Oscillation of Nonlinear First Order Neutral Differential Equations

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Abstract:

In this paper, the author established some new integral conditions for the oscillation of all solutions of nonlinear first order neutral delay differential equations. Examples are inserted to illustrate the results.

Introduction:

Consider the first order nonlinear neutral delay differential equation:

$$[x(t)+p(t)x(\tau(t))]' + q(t)f(x(\sigma(t))) = 0 \qquad (1)$$

Subject to the conditions:

$(c1)$ $p(t)\in C[R,R]$, $\tau(t)$ and $\sigma(t)$ are positive nondecreasing continuous functions, such that $\lim_{t\to\infty}\sigma(t)=\lim_{t\to\infty}\tau(t)=\infty$

$(c2)$ $q:[t_0,\infty)\to R$ is continuous function.

$(c3)$ $f:R\to R$ is continuous function with $uf(u)>0$ for $u\neq 0$,

and there is a positive constant M such that $f(u)/u^{\alpha}\geq M>0$ where α is a ratio of odd positive integers.

If we let $p(t)=\max\{\tau(t),\sigma(t)\}$ and $T\geq t_0$ then by a solution of equation (1), we mean a continuous function $x:[t_0,\infty)\to R$ such that $x(t)+p(t)x(\tau(t))$ is continuously differentiable for $t\geq t_0$, and $x(t)$ satisfies equation (1) for all $t\geq t_0$. A solution of equation (1) is said to be oscillatory if it has arbitrary large zero and nonoscillatory otherwise.

In [4], Gopasamy, Lalli, and Zhang considered the linear equation

$$(x(t)+px(t-\tau))' + q(t)x(t-\sigma) = 0 \qquad (2)$$

Where $-1<p\leq 0$ and proved that if

$$\liminf_{t\to\infty}\int_{t-\sigma}^{t} q(s)\,ds > 1+p,$$

then all solutions of equation (2) are oscillatory. For additional results on the oscillatory behavior of solutions of the linear equation (2), we refer the reader to the monographs by Bainow and Mishev [2], Erbe, Kong, and Zhang [3], and Gyǒri and Ladas [8] as well as the papers of Agarwal and Saker [1], Pahri [15], Saker and Elabbasy [17], Tanaka [18], and Zhou [21] And the references contained therein.

In [5], Graef et al. considered the nonlinear equation

$$(x(t)+px(t-\tau))' + q(t)f(x(t-\omega)) = 0, \qquad (3)$$

with f nondecreasing, sublinear, $-1<p\leq 0$, and they proved that if

$$\int_{t_0}^{\infty} q(t)\,dt = \infty,$$

then every solution of equation (3) is oscillatory. They also proved a similar result for equation (3) where f is superlinear and $p<-1$.

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Section 2, continuous some basic lemmas that are needed to prove my main results, in Section 3, there are some new integral conditions for the oscillation of all solutions of equation (1). This research includes examples to illustrate the main theorem.

Some Basic Lemmas

In this section, established some lemmas for the case $\alpha = 1$. These lemmas will be used to proved the main results.

Lemma 1 Suppose that

$q(t) \geq 0, \tau(t) < t, \sigma(t) < t, p_2 \geq p(t) \geq p_1 > 1, \sigma(t) < \tau(t) \quad for \quad t \geq t_0$

$$and \qquad \limsup_{t\to\infty} \int_t^{\sigma^{-1}(\tau(t))} q(s)\, ds > 0 \qquad (4)$$

if $x(t)$ is an eventually positive solution of equation (1) then

$$\liminf_{t\to\infty} \frac{z(\tau^{-1}(\sigma(t)))}{z(t)} < \infty \qquad (5).$$

Proof Let $x(t)$ is an eventually positive solution of equation (1) for $t \geq t_0$ then

$$z(t) = x(t) + p(t)x(\tau(t))$$

Which means, $z(t) > 0$ and so $z'(t) \leq 0$ and hence $z(t)$ is decreasing, we have

$$p(t)x(\tau(t)) = z(t) - x(t)$$
$$z(\tau^{-1}(t)) = x(\tau^{-1}(t)) + p(\tau^{-1}(t))x(t)$$
$$z(\tau^{-1}(t)) \geq p(\tau^{-1}(t))x(t) \geq p_1 x(t)$$

on the other hand

$$p_2 x(\tau(t)) \geq z(t) - x(t)$$
$$p_2 p_1 x(\tau(t)) \geq p_1 z(t) - p_1 x(t) \geq p_1 z(t) - z(t)$$

then

$$x(\tau(t)) \geq \frac{p_1 - 1}{p_2 p_1} z(t)$$

Or

$$x(\sigma(t)) \geq \frac{p_1 - 1}{p_2 p_1} z(\tau^{-1}(\sigma(t)))$$

from (1) and the last inequality we get

$$z'(t) + Mq(t)x(\sigma(t)) \leq 0$$
$$z'(t) + M\frac{p_1 - 1}{p_2 p_1} q(t) z(\tau^{-1}(\sigma(t))) \leq 0 \qquad (6)$$

by lemma (1) in [10] follow that

$$\liminf_{t\to\infty} \frac{z(\tau^{-1}(\sigma(t)))}{z(t)} < \infty$$

Lemma 2

Assume that

$q(t) \geq 0, \sigma(t) < \tau(t), p_2 \geq p(t) \geq p_1 > 1 \quad for \quad t \geq t_0$

if $x(t)$ is an eventually positive solution of equation (1) then

$$\int_t^{\sigma^{-1}(\tau(t))} q(s)\, ds \leq \frac{p_1 p_2}{M(p_1 - 1)} \qquad (7)$$

for sufficiently large t.

Proof We have $x(t) \geq 0$ and $q(t) \geq 0$, then $z'(t) \leq 0$, and $z(t) \geq 0$, if we integrating (6) from t to $\sigma^{-1}(\tau(t))$ we obtain

$$z(\sigma^{-1}(\tau(t))) - z(t) + M\frac{p_1 - 1}{p_1 p_2} \int_t^{\sigma^{-1}(\tau(t))} q(s) z(\tau^{-1}(\sigma(s)))\, ds \leq 0$$
$$z(\sigma^{-1}(\tau(t))) - z(t) + M\frac{p_1 - 1}{p_1 p_2} z(t) \int_t^{\sigma^{-1}(\tau(t))} q(s)\, ds \leq 0$$
$$z(\sigma^{-1}(\tau(t))) + (M\frac{p_1 - 1}{p_1 p_2} \int_t^{\sigma^{-1}(\tau(t))} q(s)\, ds - 1)\, z(t) \leq 0$$

then

$$M\frac{p_1 - 1}{p_1 p_2} \int_t^{\sigma^{-1}(\tau(t))} q(s)\, ds - 1 \leq 0$$

(7) follow directly from the last inequality.

Oscillation Results

In this section, get integral conditions for the oscillation of all solutions of equation (1). Consider the case $\alpha = 1$.

Theorem1 Assume that

$q(t) > 0, \sigma(t) < \tau(t), p_2 \geq p(t) \geq p_1 > 1;$

$$\int^{\infty} q(t) \ln\left[\frac{eM(p_1 - 1)}{p_1 p_2} \int_t^{\sigma^{-1}(\tau(t))} q(s)\, ds\right] dt = \infty \qquad (8),$$

and (4) holds, then every solution of (1) is oscillatory.

Proof For the sake of contradiction we may suppose that $x(t)$ is an eventually positive solution of equation (1) then $z(t) > 0$ and $z'(t) < 0$ for $t \geq t_0$, from (6) we get

$$z'(t) + M\,\frac{p_1 - 1}{p_2 p_1} q(t) z(\tau^{-1}\sigma(t))) \leq 0$$

using the integral identity $z(t_1) = z(t)\, e^{-\int_{t_1}^{t} \frac{z'(s)}{z(s)} ds}$ in (6) we obtain

$$-\frac{z'(t)}{z(t)} \geq M\,\frac{p_1 - 1}{p_2 p_1} q(t)\, e^{\int_{\tau^{-1}(\sigma(t))}^{t} -\frac{z'(s)}{z(s)} ds} \qquad (9).$$

Applying the inequality $e^{rx} \geq x + \frac{\ln(er)}{r}$ for $x > 0$ and $r > 0$ to the last inequality we get

$$-\frac{z'(t)}{z(t)} \geq \frac{M(p_1-1)}{p_1 p_2} q(t)\, e^{(\lambda(t)\frac{1}{\lambda(t)} \int_{\tau^{-1}(\sigma(t))}^{t} \frac{-z'(s)}{z(s)} ds)}$$

$$\geq \frac{M(p_1-1)}{p_1 p_2} \frac{q(t)}{\lambda(t)} [\int_{\tau^{-1}(\sigma(t))}^{t} -\frac{z'(s)}{z(s)} ds + \ln(c\lambda(t))]$$

where

$$\lambda(t) = \frac{M(p_1-1)}{p_1 p_2} \int_{t}^{\sigma^{-1}(\tau(t))} q(s)\, ds$$

it follows that

$$-\frac{z'(t)}{z(t)} \int_{t}^{\sigma^{-1}(\tau(t))} q(s)\, ds - q(t) \int_{\tau^{-1}(\sigma(t))}^{t} -\frac{z'(s)}{z(s)} ds \geq q(t) \ln(\frac{eM(p_1-1)}{p_1 p_2} \int_{t}^{\sigma^{-1}(\tau(t))} q(s)\, ds)$$

for $T \leq t \leq u, T < \tau^{-1}(\sigma(u))$ we have

$$\int_{T}^{u} \int_{t}^{\sigma^{-1}(\tau(t))} \frac{-z'(t)}{z(t)} q(s)\, ds\, dt - \int_{T}^{u} \int_{\tau^{-1}(\sigma(t))}^{t} q(t)(-\frac{z'(s)}{z(s)})\, ds\, dt \geq \int_{T}^{u} q(t) \ln(\frac{eM(p_1-1)}{p_1 p_2} \int_{t}^{\sigma^{-1}(\tau(t))} q(s)\, ds) dt \qquad (10)$$

from the increasing of order of integrating we can conclude that

$$\int_{T}^{u} \int_{\tau^{-1}(\sigma(t))}^{t} q(t)(-\frac{z'(s)}{z(s)})\, ds\, dt \geq \int_{T}^{\tau^{-1}(\sigma(u))} \int_{s}^{\sigma^{-1}(\tau(s))} q(t)(-\frac{z'(s)}{z(s)})\, dt\, ds$$

$$= \int_{T}^{\tau^{-1}(\sigma(u))} \int_{t}^{\sigma^{-1}(\tau(t))} q(s)(-\frac{z'(t)}{z(t)})\, ds\, dt$$

use the last inequality in (10) we obtain

$$\int_{\tau^{-1}(\sigma(u))}^{u} -\frac{z'(t)}{z(t)} (\int_{t}^{\sigma^{-1}(\tau(t))} q(s)\, ds)\, dt \geq \int_{T}^{u} q(t) \ln(\frac{eM(p_1-1)}{p_1 p_2} \int_{t}^{\sigma^{-1}(\tau(t))} q(s)\, ds) dt$$

using lemma 2 we have

$$\frac{p_1 p_2}{M(p_1-1)} \int_{\tau^{-1}(\sigma(u))}^{u} -\frac{z'(t)}{z(t)} dt \geq \int_{T}^{u} q(t) \ln(\frac{eM(p_1-1)}{p_1 p_2} \int_{t}^{\sigma^{-1}(\tau(t))} q(s)\, ds) dt$$

or

$$\ln \frac{z(\tau^{-1}(\sigma(u)))}{z(u)} \geq \frac{M(p_1-1)}{p_1 p_2} \int_{T}^{u} q(t) \ln(\frac{eM(p_1-1)}{p_1 p_2} \int_{t}^{\sigma^{-1}(\tau(t))} q(s)\, ds) dt$$

according to (8) we must have $\lim_{t\to\infty} \frac{z(\tau^{-1}(\sigma(t)))}{z(t)} = \infty$ which contradict lemma 1.

Example1

$$[x(t) + (a + \cos(t))x(t-\pi)]' + ((1-a) + 2\cos(t))x(t - \frac{3\pi}{2}) = 0, \quad t > 0,$$

such that $a > 1$, a sample verification yields that the conditions of theorem 1 are met. Hence all solutions of above equation oscillate for instance $x(t) = \cos(t)$ is such solution.

Theorem2 Assume that $q(t) \geq 0, \sigma(t) < \tau(t), 0 < p_2 \leq p(t) \leq p_1 \leq 1$ and there exists $k > 0$ such that

$$\frac{1}{e} \leq \int_{\tau^{-1}(\sigma(t))}^{t} q(s)\, ds < k \qquad (11)$$

then every solution of equation (1) is oscillatory.

Proof Suppose that equation (1) has an eventually positive solution $x(t)$, then from (9)

$$-\frac{z'(t)}{z(t)} \geq M\,\frac{p_1 - 1}{p_2 p_1} q(t)\, e^{\int_{\tau^{-1}(\sigma(t))}^{t} -\frac{z'(s)}{z(s)} ds}$$

let $B(t) = \exp(e \int_{\tau^{-1}(\sigma(t))}^{t} q(s)\, ds)$ from (11) we conclude that $k_1 \geq B(t) \geq e$ for some $k_1 > 0, t \geq t_0$ and we claim that (11) implies that $\lim_{t\to\infty} \int_{t_0}^{t} q(s)\, ds = \infty$, otherwise

$$\lim_{t\to\infty} \int_{t_0}^{t} q(s)\, ds < \infty.$$

So we can choose $t_1 \geq t_0$ large enough such that $\int_{t_1}^{\infty} q(s)\, ds < \frac{1}{e}$ which contradict (11) then

$$-\frac{z'(t)}{z(t)} B(t) \geq \frac{M(p_1-1)}{p_1 p_2} B(t)\, q(t)\, e^{(\frac{1}{B(t)} B(t) \int_{\tau^{-1}(\sigma(t))}^{t} -\frac{z'(s)}{z(s)} ds)}$$

using the inequality $e^{\frac{x}{r}} \geq 1+\frac{x}{r^2}, x \geq 0, r \geq 1$

the last inequality will be

$$-\frac{z'(t)}{z(t)}B(t) \geq \frac{M(p_1-1)}{p_1 p_2} B(t)\, q(t)\,(1+\frac{B(t)}{B^2(t)}\int_{\tau^{-1}(\sigma(t))}^{t} -\frac{z'(s)}{z(s)}\,ds)$$

$$-\frac{z'(t)}{z(t)}B(t) \geq \frac{M(p_1-1)}{p_1 p_2}\; q(t)\,(B(t)+\int_{\tau^{-1}(\sigma(t))}^{t} -\frac{z'(s)}{z(s)}\,ds)$$

for $T \leq t \leq u, T < \tau^{-1}(\sigma(u))$ we get

$$\int_T^u -\frac{z'(t)}{z(t)}B(t)\,dt - \frac{M(p_1-1)}{p_1 p_2}\int_T^u q(t)\int_{\tau^{-1}(\sigma(t))}^{t} -\frac{z'(s)}{z(s)}\,ds\,dt \geq \frac{M(p_1-1)}{p_1 p_2}\int_T^u q(t)\,B(t)\,dt$$

by interchange the order of the integrating we have

$$\int_T^u -\frac{z'(t)}{z(t)}B(t)\,dt - \frac{M(p_1-1)}{p_1 p_2}\int_T^{\tau^{-1}(\sigma(u))} -\frac{z'(t)}{z(t)}\int_t^{\sigma^{-1}(\tau(t))} q(s)\,ds\,dt \geq \frac{M(p_1-1)}{p_1 p_2}\int_T^u q(t)\,B(t)\,dt$$

but $B(t) = e^{(e\int_{\tau^{-1}(\sigma(t))}^{t} q(s)\,ds)}$ then

$B(\sigma^{-1}(\tau(t))) = e^{(e\int_t^{\sigma^{-1}(\tau(t))} q(s)\,ds)}, B(t) < k_1 \quad and \quad B(\sigma^{-1}(\tau(t))) \geq e$

so the last inequality lead to

$$k_1\int_T^u -\frac{z'(t)}{z(t)}\,dt - \frac{eM(p_1-1)}{p_1 p_2}\int_T^{\tau^{-1}(\sigma(u))} -\frac{z'(t)}{z(t)}\,dt \geq \frac{M(p_1-1)}{p_1 p_2}\int_T^u q(t)\,B(t)\,dt$$

where $k_1 \leq \frac{eM(p_1-1)}{p_1 p_2}$

$$\frac{eM(p_1-1)}{p_1 p_2}(\int_T^u -\frac{z'(t)}{z(t)}\,dt - \int_T^{\tau^{-1}(\sigma(u))} -\frac{z'(t)}{z(t)}\,dt) \geq \frac{eM(p_1-1)}{p_1 p_2}\int_T^u q(t)\,dt$$

then $\int_{\tau^{-1}(\sigma(u))}^{u} -\frac{z'(t)}{z(t)}\,dt \geq \int_T^u q(t)\,dt$ as $t \to \infty$ we get $\lim_{t\to\infty}\frac{z(\tau^{-1}(\sigma(t)))}{z(t)} = \infty$

which is contradict lemma 1.

Example 2 Consider the equation

$$[x(t)+\frac{1}{2}(1+\sin^2(t))x(t-2\pi)]' + \frac{3}{2}(1+\sin^2(t))x(t-\frac{5\pi}{2}) = 0, \quad t>0,$$

which satisfies all conditions of theorem 2, therefore each solution of above equation oscillate for example $x(t) = \sin(t)$ is an oscillatory solution.

Theorem 3 Suppose that

$q(t) \leq 0,\ p_1 \geq p(t) \geq p_2 > 1, \tau^{-1}(\sigma(t)) > t, \lim_{t\to\infty}\tau^{-1}(\sigma(t)) = \infty \quad for \quad t \geq t_0$

and

$$\liminf_{t\to\infty} M\,\frac{p_1-1}{p_1 p_2}\int_t^{\tau^{-1}(\sigma(t))} |q(s)|\,ds > \frac{1}{e} \qquad (12).$$

Then every solution of equation (1) is oscillatory.

Proof For the sake of contradiction we may suppose that $x(t)$ is an eventually positive solution of equation (1) then $z'(t) \geq 0$ and $z(t) > 0$ and we have

$z(t) = x(t) + p(t)x(\tau(t)$

$p(t)x(\tau(t)) = z(t) - x(t)$

$z(\tau^{-1}(t)) = x(\tau^{-1}(t)) + p(\tau^{-1}(t))x(t)$

$z(\tau^{-1}(t)) \geq p(\tau^{-1}(t))x(t) \geq p_1 x(t)$

on the other hand we have

$p_2 x(\tau(t)) \geq z(t) - x(t)$

$p_1 p_2 x(\tau(t)) \geq p_1 z(t) - p_1 x(t) \geq p_1 z(t) - z(t)$

$p_1 p_2 x(\tau(t)) \geq p_1 z(t) - z(t)$

$p_1 p_2 x(\tau(t)) \geq z(t)(p_1 - 1)$

then we get

$x(\tau(t)) \geq \frac{p_1-1}{p_1 p_2} z(t) \quad or \quad x(\sigma(t)) \geq \frac{p_1-1}{p_1 p_2} z(\tau^{-1}(\sigma(t)))$

from (1) we can get

$$z'(t) = -q(t)\frac{f(x(\sigma(t)))}{x(\sigma(t))}x(\sigma(t))$$

but we have the condition $\frac{f(x)}{x} \geq M$ if we applying it in last equation

$$-q(t)\frac{f(x(\sigma(t)))}{x(\sigma(t))}x(\sigma(t)) \geq -M\; q(t)x(\sigma(t))$$

then

$z'(t) \geq -M\; q(t)x(\sigma(t))$

$z'(t) \geq M\;|q(t)|\; x(\sigma(t)) \geq M\;\frac{p_1-1}{p_1 p_2}|q(t)|\; z(\tau^{-1}(\sigma(t)))$

$z'(t) - M\;\frac{p_1-1}{p_1 p_2}|q(t)|\; z(\tau^{-1}(\sigma(t))) \geq 0$

Which is contradict lemma 1.3.2 in [14].

Example 3 Consider the neutral equation

$$[x(t)+(3-\frac{\cos(t)}{2})x(t-\frac{3\pi}{2})]' + (\cos(2t)-4)x(t-\frac{5\pi}{4}) = 0, \quad t>0,$$

all that the conditions of theorem 3 are satisfied, hence each solutions of the above equation oscillate for example $x(t) = \cos(2t)$ is an oscillatory solution.

Theorem 4 Assume that $q(t) \le 0, 0 \le p(t) \le p < 1, \tau(t) \le t, f$ is an increasing function and

$$\int_{t_0}^{\infty} |q(s)|\, ds = \infty \qquad (13)$$

then all bounded solutions of equation (1) are oscillatory.

Proof We suppose that $x(t)$ is bounded and positive solution of equation (1) for $t \ge t_0$ which implies to $z'(t) \ge 0, z(t) \ge 0$ that means $z(t)$ is bounded increasing function and we have $z(t) \ge x(t)$ and $z(\tau(t)) \ge x(\tau(t))$ but z is increasing then $z(\tau(t)) \le z(t)$ and we have too,

$$z(t) = x(t) + p(t)\ x(\tau(t)) \le x(t) + p\ x(\tau(t))$$

$$z(\tau(t)) \le z(t) \le x(t) + p\ x(\tau(t)) \le x(t) + p\ z(\tau(t))$$

$$z(\tau(t)) \le x(t) + p\ z(\tau(t))$$

$$z(\tau(t)) - p\ z(\tau(t)) \le x(t)$$

$$(1-p)\ z(\tau(\sigma(t))) \le x(\sigma(t))$$

$\because f$ increasing we can get

$$f((1-p)z(\tau(\sigma(t)))) \le f(x(\sigma(t)))$$

$$-q(t)\ f((1-p)z(\tau(\sigma(t)))) \le -q(t)\ f(x(\sigma(t)))$$

then from equation (1) and the last inequality we obtain

$$z'(t) + q(t)\ f((1-p)z(\tau(\sigma(t)))) \ge 0$$

by integrating this inequality from t_0 to t

$$z(t) - z(t_0) + \int_{t_0}^{t} q(s)\ f((1-p)z(\tau(\sigma(s))))\, ds \ge 0$$

which implies to

$$z(t) - z(t_0) + f((1-p)z(\tau(\sigma(t_0)))) \int_{t_0}^{t} |q(s)|\ ds \ge 0$$

then if $t \to \infty$ we get contradiction.

Example 4 Consider the neutral equation

$$[x(t) + \frac{1}{4}(2+\cos(t))x(t-2\pi)]' + \frac{1}{2}(-3-\cos(t))x(t-2\pi) = 0, \quad t > 0,$$

all that the conditions of theorem 3 are satisfied, hence each solutions of the above equation oscillate for example $x(t) = \cos(t)$ is an oscillatory solution.

Theorem 5 Assume that $q(t) \ge 0, p(t) \ge 0, \tau(t) \le t, f$ is an increasing function and

$$\int_{t_0}^{\infty} q(s)\, ds = \infty \qquad (13)$$

then all bounded solutions of equation (1) are oscillatory.

Proof Suppose that $x(t)$ is an eventually positive solution of equation (1) then $z'(t) \le 0, z \ge 0$ for $t \ge t_0$ and by (1) we have

$$z'(t) = -q(t)\ f(x(\sigma(t)))$$

by integrating this equation from t_0 to t we get

$$z(t) - z(t_0) = -\int_{t_0}^{t} q(s)\ f(x(\sigma(s)))\, ds \le -f(x(\sigma(t_0))) \int_{t_0}^{t} q(s)\, ds$$

$$z(t) - z(t_0) \le -f(x(\sigma(t_0))) \int_{t_0}^{t} q(s)\, ds$$

as $t \to \infty$ we can get $\lim_{t\to\infty} z(t) = -\infty$ which is contradiction since $z(t) > 0$.

Example 5 Consider the neutral equation

$$[x(t) + (6-\cos(t))x(t-\pi)]' + (5+2\cos(t))x(t-\frac{3\pi}{2}) = 0, \quad t > 0,$$

all that the conditions of theorem 3 are satisfied, hence each solutions of the above equation oscillate for example $x(t) = \cos(t)$ is an oscillatory solution.

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تذبذب المعادلات التفاضلية المحايدة غير الخطية ذات الرتبة الأولى

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الخلاصة:

تم في هذا البحث دراسة المعادلة التفاضلية المحايدة غير الخطية من الرتبة الأولى بالصيغة

$$[x(t)+p(t)x(\tau(t))]'+q(t)f(x(\sigma(t)))=0 \qquad (1)$$

حيث توصلنا إلى إيجاد شروط ضرورية جديدة تضمن تذبذب جميع حلول المعادلة (1) وتعتبر هذه النتائج تحسين أو تعديل للنتائج التي تم التوصل إليها في البحث [1] لأن $\tau(t)$ و $\sigma(t)$ و $p(t)$ يمثلون دوال مستمرة لكل t.

حيث تضمن البحث 5 نظريات معززة بأمثلة لتأكيد وجود الحل لهذه المعادلة.