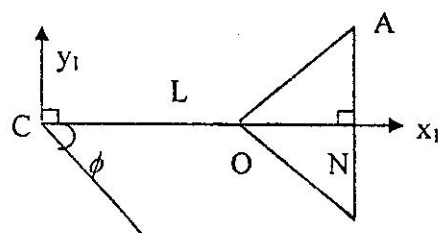

Figure 1.Coordinate system and geometry of the cross-section

3-Steady state

Consideration is given to steady motion of liquid characterized by equations of state of the form

$$T_{ik} = 2\eta e_{ik} + 4\xi e_{ij} e_{jk} \quad ...(1)$$

Where η is the viscosity coefficient and ξ is the normal stress coefficient and they are assumed to be constants for the fluid to be experimentally valid. Also T_{ik} and e_{ik} are the stress and the rate -of -strain tensors respectively, [6]. Some assumptions are made: the liquid in the pipe is an incompressible, non-Newtonian fluid, the motion in the pipe is maintained by a constant mean pressure gradient $\partial p_1 / \partial(L\phi) = -G$, where p_1 is the pressure, and since the case under consideration is steady then all time derivatives are zero.

3-1 Motion equations and boundary conditions

Let (u_1, v_1, w_1) be the velocity components in the directions of increases of the coordinates (x_1, y_1, ϕ). If $x_1 = ax, y_1 = ay$ and v is the coefficient of kinematic viscosity, we may write

$$u_1 = \frac{v}{a}\frac{\partial \psi}{\partial y}, v_1 = -\frac{v}{a}\frac{\partial \psi}{\partial x}, w_1 = \frac{v}{a}(\frac{2a}{L})^{-1/2}w \quad ..(2)$$

where $\psi(x, y)$ is the dimensionless stream function of the secondary flow in the cross-section. The equations of motion for $\psi(x, y)$ and $w(x, y)$ are

$$\nabla^2 w + (\frac{\partial \psi}{\partial x}\frac{\partial}{\partial y} - \frac{\partial \psi}{\partial y}\frac{\partial}{\partial x})w + \beta(\frac{\partial w}{\partial x}\frac{\partial}{\partial x} - \frac{\partial w}{\partial y}\frac{\partial}{\partial x})\nabla^2\psi +$$

$$2\beta\frac{\partial^2 w}{\partial x \partial y}(\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2}) + 2\beta\frac{\partial^2 \psi}{\partial x \partial y}(\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2}) = -D \quad ..(3)$$

$$\nabla^4\psi + w\frac{\partial w}{\partial y} + (\frac{\partial \psi}{\partial x}\frac{\partial}{\partial y} - \frac{\partial \psi}{\partial y}\frac{\partial}{\partial x})\nabla^2\psi$$

$$-\beta(\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}) = 0 \quad \quad ...(4)$$

where
$$\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$$

The constant D is the Dean number, defined by $D = Ga^3 (2a/L)^{\frac{1}{2}} / \rho v^3$, where ρ is the density.

The boundary conditions that are associated with the motion equation are

$w=0, \partial\psi/\partial x=\partial\psi/\partial y=0$ *on the bounda*

from which it also follows that we may take

$\psi = 0$ *on the boundary*

3-2 Method of solution

A successive approximation method, equivalent to the expansion of ψ and w in ascending power of D will be

$$\psi = D^2 \psi_1 + D^4 \psi_2 + \cdots$$

$$w = D w_o + D^3 w_1 + \cdots \quad \cdots (5)$$

used for solving the above system, which can be developed, by using

We will limit ourselves to find the solution up to the first order in D, and that enough for our purposes.

If we substitute the above expressions for ψ and w in (3) and (4) and equate coefficients of equal power in D, we obtain a series of relations from which w_o, ψ_1, w_1... can be successively found out. The out equations are

$$\nabla^2 w_o = -1 \quad \quad ..(6)$$

$$\nabla^4\psi_1 = -w_o\frac{\partial w_o}{\partial y} + \beta\frac{\partial w_o}{\partial x}\frac{\partial^2 w_o}{\partial x \partial y} + \beta\frac{\partial w_o}{\partial y}\frac{\partial^2 w_o}{\partial y^2} \quad ..(7)$$

$$\nabla^2 w_1 = -\frac{\partial \psi_1}{\partial x}\frac{\partial w_o}{\partial y} + \frac{\partial \psi_1}{\partial y}\frac{\partial w_o}{\partial x} - \beta(\frac{\partial w_o}{\partial x}\frac{\partial}{\partial y} - \frac{\partial w_o}{\partial y}\frac{\partial}{\partial x})\nabla^2\psi_1 -$$

$$2\beta(\frac{\partial^2 w_o}{\partial x \partial y})(\frac{\partial^2 \psi_1}{\partial y^2} - \frac{\partial^2 \psi_1}{\partial x^2}) - 2\beta(\frac{\partial^2 \psi_1}{\partial x \partial y})(\frac{\partial^2 w_o}{\partial x^2} - \frac{\partial^2 w_o}{\partial y^2}) \quad ..(8)$$

The solutions for the stream function ψ_1 and the components of the axial velocity w_o and w_1 are obtained by integrating their respective governing equations satisfying the corresponding boundary conditions. For clarity, we give the expressions of w_o, ψ_1 and w_1 without going through lengthy integrations:

$$w_o = -(3/8)(x^3 - xy^2 - x^2 + y^2) \quad ...(9)$$

$$\psi_1 = -(3/8)(x-1)^2(x^2 - y^2)^2$$

$$((a_1 x^4 y + a_2 x^3 y + a_3 x^2 y^3 + a_4 xy^3 + a_5 x^2 y + a_6 y^3) +$$

$$\beta(b_1 x^2 y + b_2 y^3 + b_3 xy + b_4 y)) \quad ...(10)$$

$$w_1 = -(3/8)(x-1)(x^2 - y^2)(c_1 x^4 y^2 + c_2 x^2 y^2 +$$

$$c_3 x^2 y^4 + c_4 xy^4 + c_5 x^3 y^2 + c_6 xy^2 +$$

$$c_7 y^4 + c_8 x^6 + c_9 x^5 + c_{10} x^4 + c_{11} x^3) +$$

$$\beta(c_{12} x^2 y^2 + c_{13} xy^2 + c_{14} y^2 + c_{15} x^4 + c_{16} x^3 +$$

$$c_{17} x^2 + c_{18} y^4 + c_{19} x) + \beta^2(c_{20} x^2 + c_{21} y^2 + c_{22} x)) \quad ...(11)$$

where $a_1, a_2,...b_1, b_2,...c_1, c_2,...c_{22}$ are constants. In substituting equations (9-11) in (5) we obtain an explicit form of the stream function and the axial velocity.

3-3 Streamlines on the cross-section of the pipe

The streamline projections on the cross-section of a curved pipe are represented by

$\psi = $ Constant

in derivation equations (14) and (15) from that one used in derivation of the motion equation (3 and 4) for the steady state case. The steady case can be reached from the unsteady, by setting all time derivatives to zero in equations (14) and (15), $j=1$ and reverse the sing of the stream function in equation (13) and define W_0 by $\frac{v}{a}(\frac{2a}{L})^{-1/2}$.

4-1 Method of solution

To solve the above system we will use successive approximation method (as a first step), which is equivalent to the perturbation solutions of ψ and w in ascending power of D. The solution of the above system can be developed by using

$$\psi(x,y,\tau)= D\psi_1(x,y,\tau)+D^2\psi_2(x,y,\tau)+\cdots$$
$$w(x,y,\tau)= w_0(x,y,\tau)+Dw_1(x,y,\tau)+\cdots\ldots(16)$$

We will limit ourselves to find the solution up to the first order in D, similar procedures can be used for higher order solutions, and the first order solution provide good accuracy for the purpose.

If we substitute the above expressions for ψ and w in (14) and (15) and equate coefficients of equal power in D; we obtain a series of relations from which $w_0, \psi_1, w_1, \cdots$ can be successively found. The equations are

$$\nabla^2 w_0 = \kappa^2 \frac{\partial v_0}{\partial \tau} - J\kappa^2 Cos(\tau) \quad \cdots (17)$$

$$\nabla^4 \psi_1 = \kappa^2 \frac{\partial}{\partial \tau}\nabla^2 \psi_1 + w_0 \frac{\partial w_0}{\partial y} -$$
$$\beta(\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}+\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}) \quad \cdots (18)$$

$$\nabla^2 w_1 = \kappa \frac{\partial w_1}{\partial \tau} + (\frac{\partial \psi_1}{\partial x}\frac{\partial}{\partial y}-\frac{\partial \psi_1}{\partial y}\frac{\partial}{\partial x})w_0 +$$
$$\beta(\frac{\partial w_0}{\partial x}\frac{\partial}{\partial y}-\frac{\partial w_0}{\partial y}\frac{\partial}{\partial x})\nabla^2\psi_1 + 2\beta\frac{\partial_2 w_0}{\partial x\partial y}$$
$$(\frac{\partial^2\psi_1}{\partial y^2}-\frac{\partial^2\psi_1}{\partial x^2})+$$
$$2\beta\frac{\partial_2\psi_1}{\partial x\partial y}(\frac{\partial^2 w_0}{\partial x^2}-\frac{\partial^2 w_0}{\partial y^2}) \quad \cdots (19)$$

The boundary conditions associated with the above equations are: -

$$\psi_1 = \frac{\partial\psi_1}{\partial x}=\frac{\partial\psi_1}{\partial y}=0 \quad on\ the boundary$$
$$w_n = 0, n=o,1,\cdots \quad on\ the boundary \cdots(20)$$

with constant velocity initially. By employing Galerkin variation method, it is found that the solution of equations (17-19), respectively, are given by

$$w_0 = \kappa^2(x^3 - xy^2 - x^2 + y^2)Cos(\tau)+$$
$$\kappa^4(x^3 - xy^2 - x^2 + y^2)$$
$$(n_1x^3 + n_2xy^2 + n_3x^2 + n_4y^2)Sin(\tau) \ldots(21)$$

$$\psi_1 = \kappa^4(x-1)^2(x^2-y^2)^2Cos(\tau)^2((n_5x^4y + n_6x^3y +$$
$$n_7x^2y^3 + n_8xy^3 + n_9x^2y + n_{10}y^3)+$$
$$\beta(n_{11}x^2y + n_{12}y^3 + n_{13}xy + n_{14}y))+$$
$$\kappa^6(x-1)^2(x^2-y^2)^2Cos(\tau)Sin(\tau)((n_{15}x^4y + n_{16}$$
$$x^5y + n_{17}y^7 + n_{18}x^7y + n_{19}x^6y + n_{20}y^5 +$$
$$n_{21}x^8y + n_{22}x^4y^3 + n_{23}x^4y^5 + n_{24}x^3y^5$$
$$+n_{25}x^2y^5 + n_{26}x^2y^7 + n_{27}xy^7 + n_{28}x^3 +$$
$$n_{29}y^3 + n_{30}xy^5 + n_{31}x^2y^3 + n_{32}x^5y^3$$
$$+n_{33}x^6y^3)+\beta(n_{34}y^5 + n_{35}y + n_{36}x^5y +$$
$$n_{37}x^4y + n_{38}x^3y^3 + n_{39}x^2y^3$$
$$+n_{40}x^3y + n_{41}xy^5 + n_{42}xy^3 +$$
$$n_{42}x^2y + n_{44}y^3)) \ldots(22)$$

where ψ is given by (5), which is a combination of the radial and vertical velocities.

Figures (2-10) illustrate the effect of the non-Newtonian parameter upon the secondary flow. In fig (2), where $\beta=0$ (Newtonian fluid), it is noted that there is a stagnation region and arises of two additional vortices (secondary vortex) near the outer wall of the pipe. This new result is not observed when the cross-section is square or rectangle [1,2]. For fixed value of Dean number and as β increases to 0.015 (non-Newtonian fluid), the two secondary vortices disappear, see figures (3) and (4). New stagnation region arises near the inner wall when the non-Newtonian parameter is greater than 0.015,fig (5). When β varies from 0.06 to 5, this stagnation region start to disappear and two new additional vortices appear near the inner wall of the pipe, see figures (6-10). Again this new result is not observed in the case of square and rectangular cross-section [1,2]. The fixed value of Dean number that we takes here is 0.007, it is clear that from the first equation of (5) we will get the same result regardless of this fixed value of Dean number.

3-4 The effect of D and β upon the axial velocity

The effects of Dean number and the non-Newtonian parameters upon the axial velocity are analyzed through figures (11-18) .The flow of Newtonian fluid through curved pipe is shown in figure (11). For fixed value of β (non-Newtonian fluid), it is notes that, as Dean number D increases, the flow becomes thicker, parallel to the outer wall and a boundary layer type flow is present for the entire wall, see figures (12-16), this is because of the absent of the secondary vortex. However, as β increases, the axial

velocity contours indicate that fluid of lower velocity is carried the pipe center by the secondary vortex and a substantial distortion of the boundary layer at the outer wall occurs, see figures (16-18).

4-Unsteady state

Let (u_1, v_1, w_1) be the velocity components in the directions of increases of the coordinates (x_1, y_1, ϕ).

$$-\frac{1}{L}\frac{\partial}{\partial\theta}\left(\frac{P}{\rho}\right)=JW_0\alpha Cos(\alpha t) \quad \cdots(12)$$

In this case, a sinusoidal pressure gradient in time with zero mean on the flow field is assumed such that where $JW_0\alpha$ is the amplitude of the applied pressure gradient and α is the angular frequency and t is the time. If $x_1=ax, y_1=ay, t=\tau/\alpha$ and v is the coefficient of kinematic viscosity, we may write

$$u_1=-\frac{v}{a}\frac{\partial\psi}{\partial y}, \ v_1=\frac{v}{a}\frac{\partial\psi}{\partial y}, w_1=W_0 w \ldots(13)$$

where $\psi(x,y,\tau)$ is the dimensionless stream function of the secondary flow in the cross-section. The equations of motion for $\psi(x,y,\tau)$ and $w(x,y,\tau)$ are

$$\nabla^2 w=-J\kappa^2 Cos(\tau)+\kappa^2\frac{\partial w}{\partial\tau}+$$

$$(\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x})w+\beta(\frac{\partial w}{\partial x}\frac{\partial}{\partial y}-\frac{\partial w}{\partial y}\frac{\partial}{\partial x})\nabla^2\psi+$$

$$2\beta\frac{\partial^2 w}{\partial x\partial y}(\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial x^2})+2\beta\frac{\partial^2\psi}{\partial x\partial y}(\frac{\partial^2 w}{\partial x^2}-\frac{\partial^2 w}{\partial y^2}) \ \cdots(14)$$

$$\nabla^4\psi=\kappa^2\frac{\partial}{\partial\tau}\nabla^2\psi+Dv\frac{\partial w}{\partial y}+$$

$$(\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x})\nabla^2\psi-$$

$$\beta D(\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}) \ \cdots(15)$$

Three parameters, a non-dimensional frequency parameter $\kappa=a(\alpha/v)^{(1/2)}$, the non-Newtonian parameter $\beta=\xi/\rho a^2$ and Dean number $D=2W_0^2 a^3/Lv^2$ control these equations. For mathematical convenient, we use different approach

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in derivation equations (14) and (15) from that one used in derivation of the motion equation (3 and 4) for the steady state case. The steady case can be reached from the unsteady, by setting all time derivatives to zero in equations (14) and (15), $j = 1$ and reverse the sing of the stream function in equation (13) and define W_0 by $\frac{v}{a}(\frac{2a}{L})^{-1/2}$.

4-1 Method of solution

To solve the above system we will use successive approximation method (as a first step), which is equivalent to the perturbation solutions of ψ and w in ascending power of D. The solution of the above system can be developed by using

$$\psi(x,y,\tau) = D\psi_1(x,y,\tau) + D^2\psi_2(x,y,\tau) + \cdots$$
$$w(x,y,\tau) = w_0(x,y,\tau) + Dw_1(x,y,\tau) + \cdots \cdots (16)$$

We will limit ourselves to find the solution up to the first order in D, similar procedures can be used for higher order solutions, and the first order solution provide good accuracy for the purpose.

If we substitute the above expressions for ψ and w in (14) and (15) and equate coefficients of equal power in D; we obtain a series of relations from which $w_0, \psi_1, w_1, \cdots$ can be successively found. The equations are

$$\nabla^2 w_o = \kappa^2 \frac{\partial v_o}{\partial \tau} - J\kappa^2 Cos(\tau) \quad \cdots (17)$$

$$\nabla^4 \psi_1 = \kappa^2 \frac{\partial}{\partial \tau}\nabla^2\psi_1 + w_o\frac{\partial w_o}{\partial y} -$$

$$\beta(\frac{\partial w_o}{\partial x}\frac{\partial^2 w_o}{\partial x\partial y} + \frac{\partial w_o}{\partial y}\frac{\partial^2 w_o}{\partial y^2}) \cdots (18)$$

$$\nabla^2 w_1 = \kappa\frac{\partial w_1}{\partial\tau} + (\frac{\partial\psi_1}{\partial x}\frac{\partial}{\partial y} - \frac{\partial\psi_1}{\partial y}\frac{\partial}{\partial x})w_o +$$

$$\beta(\frac{\partial w_o}{\partial x}\frac{\partial}{\partial y} - \frac{\partial w_o}{\partial y}\frac{\partial}{\partial x})\nabla^2\psi_1 + 2\beta\frac{\partial_2 w_o}{\partial x\partial y}$$

$$(\frac{\partial^2\psi_1}{\partial y^2} - \frac{\partial^2\psi_1}{\partial x^2}) +$$

$$2\beta\frac{\partial_2\psi_1}{\partial x\partial y}(\frac{\partial^2 w_o}{\partial x^2} - \frac{\partial^2 w_o}{\partial y^2}) \quad \cdots (19)$$

The boundary conditions associated with the above equations are: -

$$\psi_1 = \frac{\partial\psi_1}{\partial x} = \frac{\partial\psi_1}{\partial y} = 0 \quad on\ the\ boundary$$

$$w_n = 0, n = o,1,\cdots \quad on\ the\ boundary \cdots(20)$$

with constant velocity initially. By employing Galerkin variation method, it is found that the solution of equations (17-19), respectively, are given by

$$w_0 = \kappa^2(x^3 - xy^2 - x^2 + y^2)Cos(\tau) +$$
$$\kappa^4(x^3 - xy^2 - x^2 + y^2)$$
$$(n_1x^3 + n_2xy^2 + n_3x^2 + n_4y^2)Sin(\tau) \quad \ldots(21)$$

$$\psi_1 = \kappa^4(x-1)^2(x^2-y^2)^2Cos(\tau)^2((n_5x^4y + n_6x^3y +$$
$$n_7x^2y^3 + n_8xy^3 + n_9x^2y + n_{10}y^3) +$$
$$\beta(n_{11}x^2y + n_{12}y^3 + n_{13}xy + n_{14}y)) +$$
$$\kappa^6(x-1)^2(x^2-y^2)^2Cos(\tau)Sin(\tau)((n_{15}x^4y + n_{16}$$
$$x^5y + n_{17}y^7 + n_{18}x^7y + n_{19}x^6y + n_{20}y^5 +$$
$$n_{21}x^8y + n_{22}x^4y^3 + n_{23}x^4y^5 + n_{24}x^3y^5$$
$$+ n_{25}x^2y^5 + n_{26}x^2y^7 + n_{27}xy^7 + n_{28}x^3 +$$
$$n_{29}y^3 + n_{30}xy^5 + n_{31}x^2y^3 + n_{32}x^5y^3$$
$$+ n_{33}x^6y^3) + \beta(n_{34}y^5 + n_{35}y + n_{36}x^5y +$$
$$n_{37}x^4y + n_{38}x^3y^3 + n_{39}x^2y^3$$
$$+ n_{40}x^3y + n_{41}xy^5 + n_{42}xy^3 +$$
$$n_{42}x^2y + n_{44}y^3)) \quad \ldots(22)$$

$$w_1 = \kappa^6((x-1)(x^2-y^2)Cos(\tau)^3((n_{45}x^8 y^2 +$$
$$n_{46}x^4 y^6 + n_{47}x^6 y^4 + n_{48} x^7 y^2 + n_{49}x^5 y^4 +$$
$$n_{50}x^3 y^6 + n_{51}xy^8 + n_{52}y^8 + n_{53}x^4 y^8 +$$
$$n_{54}x^3 y^8 + n_{55}x^9 y^2 + n_{56}x^{10} y^2 + n_{57}x^6$$
$$y^6 + n_{58}x^7 y^4 + n_{59}x^2 y^8 + n_{60}x^5 y^6 +$$
$$n_{61}x^8 y^4 + n_{62}x^7 + n_{63}x^8 + n_{64}x^4 y^4 + n_{65}$$
$$x^3 y^4 + n_{66}x^2 y^6 + n_{67}xy^6 + n_{68}x^5 y^2 +$$
$$n_{69}x^6 y^2 + n_{70}x^9 + n_{71}x^{11} + n_{72}x^{10} + n_{73}x^{12})$$
$$+ \beta(n_{74}x + n_{75}x^2 y^8 + n_{76}xy^8 + n_{77}x^8 y^2 +$$
$$n_{78}x^6 y^4 + n_{79}x^4 y^6 + n_{80}x^9 + n_{81}x^{10} + n_{82}$$
$$y^8 + n_{83}xy^2 + n_{84}x^5 y^4 + n_{85}x^7 y^2 + n_{86} x^3 y^6 +$$
$$n_{87}x^2 + n_{88}x^3 + n_{89}y^2 + n_{90}x^4 + n_{91}$$
$$x^2 y^2 + n_{92}x^6 y^2 + n_{93}x^5 y^2 + n_{94}x^4 y^2 +$$
$$n_{95}x^4 y^4 + n_{96}x^3 y^4 + n_{97}x^2 y^4 + n_{98}x^2 y^6 +$$
$$n_{99}xy^6 + n_{100}x^3 y^2 + n_{101}xy^4 + n_{102}y^4 + n_{103}x^8 +$$
$$n_{104}x^7 + n_{105}x^6 + n_{106}y^6 + n_{107}x^5)$$
$$+ \beta^2(n_{108}x^2 + n_{109}y^2 + n_{110}x + n_{111})) \dots(23)$$

Where $n_1, n_2, \dots n_{111}$ are constants. In equations (21-23), if we set $\tau = 0, j = 1$ and $\kappa = 1$ we obtain the corresponding solution in the case of steady state (equations (9-11)), and in addition to that if $\beta = 0$ we obtain the corresponding solution for Newtonian fluid flow in curved pipe with triangular cross-section. The solution for stream function ψ and the axial velocity w can be obtained by substituting equations (21), (22) and (23) into (16).

4-2 Streamlines on the cross-section of the pipe

The streamline projections on the cross-section of a curved pipe are represented by

ψ = Constant

where ψ is given by (16), which is a combination of the radial and vertical velocities. Figures (19-33) illustrate the effect of the non-dimensional parameters, $\kappa, \tau, and \beta$, which controlled the motion equations upon the stream

function. It is found through all of these figures that there are two symmetrical regimes of the secondary flow in the upper and lower half of the cross-section. Also, the shape of streamlines, closed curves, is changes and accordingly the center of the vortices have been displaced. In addition to that it is noted that the intensity of the stream function has its maximum value in the middle of the upper or the lower half of the cross-section and decreases gradually whenever we moves toward the walls. The intensity of the main vortices will mentioned with title of the figure and for the secondary vortex on the graph.

The effect of the frequency parameter κ is shown through figures (19-22). The dimensionless time τ was set at 0.39, the non-Newtonian parameter β at 0.01 and κ varies from 1.2 to 5. It is found that there is generation of stagnation region near the outer wall and as κ increases a new secondary vortex (move in opposed direction of the main vertices) replaced that region, i.e. transition from two vortex structure to four vortex structure occur at κ between 1.2 and 1.5, see figures (19) and (20). With the increasing of κ, the secondary vortex become beiger and pushes the main vortices toward the outer wall of the pipe, see figures (21) and (22). The intensity of the stream function increases as κ increase.

The effect of dimensionless time τ has been found same as the effect of the frequency parameter with exceptions that, as τ increases the new secondary vortex disappeared gradually and the intensity of the stream function increases, see figures (23-26).

Finally, the effect of the non-Newtonian parameter β was analyzed by setting the frequency parameter at 1.32, the dimensionless time at 0.39

and β varies from 0.02 to 0.11. It is found that as β' increase, there is a transition from the structure to four structure at β < 0.02, the new secondary vortex become beggar as β increases and pushes the main vortices toward the outer wall of the cross-section. At β >0.03, the main vortices gradually become smaller and disappears, i.e. the new secondary vortex occupy all the space of the upper and lower half of the cross-section, see figures (27-33).

In all of the above cases the Dean number was set at 0.01 and its effect same as mentioned in section (3-3).

4-3 The effect of D, τ, β, and κ on axial velocity

The effect of the dimensionless parameter that controlled the motion equation upon the axial velocity is illustrated through figures (34-45).

In order to see the effect of Dean number upon the axial velocity, the dimensionless time τ was set at 0.5,the non-Newtonian parameter β at 0.01,the frequency parameter κ at 1.32 and Dean number varies from 0 to 20. It is found, as Dean number increase, there is small displacement in center of the vortex toward the outer wall and the intensity of the axial velocity for the same points decreased, see figures (34-36).

When the flow reach the fully development state, it noted that as the time τ increase there is increase in the intensity of the axial velocity, see figures (37-39).

The effect of β and κ are similar in sense that there is displacement in the center of the axial velocity toward the inner wall, but the displacement when β increase is greater than when κ increased, see figures (40-45).

Steady state figures
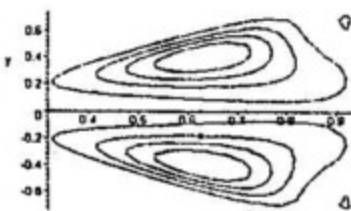
Fig(2).Secondary flow for D=0.007,β=0. (Newtonian fluid)Ψ=-1.6E-09 to -2.02E-10.
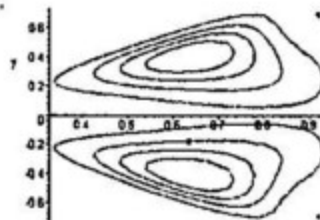
Fig(3).Secondary flow for D=0.007, β=0.01, Ψ =-1.58E-09 to -1.87E-10
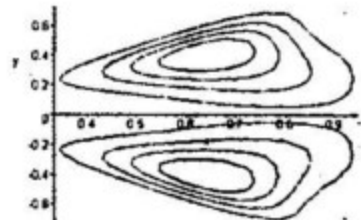
Fig(4).Secondary flow for D=0.007, β=0.015, Ψ =-1.53E-09 to -2.06E-10
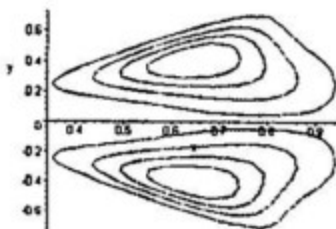
Fig(5).Secondary flow for D=0.007,β=0.02, Ψ =-1.57E-09 to -2.37E-10
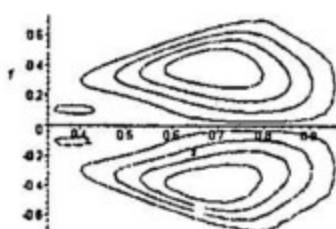
Fig(6).Secondary flow for D=0.007, β=0.06, Ψ =-1.58E-09 to -2.12E-10
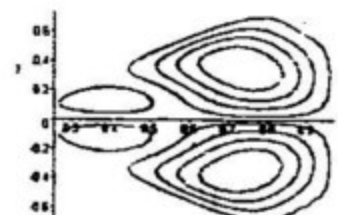
Fig(7).Secondary flow for D=0.007, β=0.1, Ψ = -1 64E-09 to -2.414E-10
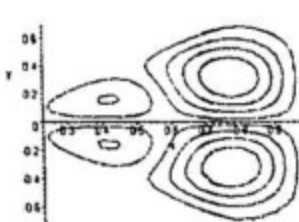
Fig(8).Secondary flow for D=0.007,β=0.2,   Fig(9).Secondary flow for D=0.007,   Fig(10).Secondary flow for D=0.007,
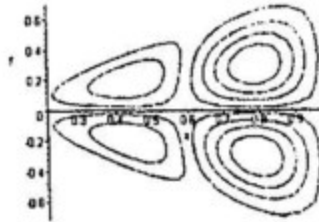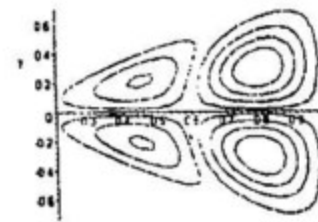
Ψ =-1.91E-09 to -2.28E-10   β=1, Ψ =-3.06E-09 to -5.60E-10   β=5, Ψ =-5.83E-08 to -8.03E-09
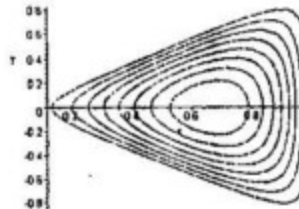
Fig (11).The axial velocity for D=0.007,β=0   Fig (12). The axial velocity for D=3,   Fig (13). The axial velocity for D=4
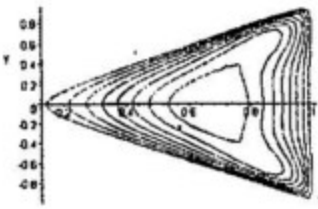
β=0w=3.47E-04 to 4.43E-05,(Newtonian fluid)   β=0.01,w=.18 to 1.42E-02   β=0.01, w=.13 to 1.41E-02
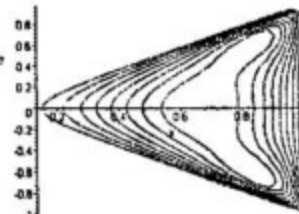
Fig (14). The axial velocity for D=4.5,   Fig (15). The axial velocity for D=8   Fig (16). The axial velocity for D=8
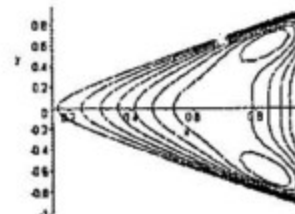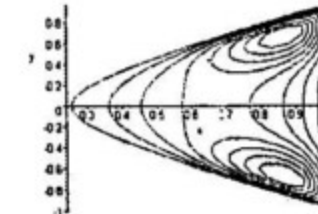
β=0.01, w=.12 to 1.35E-02   β=0.01, w= .1 to 1.06E-02   β=0.01, w=.8 to -.14
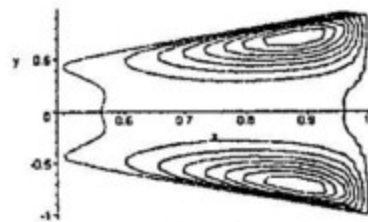
Fig(17).The axial velocity for D=8,   Fig(18).The axial velocity for D=8,
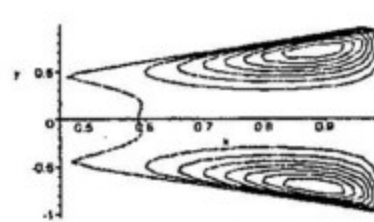
β=0.08,w=.93 to 0.02   β=0.0837, w=0.9 to 2.71E-03

Unsteady state figures
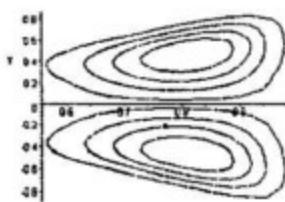
Fig(19).Secondary flow for D=0.01,   Fig(20).Secondary flow for D=0.01,   Fig(21).Secondary flow for D=0.01,
κ=1.2,τ=0.39,β=0.01,Ψ=6.32E-07 to   κ=1.5,τ=0.39,β=0.01,Ψ=1.82E-06 to   κ=2, τ=0.39,β=0.01,Ψ=8.24E-06 to
9.53E-08   2.73E-07   1.04E-06
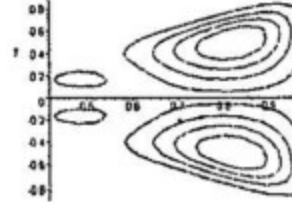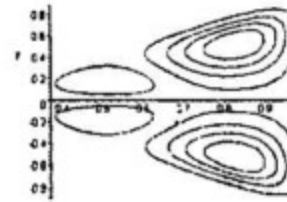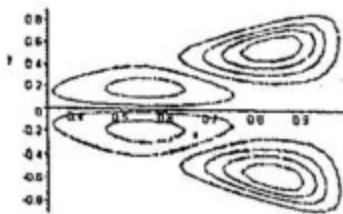
Fig(22).Secondary flow for D=0.01, κ=5,τ=0.39,β=0.01,Ψ=1.55E-03 to 2.53E-04
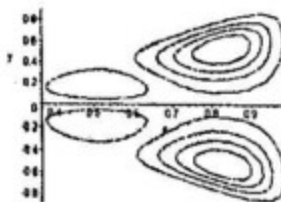
Fig(23).Secondary flow for D=0.01, κ=1.32,τ=1.85,β=0.01,Ψ=3.52E-07 to 1.03E-08
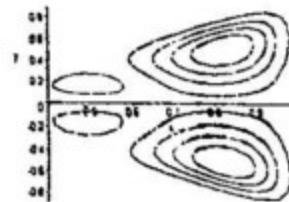
Fig(24).Secondary flow for D=0.01, κ=1.32, τ=2.5,β=0.01,Ψ=4.43E-07 to 1.13E-07
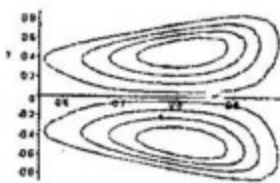
Fig(25).Secondary flow for D=0.01, κ=1.32,τ=2.8,β=0.01,Ψ=6.07E-07 to 2.17E-07
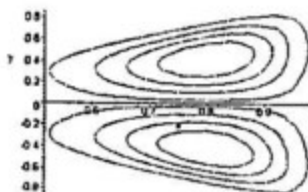
Fig(26).Secondary flow for D=0.01, κ=1.32,τ=3,β=0.01,Ψ=8.36E-07 to 2.16E-07
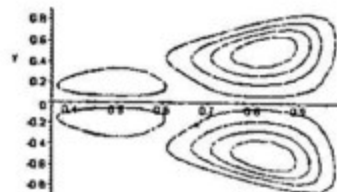
Fig(27).Secondary flow for D=0.01, κ=1.32, τ=0.39,β=0.02,Ψ=8.71E-07 to 1.21E-07
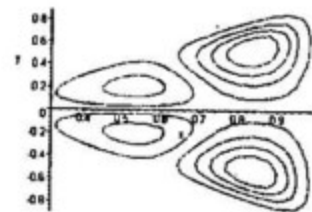
Fig(28).Secondary flow for D=0.01, κ=1.32,τ=0.39,β=0.03,Ψ=7.3E-07 to 9.97E-08
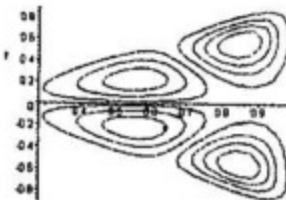
Fig(29).Secondary flow for D=0.01, κ=1.32,τ=0.39,β=0.04,Ψ=6.62E-07 to 7.44E-08
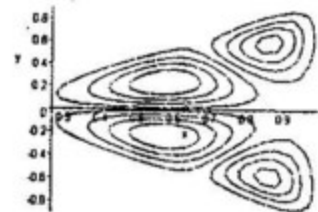
Fig(30).Secondary flow for D=0.01, κ=1.32, τ=0.39,β=0.05,Ψ=6.19E-07 to 8.17E-08

Fig(31).Secondary flow for D=0.01, κ=1.32,τ=0.39,β=0.08,Ψ=1.19E-06 to 1.8E-07
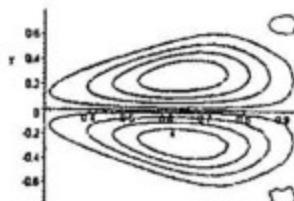
Fig(32).Secondary flow for D=0.01, κ=1.32, τ=0.39,β=0.1,Ψ=1.61E-06 to 2.13E-07
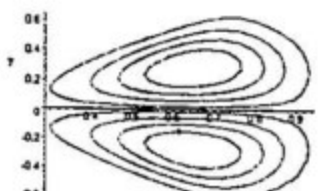
Fig(33).Secondary flow for D=0.01, κ=1.32,τ=0.39,β=0.11,Ψ=1.8E-06 to 2.98E-07
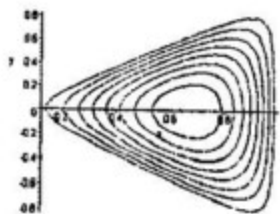
Fig(34).Secondary flow for D=0, κ=1.32,τ=0.5,β=0.01,w=.20847 to 2.641245E-02
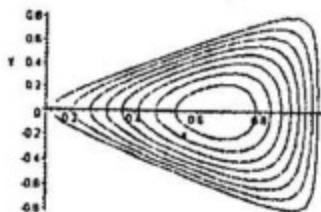
Fig(35).Secondary flow for D=10, κ=1.32,τ=0.5,β=0.01,w=.20844 to 2.641242E-02
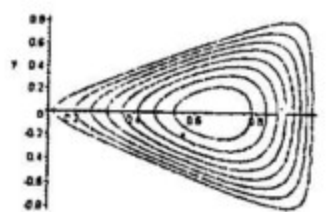
Fig(36).Secondary flow for D=20, κ=1.32, τ=0.5,β=0.01,w=.20840 to 2.641239E-02
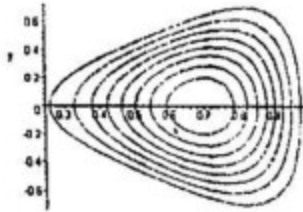
Fig(37).Secondary flow for D=10,
κ=1.32,τ=1.57,β=0.01,w=1.66E-02
to 2.17E-03
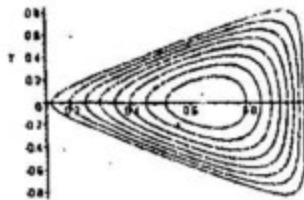
Fig(38).Secondary flow for D=10,
κ=1.32,τ=2.1,β=0.01,w=.101
to 1.25E-02
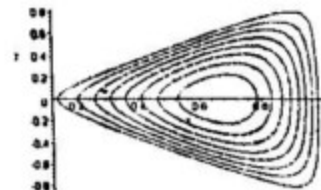
Fig(39).Secondary flow for D=10,
κ=1.32, τ=3.14,β=0.01,w=.231
to 2.88E-02
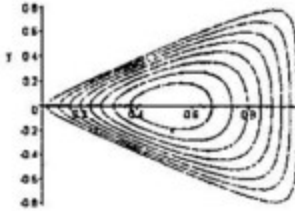
Fig (40).Secondary flow for D=10,
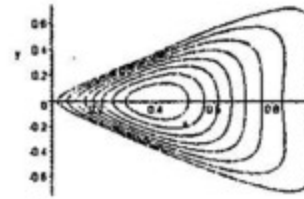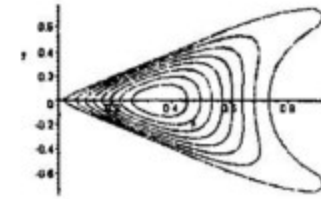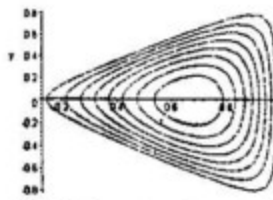κ=1.32,τ=1.05,β=0.2,w=.144
to 4.519E-02

Fig(41).Secondary flow for D=10,
κ=1.32,τ=1.05,β=0.3,w=.177
to .106

Fig(42).Secondary flow for D=10,
κ=1.32, τ=1.05,β=0.4w,=1.464
to 1.134

Fig(43).Secondary flow for D=10,
κ=1.2,τ=1.05,β=0.01,w=.104 to
1.274E-02
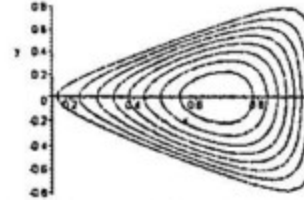
Fig(44).Secondary flow for D=1
κ=2,τ=1.05,β=0.01,w=.337 to
3.980E-02
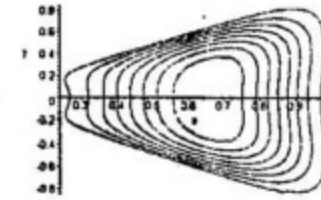
Fig(45).Secondary flow for D=10,
κ=7, τ=1.05,β=0.01,=13.181 to
2.715

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الجريان المستقر واللامستقر لمائع لانيوتيني في الأنابيب المنحنية ذات المقاطع المثلثية الشكل

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المستخلص

يتناول هذا البحث دراسة عددية لجريان مستقر واللامستقر لمائع لانيوتيني في منطقة محاطة بمثلث قائم الزاوية . وقد تناول البحث بصورة خاصة الجريان الثانوي في المقطع العرضي الناتج من الضغط المتدرج ففي الحالة الأولى تم تحليل الجريان المستقر حيث وجد أن معادلات الحركة، التي تصف حركة المائع، يتحكم بها وسيطين عديمة الأبعاد وهما ، عدد دين و الوسيط اللانيوتيني . وفي الحالة الثانية تم تحليل الجريان المستقر حيث وجد أن معادلات الحركة يتحكم بها بالإضافة إلى الوسيطين المذكورة أعلاه وسيط ثالث وهو وسيط التردد . وقد تم أيجاد الحل في الحالة الأولى بعد نشر المتغيرات من خلال عدد دين بينما في الحالة الثانية بعد نشرها من خلال عدد دين أعيد نشرها من خلال وسيط التردد. المعادلات الناتجة تم حلها باستخدام طريقة كرين . في كلتا الحالتين قمنا بدراسة تأثير الوسائط المذكورة أعلاه على كل من الحركة الثانوية والسرعة المحورية.