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Exploration of CPCD number for power graph

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Abstract:

Recently, complementary perfect corona domination in graphs was introduced. A dominating set S of a graph G is said to be a complementary perfect corona dominating set (CPCD – set) if each vertex in $< S >$ is either a pendent vertex or a support vertex and $< V - S >$ has a perfect matching. The minimum cardinality of a complementary perfect corona dominating set is called the complementary perfect corona domination number and is denoted by γ_{cpc}(G). In this paper, our parameter hasbeen discussed for power graphs of path and cycle.

Keywords: Corona domination number, Cycle, Path, Pendent vertex, Perfect matching, Support vertex.

Introduction:

A graph $G = (V, E)$ where V, E are the vertex set and edge set. The graphs used in this paper were simple, undirected, and non-trivial. P_n denotes the path on n vertices with the vertex set $V(P_n) = (v_1, v_2, v_3, \dots, v_n)$ and the edge set $E(P_n) = \{v_i v_{i+1}: i = 1,2, \dots, n-1\}$. A cycle on n vertices is denoted by C_n with vertex set $V(C_n) = \{v_1, v_2, v_3, \dots, v_n\}$ and edge set $E(C_n) = \{v_i v_{i+1}: i = 1,2, \dots, n-1\} \cup \{v_1, v_n\}$. The concept of corona domination in graphs was introduced by G. Mahadevan et al.[1], and the complementary perfect dominating set was presented by Paulraj Joseph et al. Lately the complementary connected perfect domination number of a graph was studied.[2] In recent years many authors have discussed many interesting results by imposing various conditions on the dominating sets they are restrained and secured[3,4], hn-domination[4,5], tadpole domination[6], Power graphs were also discussed frequently[7,8]. By considering the existing corona domination parameter and imposing a perfect matching condition in the complement, the authors obtained a new parameter called the complementary perfect corona dominating set. A dominating set $S \subseteq V$ is said to be a complementary perfect corona dominating set (CPCD-set) if every vertex in $< S >$ is either a pendent vertex or a support vertex and $< V - S >$ has a perfect matching. The minimum cardinality of a CPCD-set is called a complementary perfect corona domination number and is denoted by $\gamma_{CPC}(G)$. A graph is considered complete if each pair of distinct vertices in G are adjacent and is denoted by K_n. For $n \geq 3$, the graph $K_1 + C_n$ is defined as the wheel $W_{1,n-1}$. G^k is a graph with the same number of vertices as G, and two vertices are adjacent when $d(v_1, v_2) = k$. A $F_{n,m}$ fan graph is defined as $K_n + P_m$. The author began this article by determining the exact CPCD value of the cycle C_n for any $n \neq 7$. The CPCD value for the power graph of the path and cycle are examined in this article because the exact CPCD value for path and path-related graphs are already determined.

Main Results:

Theorem.1: For a cycle C_n, $n \neq 7$,

$$\gamma_{CPC}(C_n) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil & if\, n \equiv 0 \; or \; 1 \; (mod\; 4) \\ \left\lceil \frac{n}{2} \right\rceil + 1 & otherwise. \end{cases}$$

Proof: Let $C_n = (v_1, v_2, \dots, v_n, v_1), n \neq 7$ and let $S_1 = \{v_i : i \equiv 1 \; or \; 2 \; (mod\; 4)\}$. Assume

$$S = \begin{cases} S_1 \cup \{v_{n-4}, v_n\} - \{v_{n-2}\} & if\, n \equiv 3 \; (mod\; 4) \\ S_1 & otherwise. \end{cases}$$

Then S is a CPCD-set of C_n and hence

$\gamma_{CPC}(C_n) \le |S| = \begin{cases} \lceil \frac{n}{2} \rceil & if n \equiv 0 \ or \ 1 \ (mod \ 4) \\ \lceil \frac{n}{2} \rceil + 1 & otherwise. \end{cases}$.

Let S' be a CPCD-set of C_n. Since any dominating set D of cardinality at most

$k = \begin{cases} \lceil \frac{n}{2} \rceil - 1 & if n \equiv 0 \ or \ 1 \ (mod \ 4) \\ \lceil \frac{n}{2} \rceil & otherwise, \end{cases}$ either

contains an isolated vertex in $< D >$ or $|V - D|$ is odd, having

$|S'| \ge k + 1 = \begin{cases} \lceil \frac{n}{2} \rceil - 1 & if n \equiv 0 \ or \ 1 \ (mod \ 4) \\ \lceil \frac{n}{2} \rceil & otherwise. \end{cases}$

Theorem.2: For a path P_n, n be odd, k be even, and $n \ge 3 + k$,

$$\gamma_{CPC}(P_n^k) = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 1 \ or \ 2 \ or \ldots ork (mod \ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 0 \ or \ \frac{3k+2}{2} \ or \ \frac{3k+4}{2} \ or \ldots or \ 3k (mod \ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 2 & if n \equiv k+1 \ ork+2 \ or \ldots or \ \frac{3k}{2} (mod \ 3k+1). \end{cases}$$

Proof: Let $P_n = (v_1, v_2, \ldots, v_n), n$ be odd, k be even and $n \ge 3 + k$, let $S_1 = \{v_i : i \equiv k + 1 \ or \ 2k + 1 \ (mod \ 3k + 1)\}$. Assume

$$S = \begin{cases} S_1 \cup \{v_n\} & if n \equiv 1 \ or \ 2k+2 \ or \ 2k+3 \ or \ldots or \ 3k \ (mod \ 3k+1) \\ S_1 \cup \{v_{n-i}\} & if n \equiv i (mod \ 3k+1); 1 \le i \le k \\ S_1 \cup \{v_{n-1}, v_1\} & if n \equiv k+1 \ (mod \ 3k+1) \\ S_1 \cup \{v_n, v_1\} & if n \equiv k+2 \ ork+3 \ or \ldots or \ 2k \ (mod \ 3k+1) \\ S_1\{v_1\} & if n \equiv 2k+1 \ (mod \ 3k+1). \end{cases}$$

Then S is a CPCD-set of P_n^k and hence

$$\gamma_{CPC}(P_n^k) \le |S| = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 1 \ or \ 2 \ or \ldots ork \ (mod \ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 0 \ or \ \frac{3k+2}{2} \ or \ \frac{3k+4}{2} \ or \ldots or \ 3k \ (mod \ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 2 & if n \equiv k+1 \ ork+2 \ or \ldots or \ \frac{3k}{2} (mod \ 3k+1). \end{cases}$$

Let S' be a CPCD-set of P_n^k. Since any dominating set $D$of cardinality at most

$$k = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil - 1 & if n \equiv 1 \ or \ 2 \ or \ldots ork \ (mod \ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 0 \ or \ \frac{3k+2}{2} \ or \ \frac{3k+4}{2} \ or \ldots or \ 3k \ (mod \ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv k+1 \ ork+2 \ or \ldots or \ \frac{3k}{2} (mod \ 3k+1) \end{cases}$$

either contains an isolated vertex in $< D >$ or $|V - D|$ is odd, having

$$|S'| \geq k+1 = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 1 \; or \; 2 \; or \ldots or k \; (mod\; 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 0 \; or \frac{3k+2}{2} or \frac{3k+4}{2} or \ldots or \; 3k \; (mod\; 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 2 & if n \equiv k+1 \; or k+2 \; or \ldots or \frac{3k}{2} (mod\; 3k+1). \end{cases}$$

Theorem.3: For a path P_n, n, k are even, $n \geq k$,

$$\gamma_{CPC}(P_n^k) = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 0 \; or \frac{3k+2}{2} or \frac{3k+4}{2} or \ldots or \; 3k \; (mod\; 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 1 \; or \; 2 \; or \ldots or \frac{3k}{2} (mod\; 3k+1). \end{cases}$$

Proof: Let $P_n = (v_1, v_2, \ldots, v_n), n, k$ be an even and $n \geq k$, let $S_1 = \{v_i : i \equiv k+1 \; or \; 2k+1 \; (mod\; 3k+1)\}$. Assume

$$S = \begin{cases} S_1 & if n \equiv 0 \; or \; 2k+1 or \; 2k+2 \; or \ldots or \; 3k \; (mod\; 3k+1) \\ S_1 \cup \{v_{n-1}, v_n\} & if n \equiv 1 \; or \; 2 \; or \ldots k \; (mod\; 3k+1) \\ S_1 \cup \{v_{n-1}\} & if n \equiv k+1 \; (mod\; 3k+1) \\ S_1 \cup \{v_n\} & if n \equiv k+2 \; or k+3 \; or \ldots or \; 2k \; (mod\; 3k+1). \end{cases}$$

Then S is a CPCD-set of P_n^k and hence

$$\gamma_{CPC}(P_n^k) \leq |S| = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 0 \; or \frac{3k+2}{2} or \frac{3k+4}{2} or \ldots or \; 3k \; (mod\; 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 1 \; or \; 2 \; or \ldots or \frac{3k}{2} (mod\; 3k+1). \end{cases}$$

Let S' be a CPCD-set of P_n^k. Since any dominating set $D$of cardinality at most

$$k = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil - 1 & if n \equiv 0 \; or \frac{3k+2}{2} or \frac{3k+4}{2} or \ldots or \; 3k \; (mod\; 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 1 \; or \; 2 \; or \ldots or \frac{3k}{2} (mod\; 3k+1). \end{cases}$$

either contains an isolated vertex in $< D >$ or $|V - D|$ is odd, having

$$|S'| \geq k+1 = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 0 \; or \frac{3k+2}{2} or \frac{3k+4}{2} or \ldots or \; 3k \; (mod\; 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 1 \; or \; 2 \; or \ldots or \frac{3k}{2} (mod\; 3k+1). \end{cases}$$

Theorem.4: For a path P_n, k be odd and $n \geq 3+k$,

$$\gamma_{CPC}(P_n^k) = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 0 \; or \; 1 \; or \; 3 \; or \ldots k or k+5 \; or k+7 \; or \ldots or \; 2k \\ & or \; 2k+2 \; or \; 2k+4 \; or \ldots or \; 3k-1 \; (mod\; 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 2 \; or \; 4 \; or \; 6 \; or \ldots or k+3 \; or k+6 \; or k+8 \\ & \ldots 2k+1 \; or \; 2k+3 \; or \ldots or \; 3k \; (mod\; 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 2 & if n \equiv k+2 \; or k+4 \; (mod\; 3k+1). \end{cases}$$

Proof: Let $P_n = (v_1, v_2, \dots, v_n), n, k$ be an even and $n \geq k$, let $S_1 = \{v_i : i \equiv k+1 \ or \ 2k+1 \ (mod\ 3k+1)\}$. Assume

$$S = \begin{cases} S_1 \quad if n \equiv 0 \ or \ 2k+2 \ or \ 2k+4 \ or \dots or \ 3k-1 \ (mod\ 3k+1) \\ S_1 \cup \{v_{n-i}\} \quad if n \equiv i or k+1 \ (mod\ 3k+1) where i = 1,3,5,\dots,k \\ S_1 \cup \{v_{n-1}, v_n\} \quad if n \equiv 2 \ or \ 4 \ or \dots or k-1 \ (mod\ 3k+1) \\ S_1 \cup \{v_1, v_n\} \quad if n \equiv k+2 \ or k+4 \ or \dots 2k-1 \ (mod\ 3k+1) \\ S_1 \cup \{v_n\} \quad if n \equiv k+3 \ or k+5 \dots 2k or \ 2k+3 \ or \ 2k+5 \dots 3k \ (mod\ 3k+1) \\ S_1 \cup \{v_1\} \quad if n \equiv 2k+1 \ (mod\ 3k+1) \end{cases}$$

Then S is a CPCD-set of P_n^k and hence

$$\gamma_{CPC}(P_n^k) \leq |S| = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil if n \equiv 0 \ or \ 1 \ or \ 3 \ or \dots k or k+5 \ or k+7 \ or \dots or \ 2k \\ \quad or \ 2k+2 \ or \ 2k+4 \ or \dots or \ 3k-1 \ (mod\ 3k+1\ ) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 \quad if n \equiv 2 \ or \ 4 \ or \ 6 \ or \dots or k+3 \ or k+6 \ or k+8 \\ \quad \dots 2k+1 \ or \ 2k+3 \ or \dots or \ 3k \ (mod\ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 2 \quad if n \equiv k+2 \ or k+4 \ (mod\ 3k+1). \end{cases}$$

Let S' be a CPCD-set of P_n^k. Since any dominating set $D$of cardinality at most

$$k = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil - 1 \quad if n \equiv 0 \ or \ 1 \ or \ 3 \ or \dots k or k+5 \ or k+7 \ or \dots or \ 2k \\ \quad or \ 2k+2 \ or \ 2k+4 \ or \dots or \ 3k-1 \ (mod\ 3k+1\ ) \\ \left\lceil \frac{2n}{3k+1} \right\rceil if n \equiv 2 \ or \ 4 \ or \ 6 \ or \dots or k+3 \ or k+6 \ or k+8 \\ \quad \dots 2k+1 \ or \ 2k+3 \ or \dots or \ 3k \ (mod\ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 \quad if n \equiv k+2 \ or k+4 \ (mod\ 3k+1). \end{cases}$$

either contains an isolated vertex in $< D >$ or $|V - D|$ is odd, having

$$|S'| \geq k+1 = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil if n \equiv 0 \ or \ 1 \ or \ 3 \ or \dots k or k+5 \ or k+7 \ or \dots or \ 2k \\ \quad or \ 2k+2 \ or \ 2k+4 \ or \dots or \ 3k-1 \ (mod\ 3k+1\ ) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 \quad if n \equiv 2 \ or \ 4 \ or \ 6 \ or \dots or k+3 \ or k+6 \ or k+8 \\ \quad \dots 2k+1 \ or \ 2k+3 \ or \dots or \ 3k \ (mod\ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 2 \quad if n \equiv k+2 \ or k+4 \ (mod\ 3k+1). \end{cases}$$

Theorem.5: For a cycleC_n, n be odd, k be even, and $n \geq 3+k$,

$$\gamma_{CPC}(C_n^k) = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil \quad if n \equiv 1 \ or \ 2 \ or \dots or k \ (mod\ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 \quad if n \equiv 0 \ or \frac{3k+2}{2} or \frac{3k+4}{2} or \dots or \ 3k \ (mod\ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 2 \quad if n \equiv k+1 \ or k+2 \ or \dots or \frac{3k}{2} (mod\ 3k+1). \end{cases}$$

Proof: Let $C_n = (v_1, v_2, \dots, v_n, v_1), n$ be odd, k be even and $n \geq 3 + k$, let $S_1 = \{v_i : i \equiv k + 1\ or\ 2k + 1\ (mod\ 3k + 1)\}$. Assume

$$S = \begin{cases} S_1 \cup \{v_n\} & if n \equiv 1\ or\ 2k + 2\ or\ 2k + 3\ or \dots or\ 3k\ (mod\ 3k + 1) \\ S_1 \cup \{v_{n-i}\} & if n \equiv i (mod\ 3k + 1); 1 \leq i \leq k \\ S_1 \cup \{v_{n-1}, v_1\} & if n \equiv k + 1\ (mod\ 3k + 1) \\ S_1 \cup \{v_n, v_1\} & if n \equiv k + 2\ or k + 3\ or \dots or\ 2k\ (mod\ 3k + 1) \\ S_1\{v_1\} & if n \equiv 2k + 1\ (mod\ 3k + 1). \end{cases}$$

Then S is a CPCD-set of C_n^k and hence

$$\gamma_{CPC}(C_n^k) \leq |S| = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 1\ or\ 2\ or \dots or k\ (mod\ 3k + 1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 0\ or \frac{3k+2}{2} or \frac{3k+4}{2} or \dots or\ 3k\ (mod\ 3k + 1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 2 & if n \equiv k + 1\ or k + 2\ or \dots or \frac{3k}{2} (mod\ 3k + 1). \end{cases}$$

Let S' be a CPCD-set of C_n^k. Since any dominating set $D$of cardinality at most

$$k = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil - 1 & if n \equiv 1\ or\ 2\ or \dots or k\ (mod\ 3k + 1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 0\ or \frac{3k+2}{2} or \frac{3k+4}{2} or \dots or\ 3k\ (mod\ 3k + 1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv k + 1\ or k + 2\ or \dots or \frac{3k}{2} (mod\ 3k + 1) \end{cases}$$

either contains an isolated vertex in $< D >$ or $|V - D|$ is odd, having

$$|S'| \geq k + 1 = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 1\ or\ 2\ or \dots or k\ (mod\ 3k + 1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 0\ or \frac{3k+2}{2} or \frac{3k+4}{2} or \dots or\ 3k\ (mod\ 3k + 1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 2 & if n \equiv k + 1\ or k + 2\ or \dots or \frac{3k}{2} (mod\ 3k + 1). \end{cases}$$

Theorem.6: For a cycleC_n, n, k are even, $n \geq k$,

$$\gamma_{CPC}(C_n^k) = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 0\ or \frac{3k+2}{2} or \frac{3k+4}{2} or \dots or\ 3k\ (mod\ 3k + 1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 1\ or\ 2\ or \dots or \frac{3k}{2} (mod\ 3k + 1). \end{cases}$$

Proof: Let $C_n = (v_1, v_2, \dots, v_n, v_1), n, k$ be an even and $n \geq k$, let $S_1 = \{v_i : i \equiv k + 1\ or\ 2k + 1\ (mod\ 3k + 1)\}$. Assume

$$S = \begin{cases} S_1 & if n \equiv 0\ or\ 2k + 1 or\ 2k + 2\ or \dots or\ 3k\ (mod\ 3k + 1) \\ S_1 \cup \{v_{n-1}, v_n\} & if n \equiv 1\ or\ 2\ or \dots k\ (mod\ 3k + 1) \\ S_1 \cup \{v_{n-1}\} & if n \equiv k + 1\ (mod\ 3k + 1) \\ S_1 \cup \{v_n\} & if n \equiv k + 2\ or k + 3\ or \dots or\ 2k\ (mod\ 3k + 1). \end{cases}$$

Then S is a CPCD-set of C_n^k and hence

$$\gamma_{CPC}(C_n^k) \le |S| = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 0 \ or \ \frac{3k+2}{2} \ or \ \frac{3k+4}{2} \ or \ ... \ or \ 3k \ (mod \ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 1 \ or \ 2 \ or \ ... \ or \ \frac{3k}{2} (mod \ 3k+1). \end{cases}$$

Let S' be a CPCD-set of C_n^k. Since any dominating set $D$of cardinality at most

$$k = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil - 1 & if n \equiv 0 \ or \ \frac{3k+2}{2} \ or \ \frac{3k+4}{2} \ or \ ... \ or \ 3k \ (mod \ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 1 \ or \ 2 \ or \ ... \ or \ \frac{3k}{2} (mod \ 3k+1). \end{cases}$$

either contains an isolated vertex in $< D >$ or $|V - D|$ is odd, having

$$|S'| \ge k + 1 = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil & if n \equiv 0 \ or \ \frac{3k+2}{2} \ or \ \frac{3k+4}{2} \ or \ ... \ or \ 3k \ (mod \ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 1 \ or \ 2 \ or \ ... \ or \ \frac{3k}{2} (mod \ 3k+1). \end{cases}$$

Theorem.7: For a cycleC_n, k be odd and $n \ge 3 + k$,

$$\gamma_{CPC}(C_n^k) = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil if n \equiv 0 \ or \ 1 \ or \ 3 \ or \ ... \ k or k+5 \ or k+7 \ or \ ... \ or \ 2k \\ \quad or \ 2k+2 \ or \ 2k+4 \ or \ ... \ or \ 3k-1 \ (mod \ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 2 \ or \ 4 \ or \ 6 \ or \ ... \ or k+3 \ or k+6 \ or k+8 \\ \quad ... \ 2k+1 \ or \ 2k+3 \ or \ ... \ or \ 3k \ (mod \ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 2 & if n \equiv k+2 \ or k+4 \ (mod \ 3k+1). \end{cases}$$

Proof: Let $C_n = (v_1, v_2, \dots, v_n, v_1), n, k$ be an even and $n \ge k$, let $S_1 = \{v_i : i \equiv k+1 \ or \ 2k+1 \ (mod \ 3k+1)\}$. Assume

$$S = \begin{cases} S_1 & if n \equiv 0 \ or \ 2k+2 \ or \ 2k+4 \ or \ ... \ or \ 3k-1 \ (mod \ 3k+1) \\ S_1 \cup \{v_{n-i}\} & if n \equiv i or k+1 \ (mod \ 3k+1) where i = 1,3,5, \dots, k \\ S_1 \cup \{v_{n-1}, v_n\} & if n \equiv 2 \ or \ 4 \ or \ ... \ or k-1 \ (mod \ 3k+1) \\ S_1 \cup \{v_1, v_n\} & if n \equiv k+2 \ or k+4 \ or \ ... \ 2k-1 \ (mod \ 3k+1) \\ S_1 \cup \{v_n\} & if n \equiv k+3 \ or k+5 \ ... \ 2k or \ 2k+3 \ or \ 2k+5 \ ... \ 3k \ (mod \ 3k+1) \\ S_1 \cup \{v_1\} & if n \equiv 2k+1 \ (mod \ 3k+1) \end{cases}$$

Then S is a CPCD-set of C_n^k and hence

$$\gamma_{CPC}(C_n^k) \le |S| = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil if n \equiv 0 \ or \ 1 \ or \ 3 \ or \ ... \ k or k+5 \ or k+7 \ or \ ... \ or \ 2k \\ \quad or \ 2k+2 \ or \ 2k+4 \ or \ ... \ or \ 3k-1 \ (mod \ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 & if n \equiv 2 \ or \ 4 \ or \ 6 \ or \ ... \ or k+3 \ or k+6 \ or k+8 \\ \quad ... \ 2k+1 \ or \ 2k+3 \ or \ ... \ or \ 3k \ (mod \ 3k+1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 2 & if n \equiv k+2 \ or k+4 \ (mod \ 3k+1). \end{cases}$$

Let S' be a CPCD-set of C_n^k. Since any dominating set $D$of cardinality at most

$$k = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil if n \equiv 0 \ or \ 1 \ or \ 3 \ or \ ... kork + 5 \ ork + 7 \ or \ ... or \ 2k \\ \quad or \ 2k + 2 \ or \ 2k + 4 \ or \ ... or \ 3k - 1 \ (mod \ 3k + 1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 \quad if n \equiv 2 \ or \ 4 \ or \ 6 \ or \ ... ork + 3 \ ork + 6 \ ork + 8 \\ \quad ... 2k + 1 \ or \ 2k + 3 \ or \ ... or \ 3k \ (mod \ 3k + 1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 2 \quad if n \equiv k + 2 \ ork + 4 \ (mod \ 3k + 1). \end{cases}$$

either contains an isolated vertex in $< D >$ or $|V - D|$ is odd, having

$$|S'| \geq k + 1 = \begin{cases} \left\lceil \frac{2n}{3k+1} \right\rceil if n \equiv 0 \ or \ 1 \ or \ 3 \ or \ ... kork + 5 \ ork + 7 \ or \ ... or \ 2k \\ \quad or \ 2k + 2 \ or \ 2k + 4 \ or \ ... or \ 3k - 1 \ (mod \ 3k + 1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 1 \quad if n \equiv 2 \ or \ 4 \ or \ 6 \ or \ ... ork + 3 \ ork + 6 \ ork + 8 \\ \quad ... 2k + 1 \ or \ 2k + 3 \ or \ ... or \ 3k \ (mod \ 3k + 1) \\ \left\lceil \frac{2n}{3k+1} \right\rceil + 2 \quad if n \equiv k + 2 \ ork + 4 \ (mod \ 3k + 1). \end{cases}$$

Observation:

1. $\gamma_{CPC}(K_n^k) = 2 \ \ if n is even$
2. $\gamma_{CPC}(F_{n,m}^k) = 2 \ \ if n + m is even$
3. $\gamma_{CPC}(W_{1,n-1}^k) = 2 \ \ if n is even$

Conclusion:

In this article, the CPCD number of graphs was discussed and obtained the number for the cycle and power graph of path and cycle. The authors investigated this number for many product-related graphs and some more notable types of graphs which will be reported in successive papers.

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- Conflicts of Interest: None.
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Authors Contribution Statement:

G.M and S. A conceived and developed the theory. S. Anuthiya and C. Sivagnanam verified the results. C. S supervised the findings of this work.All the authors accepted the final manuscript

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استكشاف رقم CPCD للرسم البياني للطاقة

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الخلاصة:

في الآونة الأخيرة ، تم تقديم هيمنة الإكليل المثالية التكميلية في الرسوم البيانية.يقال إن المجموعة المهيمنة S من الرسم البياني G هي مجموعة مهيمنة كاملة على الهالة (CPCD - مجموعة) إذا كان كل رأس في <S> إما رأس معلق أو رأس دعم <V-S> وله تطابق تام. يطلق على الحد الأدنى من الكاردينالية لمجموعة الإكليل المثالية التكميلية المهيمنة رقم هيمنة الإكليل المثالي التكميلي ويرمز له ب $\gamma_{cpc}(G)$.في هذه الورقة ، تمت مناقشة معلمتنا للرسوم البيانية للطاقة للمسار والدورة.

الكلمات المفتاحية: الرقم الاكليلي المهيمن، دوري، مسار، رؤوس معلقة، مطابقة مثالية، قمة الرأس المدعومة.