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Momentum Ranking Function of Z-Numbers and its Application to Game Theory

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Abstract:
After Zadeh introduced the concept of z-number scientists in various fields have shown keen interest in applying this concept in various applications. In applications of z-numbers, to compare two z-numbers, a ranking procedure is essential. While a few ranking functions have been already proposed in the literature there is a need to evolve some more good ranking functions. In this paper, a novel ranking function for z-numbers is proposed- "the Momentum Ranking Function"(MRF). Also, game theoretic problems where the payoff matrix elements are z-numbers are considered and the application of the momentum ranking function in such problems is demonstrated.

Keywords: Momentum Ranking Function, z-Games, z-number, z-payoff matrix, z-saddle point.

Introduction:
Operations research techniques are widely applied in many real-life problems. However, while collecting data for the construction of the appropriate mathematical model, it is frequently seen that the data available is imprecise and not fully reliable. In 2011, Zadeh[1] developed the concept of fuzzy numbers and introduced z -numbers. z-numbers contain both information and an estimate of the reliability of the information. Hence z numbers have the potential for application in various types of operations research problems. Zainb Hassan Radhy[2] and et.al. dealt fuzzy assignment model by using a linguistic variable. Computations involving z-numbers were studied by Shahila Bhanu and Velammal [3]. In dealing with applications of z-numbers there are two challenges 1. How to perform arithmetic operations on z numbers? 2. How to compare z numbers?

The first challenge can be overcome by using the novel R-type arithmetic operations introduced by Stephen[4]. The second problem–the problem of ranking or ordering z-numbers is a topic of interest to those who wish to study applications of z-numbers. So far, a few ranking methods have been proposed.

Rasha Jalal Mitlif [5] describes an efficient algorithm for Fuzzy Linear Fractional Programming Problems via Ranking Function. Siddhartha Sankar Biswas[6] defines Z_1 as strongly greater than Z_2 as component-wise comparisons. Wen Jiang, ChunheXie, Yu Luo, and YongchuanTang[7] proposed a new method for ranking Z-numbers by evaluating generalized fuzzy numbers. Iden Hassan Hussein and Rasha Jalal Mitilif [8] introduced a ranking function to solve fuzzy multiple objective functions. Amir HoseinMahmoodiI D et. al.[9] gave the comparison of linguistic z-numbers with the max-score rule. Bingyi Kanga, Gyan Chhipi-Shrestha, Yong Denga, Kasun Hewage, RehanSadiq[10] gave the evolutionary games with the z-number. Iden Hassan Hussein, Zainab Saad Abood[11] solved fuzzy game problems by using three different ranking functions. Mujahid Abdullahi[12] and others gave a new ranking method for Z-by converting z-number into a fuzzy number, and then the centroid, point method, and decision rules are utilized to rank the obtained fuzzy numbers. Parameswari[13] proposed a Lexicographic order-based ranking on z numbers. However, there is scope for further research in this area.

In this paper, a novel ranking function for z-numbers is proposed- the momentum ranking function. Also, game theoretic problems where the payoff matrix elements are z numbers, that is, z-payoff matrix are considered and the application of momentum ranking function in such problems is demonstrated.

Preliminary Definitions:

Definition 1: Formal definition of z-number
Consider an ordered pair of (C, D) where C is a fuzzy set defined on the real line and D is a fuzzy number whose support is contained in [0,1]. Then (C, D) is called a z-number.

Definition 2: Formal definition of z- valuation
Let X be an uncertain variable. The z-valuation 'X is z (C, D)' is equivalent to an assignment statement "X is (C, D)". It means that FEP (X is C) is D. That is, Possibility (FEP(x∈ C)=s) = $\mu_D(s)$.

Definition 3: Ranking Function
A ranking function r_k on a set of fuzzy numbers F, a real-valued function, $A_1 \leq A_2$ if and only if $r_k(A_1) \leq r_k(A_2)$, where $A_1, A_2 \in$ F.

Definition 4: MIN R Operation
Let * be any one of the basic arithmetic operations addition, subtraction, multiplication, or division. Let R_k be any suitably chosen ranking function. Then the MIN R operation is defined by
(A, B) (*,MIN) (C, D)=(A*C, MIN(B, D)), where A*C is calculated by using the extension principle, and MIN(B, D) = $\begin{cases} B \text{ if } R_k(B) < R_k(D) \\ D \text{ if} R_k(D) < R_k(B) \end{cases}$

Definition 5: Sum and Difference of two trapezoidal z-numbers by MIN R
Let Z_1 = (A_1, B_1) = $\big((a_{11}, a_{12}, a_{13}, a_{14}),$ $(b_{11}, b_{12}, b_{13}, b_{14})\big)$, and
Z_2 = (A_2, B_2) = $\big((a_{21}, a_{22}, a_{23}, a_{24}),\ (b_{21}, b_{22}, b_{23}, b_{24})\big)$ be any two Z-numbers whose components are trapezoidal numbers.
$Z_1(+,\mathrm{MIN})Z_2 = ((a_{11} + a_{21},\ a_{12} +\ a_{22},\ a_{13} + a_{23},\ a_{14} + a_{24}),\ MIN(B_1,\ B_2))$.
and $Z_1(-,\mathrm{MIN})Z_2 = ((a_{11} - a_{24},\ a_{12} -\ a_{23}, a_{13} - a_{22},\ a_{14} - a_{21}), MIN(B_1, B_2))$

Definition 6: MIN R type Operations on z-Intervals
Let $z_1 = ([a_1, a_2], [b_1, b_2])$ *and*
$z_2 = ([c_1, c_2], [d_1, d_2])$ then
$([a_1, a_2], [b_1, b_2])\ (+, MIN)([c_1, c_2], [d_1, d_2]) = ([a_1 +\ c_1, a_2 +\ c_2], MIN([b_1, b_2], [d_1, d_2])$.
$([a_1, a_2], [b_1, b_2])\ (-, MIN)\ ([c_1, c_2], [d_1, d_2]) = ([a_1 - c_2, a_2 - c_1], MIN([b_1, b_2], [d_1, d_2])$
$([a_1, a_2], [b_1, b_2])(., MIN)([c_1, c_2], [d_1, d_2]) = \left(\begin{bmatrix} \min(a_1c_1, a_1c_2, a_2c_1, a_2c_2) \\ , \max(a_1c_1, a_1c_2, a_2c_1, a_2c_2) , \end{bmatrix}, MIN([b_1, b_2], [d_1, d_2]\right)$
.
$([a_1, a_2], [b_1, b_2])(/, MIN)([c_1, c_2], [d_1, d_2]) = \left(\begin{bmatrix} \min\left(\frac{a_1}{c_1}, \frac{a_1}{c_2}, \frac{a_2}{c_1}, \frac{a_2}{c_2}\right), \\ \max(\frac{a_1}{c_1}, \frac{a_1}{c_2}, \frac{a_2}{c_1}, \frac{a_2}{c_2} \end{bmatrix}, MIN([b_1, b_2], [d_1, d_2])\right),$
provided that $0 \notin [c_1, c_2]$.

Momentum Ranking Function of z-number
Definition 7: Momentum Ranking Function [MRF]
Let r_1 and r_2 be any two ranking functions for fuzzy numbers. Then for the z-number (A, B), define the Momentum Ranking Function[MRF] by $MRF(z) = \mathrm{M}(r_1, r_2)(z) = r_1(A)r_2(B)$
The MRF function can then be used to rank or order a list of z- numbers.

Example 1:
Consider the z-number ((1,2,3,5),(.75,.8,.9,1)), Here A = (1,2,3,5), and B = (.75,.8,.9,1)
- "Choose two-ranking function: r_1 as Center of Gravity method, and $r_2$as Median method" with $r_1(C) = \frac{(c^2 + d^2 + cd - a^2 - b^2 - ab)}{3(c+d-a-b)}$ and $r_2(C) = \frac{a + b + c + d}{4}$ for the trapezoidal number C(a,b,c,d).
- $r_1(A) = \frac{3^2 + 5^2 + 3\times5 - 1^2 - 2^2 - 1\times2}{3(3+5-1-2)}$ = 2.8 and $r_2(B) = \frac{.75 + .8 + .9 + 1}{4}$ = .8625, So, $\mathrm{M}(r_1, r_2)(z) = r_1(A)r_2(B)$ = 2.415

Definition 8: Ordering of z-numbers using the MRF -$\mathbf{M(r_1, r_2)z}$
Choose any two ranking functions r_1 and r_2 . Let $z_1 = (A_1, B_1), z_2 = (A_2, B_2)$ be two z-numbers. Define $z_1 \leq z_2$ if and only if
$\mathrm{M}(r_1, r_2)(A_1, B_1) \leq \mathrm{M}(r_1, r_2)(A_2, B_2)$ or Simply $MRF(z_1) \leq MRF(z_2)$.

Examples 2:
1) Let $Z_1 = (A_1, B_1)$ = ((3,5,6,7),(.8,.85,.9,.95)) and $Z_2 = (A_2, B_2)$ = ((3,4,5,6),(.7,.8,.85,.9))
   Here, $\mathrm{M}(r_1, r_2)(A_1, B_1) = r_1(A_1) \times r_2(B_1) = 5.2 \times .875 = 4.8125$
   $\mathrm{M}(r_1, r_2)(A_2, B_2) = r_1(A_2) \times r_2(B_2) = 4.5 \times .8125 = 3.65625$. Hence, $Z_2 \leq Z_1$.
2) Let $Z_1 = (A_1, B_1)$ = ((3,5,6,7),(.7,.75,.8,.85)) and $Z_2 = (A_2, B_2)$ = ((3,4,5,6),(.85,.9,.95,1))
   Here, $\mathrm{M}(r_1, r_2)(Z_1) = \mathrm{M}(r_1, r_2)(A_1, B_1) = r_1(A_1) \times r_2(B_1) = 5.2 \times .775 = 4.03$
   $\mathrm{M}(r_1, r_2)(Z_2) = \mathrm{M}(r_1, r_2)(A_2, B_2) = r_1(A_2) \times r_2(B_2) = 4.5 \times .925 = 4.1625$. Hence, $Z_1 \leq Z_2$.

Definition 9: z-Games
Two persons zero sum z-game is a two-person zero-sum game with the elements of the payoff matrix z-numbers.

Application to Game Theory by using MRF

Proposed Method: (z-Games with z-Saddle Point)

Consider a two-person zero-sum z-game with a z-payoff matrix: **Player1** **Player 2**

$$\begin{array}{c} \\ P_1 \\ \vdots \\ P_m \end{array}\begin{array}{c} \boldsymbol{Q_1} \cdots \quad \boldsymbol{Q_n} \\ \begin{bmatrix} z_{11} \cdots & z_{1n} \\ \vdots & \vdots \\ z_{m1} \cdots & z_{mn} \end{bmatrix} \end{array}$$

Player 1 has m strategies and player 2 has n strategies. The entry z_{ij} gives information regarding the z-payoff to player 1 when strategy i is used by the first player and strategy j is used by the second player. If $z_{ij} = (A_{ij}, B_{ij})$, then A_{ij} is the fuzzy estimate of player 1's gain and B_{ij} is the reliability of this estimate.

Step 1: Choose any two ranking functions r_1 and r_2, the MRF function $M(r_1, r_2)$ can be used to order the entries in any row or column. Hence the maximum of each column and the minimum of every row can be found. If a row minimum coincides with a column maximum, then it can be considered as a z-saddle point, then continue, otherwise, if a z-saddle point does not exist, then go to Step 4

Step 2: If (l, m) is a z-saddle point then the optimal strategy for players 1 and 2 is (P_l, Q_m), and the value of the game is (A_{lm}, B_{lm}), where B_{lm} is calculated by using step 3.

Step 3: However, what is the reliability of this estimate.? Since all the entries of the z-payoff matrix play a role in the computation, the reliability of the expected gain is
$B_{lm} = B_{MIN} = \min\{ B_{ij} | 1 \le i \le m, 1 \le j \le n\}$,
where the minimum of fuzzy numbers is calculated by ordering the list of fuzzy numbers by the r_2 ranking function. Hence the optimum strategy for player 1 at the z-saddle point (l, m) is(A_{lm}, B_{MIN}).

Step 4: If the z-saddle point of the given z-game does not exist and the z-payoff is 2×2, then go to Step 6, or if the given z-payoff matrix is of any m×n, where $m \ge 3$ and $n \ge 3$, then continue the following steps.

Step 5: Reduce the given z-payoff matrix into 2 × 2 by using the z-dominance property:

(a) Find the value of $M(r_1, r_2)(z_{ij}), i = 1$ to m, and $j = 1$ to n by using any two-ranking functions r_1 and r_2.

(b) All the ranks of the k^{th} row or any convex linear combination of two or more strategies (rows) are less than or equal to the corresponding ranks of any other r^{th} row, then the k^{th} row is dominated by the r^{th} row.

(c) All the ranks of the k^{th} column or any convex linear combination of two or more strategies (columns) are greater than or equal to the corresponding ranks of any other r^{th} column, then the k^{th} column is dominated by the r^{th} column.

(d) To reduce the size of the z-payoff matrix, delete the dominated rows or columns.

(e) Do the above steps (b) to (d) repeatedly until get the z-game's z-payoff as 2 ×2.

Step 6: For any 2×2 two-person zero-sum z-game without any z-saddle point having the payoff matrix as:

$$\begin{array}{c} \\ P_1 \\ P_2 \end{array}\begin{array}{c} \quad Q_1 \quad Q_2 \\ \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \end{array},$$

the optimum mixed strategies for player 1 are $zS_P = \begin{bmatrix} P_1 & P_2 \\ zp_1 & zp_2 \end{bmatrix}$ and for player 2 are $zS_Q = \begin{bmatrix} Q_1 & Q_2 \\ zq_1 & zq_2 \end{bmatrix}$, where

$zp_1 = \frac{z_{22}(-,\ MIN)\ z_{21}}{[z_{11}(+,MIN)\ z_{22}]\ (-,\ MIN)\ [z_{12}(+,MIN)\ z_{21}]}$,

$zp_2 = \frac{z_{11}(-,\ MIN)\ z_{12}}{[z_{11}(+,MIN)\ z_{22}]\ (-,\ MIN)\ [z_{12}(+,MIN)\ z_{21}]}$ and

$zq_1 = \frac{z_{22}(-,\ MIN) z_{12}}{[z_{11}(+,MIN)\ z_{22}]\ (-,\ MIN)\ [z_{12}(+,MIN)\ z_{21}]}$,

$zq_2 = \frac{z_{11}(-,\ MIN) z_{21}}{[z_{11}(+,MIN)\ z_{22}]\ (-,\ MIN)\ [z_{12}(+,MIN)\ z_{21}]}$ and

the value of the z-game is

$zv = \frac{z_{11}z_{22}(-,MIN)z_{12}z_{21}}{[z_{11}(+,MIN)\ z_{22}]\ (-,\ MIN)\ [z_{12}(+,MIN)\ z_{21}]}$.

Numerical Computation

Example 3: Consider the following z-payoff, present in Table 1:

Table 1. z-payoff

	Player 2		
Player 1	$Z_{11} = (A_{11}, B_{11})$ = ((-1,2,3,4),(.6,.7,.8,.9))	$Z_{12} = (A_{12}, B_{12})$ ((1,2,3,6),(.7,.8,.9,.95))	$Z_{13} = (A_{13}, B_{13})$ ((-1,1,2,3),(.6,.7,.8,.9))
	$Z_{21} = (A_{21}, B_{21})$ = ((-2,-1,1,2),(1,1,1,1))	$Z_{22} = (A_{22},B_{22})$ = ((-2,-4,-6,-6),(.6,.8,.9,.95))	$Z_{23} = (A_{23}, B_{23})$ = ((-2,-3,-4,-5),(.75,.85,.9,.95))
	$Z_{31} = (A_{31}, B_{31})$ = ((-2,-1,3,6),(.6,.7,.8,.9))	$Z_{32} = (A_{32}, B_{32})$ = ((3,4,6,7),(.85,.9,.95,1))	$Z_{33} = (A_{33}, B_{33})$ = ((-4,-3,0,2),(.85,.9,.95,1))

Choose the rank $r_1(C) = r_1(a,b,c,d) = \frac{(c^2+d^2+cd-a^2-b^2-ab)}{3(c+d-a-b)}$ and $r_2(C) = \frac{a+b+c+d}{4}$ for the trapezoidal fuzzy number C(a,b,c,d) and find MRF(Z_{ij}), i = 1 to 3, j = 1 to 3, then put it in Table 2.

Table 2. The rank of the above z-payoff using MRF:

	Player 2		
Player 1	MRF(A_{11}, B_{11}) = 1.4	MRF (A_{12}, B_{12}) = 2.6	MRF (A_{13}, B_{13}) = 0.9
	MRF (A_{21}, B_{21}) = 0	MRF (A_{22},B_{22}) = -3.6	MRF(A_{23}, B_{23}) = -3.0
	MRF (A_{31}, B_{31}) = 1.2	MRF (A_{32}, B_{32}) = 4.6	MRF (A_{33}, B_{33}) = -1.1

Ranking matrix for the given z-payoff:

$$\begin{array}{cc} & \begin{array}{ccc} \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \quad (\boldsymbol{Row\ minimum}) \\ \begin{array}{c} \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{bmatrix} \mathbf{1.4} & \mathbf{2.6} & \mathbf{0.9} \\ \mathbf{0} & \mathbf{-3.6} & \mathbf{-3.0} \\ \mathbf{1.2} & \mathbf{4.6} & \mathbf{-1.1} \end{bmatrix} \begin{array}{c} \mathbf{0.9} \\ \mathbf{-3.6} \\ \mathbf{-1.1} \end{array} \\ (\boldsymbol{Column\ Maximum}) & \begin{array}{ccc} \mathbf{1.4} & \mathbf{4.6} & \mathbf{0.9} \end{array} \end{array}$$

From the above table, Maximum of Rowminimum = 0.9 = Minimum of Column maximum. So, the position of a z-saddle point is (1,3) and the Strategy 1 is the optimal strategy for Player 1 and Strategy 3 is the optimal strategy for Player 2. Also, the reliability of the expected gain is in Table 3, $B_{MIN} = \min\{B_{ij} | 1 \le i \le 3, 1 \le j \le 3\}$, where the minimum of fuzzy numbers is calculated by ordering the list of fuzzy numbers by the r_2 ranking function. Hence, the expected z-payoff to player 1 at the z-saddle point is (A_{13}, B_{MIN}), where A_{13} = (-1,1,2,3), B_{MIN}= min {B_{11}, B_{12}, B_{13}, B_{21}, B_{22}, B_{23}, B_{31}, B_{32}, B_{33}}

Table 3. Calculation of B_{MIN}

B_{ij}= (a,b,c,d)	$r_2(B_{ij}) = (a+b+c+d)/4$	$B_{MIN} = \min\{B_{ij} \| 1 \le i \le 3, 1 \le j \le 3\}$
B_{11} = (.6,.7,.8,.9)	.75	Comparing with all $r_2(B_{ij})$, .75 is minimum, So, B_{MIN} = B_{11} or B_{13} or B_{31} = (.6,.7,.8,.9) Take $B_{MIN} = (.6,.7,.8,.9) = B_{11}$
B_{12} = (.7,.8,.9,.95)	.84	
B_{13} =(.6,.7,.8,.9)	.75	
B_{21} = (1,1,1,1)	1	
B_{22} = (.6,.8,.9,.95)	.81	
B_{23} = (.75,.85,.9,.95)	.86	
B_{31} = (.6,.7,.8,.9)	.75	
B_{32} = (.85,.9,.95,1)	.93	
B_{33} = (.85,.9,.95,1)	.93	

The expected z-payoff to the player 1 is (A_{13}, B_{MIN}))= ((-=((1,1,2,3),(.6,.7,.8,.9)) = the value of the z-game.

Example 4: Solve the following z-game:

$$\begin{array}{ccc} & Q_1 & Q_2 \\ \begin{array}{c} P_1 \\ P_2 \end{array} & \multicolumn{2}{c}{\begin{bmatrix} ([8,10],[0.75,.8]) & ([6,8],[.8,.85]) \\ ([4,6],[.8,.9]) & ([10,12],[.75,.9]) \end{bmatrix}} \end{array}$$

Solution:

Ranking of above z-payoff using MRF

Choose the ranking function $r_1(A) = r_1([a,b]) = \frac{(a+b)}{2}$ and $r_2([c,d]) = \frac{(c+d)}{2}$ for the z-interval z = (A, B), where A = [a, b] and B = [c, d].

$$\begin{array}{ccc} & Q_1 & Q_2 \\ \begin{array}{c} P_1 \\ P_2 \end{array} & \multicolumn{2}{c}{\begin{bmatrix} MRF([8,10],[0.75,.8]) & MRF([6,8],[.8,.85]) \\ MRF([4,6],[.8,.9]) & MRF([10,12],[.75,.9]) \end{bmatrix}} \end{array} = \begin{array}{cccc} & Q_1 & Q_2 & Row\ Minimum \\ \begin{array}{c} P_1 \\ P_2 \end{array} & \multicolumn{2}{c}{\begin{bmatrix} 6.975 & 5.775 \\ 4.25 & 9.075 \end{bmatrix}} & \begin{array}{c} \mathbf{5.775} \\ 4.25 \end{array} \\ Column\ Maximum & \mathbf{6.975} & 9.075 & \end{array}$$

Maximum of RowMinimum $\neq$ Minimum of Column Maximum

So, the z-saddle point does not exist.

Now, $zp_1 = \frac{z_{22}(-,\ MIN)\ z_{21}}{[z_{11}(+,MIN)\ z_{22}]\ (-,\ MIN)\ [z_{12}(+,MIN)\ z_{21}]}$
$= \frac{([4,8],[.75,.9])}{([4,12],[.75,.8])}$
$= \left(\left[\min\left(\frac{4}{4},\frac{4}{12},\frac{8}{4},\frac{8}{12}\right), max\left(\frac{4}{4},\frac{4}{12},\frac{8}{4},\frac{8}{12}\right)\right], MIN(.825,.775)\right) = ([0.3,2],[.75,.8])$

The optimum mixed strategy for player 1 is $zp_1 = ([0.3,2],[.75,.8])$, zp_2= ([0,1],[.75,.8]), and the optimum mixed strategy for player 2 is $zq_1 =$ ([0.5,1.5], [.75,.8]),$zq_2 =$ ([0.5,1.5], [.75,.8]) and the value of the game zv=([6.7,30],[,75,.8]).

Conclusion:

A novel ranking procedure for z-numbers has been presented here. Its application to two-person zero-sum z-games has been highlighted. MRF is easy to implement and will prove to be a very useful tool in z versions of various optimization problems.

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Authors' declaration:

- Conflicts of Interest: None.
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Authors' Contribution Statement:

GV gave the concept for the paper and suggested the problem. KP analyzed the problem and derived the solution and interpreted the result. KP drafted the paper. GV revised and proofread the manuscript.

Under the guidance of G.V., this work was newly introduced and developed by K.P. Both authors read the manuscript carefully and approved the final manuscript.

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دالة ترتيب الزخم لأرقام - Z وتطبيقاتها في نظرية الألعاب

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[2]قسم الرياضيات، كلية سري ميناكشي الحكومية للفنون للبنات (أ)، مادوراي، تاميل نادو، الهند.

الخلاصة:

بعد أن قدم زاده مفهوم ارقام - z أبدى العلماء في مختلف المجالات اهتماما كبيرا بتطبيق هذا المفهوم على مختلف التطبيقات. في تطبيقات أرقام - z ، لمقارنة رقمين z ، يعد إجراء الترتيب ضروريا. في حين تم بالفعل اقتراح عدد قليل من وظائف الترتيب في الأدبيات ، هناك حاجة لتطوير بعض وظائف الترتيب الجيدة. في هذه الورقة ، تم اقتراح دالة ترتيب جديدة لأرقام - z "وظيفة ترتيب الزخم" (MRF). أيضا ، تم النظر في المشكلات النظرية للعبة حيث تكون عناصر مصفوفة العائد هي أرقام - z وتم توضيح تطبيق وظيفة ترتيب الزخم في مثل هذه المشكلات.

الكلمات المفتاحية: دالة ترتيب الزخم، العاب–z، اعداد – z، مصفوفة العائد–z، النقطة السرجية–z.