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Optimum System Design Using Rough Interval Multi-Objective De Novo Programming

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Abstract

The Multi-objective de novo programming method is an effective tool to deal with the optimal system design by determining the optimal level of resources allocation (RA) to improve the value of the objective functions according to the price of resources (the conditions are certainty). This paper suggested a new approach for solving uncertainty of De novo programming problems (DNP) using a combination model consisting of a rough interval multi-objective programming (RIMOP) and DNP, where coefficients of decision variables of objective functions and constraints are rough intervals (RIC). Three methods are used to find the optimal system design for the proposed model, the first method is the weighted sum method (WSM) which is used before reformulating RIMOP (bi of constraints is known), WSM gives one ideal solution among the feasible solutions under each bound of sub-problem, the second method is Zeleny's approach and the third method is the optimal path- ratios, methods (two and three) are used after formulating (RIMODNP) (bi of constraints is unknown), Zeleny's approach gives one (alternative) optimal system design under each bound of sub-problem, while the optimal path-ratios method: after checking the bounds according to Shi's theorem, determines whether the bounds of the proposed model are feasible or not, and then use the method, this method uses three types of ratios gives three (alternatives) under each bound of sub-problem. From the results, it is clear that the optimal path-ratios method is more efficient than others in solving the proposed model because it provides alternatives to the decision-maker (DM), it is noted that the proposed model is compatible with the conditions and theories of RIC. As a result, the proposed model is very suitable for conditions of uncertainty. Finally, applied example is also presented for the proposed model application.

Keywords: De novo programming, Multi-objective linear programming, Optimum-path ratios, Optimal system design, Rough interval linear programming.

Introduction

Multi-objective linear programming (MOLP) techniques play an important role in solving decision problems, which involve more than one objective function[1]. These techniques use the priority factor or weighting factor according to the information obtained from the decision maker and provide the decision maker with solutions. However, since there are multi-objectives, it is very difficult to

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obtain optimal solutions using these techniques. Therefore, one must look for satisfactory or bargain solutions[2,3].

In mathematical techniques, both method of solution and constraints affect the solution[4]. If the constraint resources are not used to their full potential in a mathematical model, the unused resources reduce the level of achievement of the goals. Therefore, it is very important to ensure that all objectives are achieved at optimal levels, and that constrained resources are used to their full potential[5].

De novo programming (DNP) was conducted by Zeleny[6], which represents an ideal system rather than an optimization of a specific system. Today's production and management systems are necessarily more flexible. It must be quickly designed and redesigned, disassembled and reassembled again, which requires continuous reorganization of resources to ensure the feature. Because all systems are built within their boundaries, they lack alternatives, options, and design variables in their creation environment. Therefore, when redesigning, reconfiguring or optimizing the system, its limitations and limitations must be worked out as well. It is not enough to reshape it on the basis of a specific system with its priorities and options. Therefore, system design requires creation of alternatives rather than selection. As opposed to optimizing a specific system as standard methods do, see[7-9]

There are plenty of studies on DNP methods and applications under certainty for instance, the researchers[7] proposed a new approach to project portfolio design based on a systematic combination of the data envelop analysis (DEA) model and DNP optimization approach, the proposed model provides optimal project portfolio design with minimal budget as well, authors[8] generalized the DNP approach to find the optimal design for production system, suggesting more types of restrictions possible, in particular $' \geq ', =$.

The worker[9] used lexical-objective programming to find solutions of MODNP problem with positive ideal solutions. New authors[10] suggested a new approach min-max GP for solving MODNP, they compare it with Umarusman's problem, they found that the solution gotten by min-max GP approach are better than Umarusman's problem in same weights used.

Participants[11]were used DNP to the planning of urban parks in Taichung city, Taiwan. They found that the DNP increases the total utility of metropolitan parks by move resources from the economic and ecological, thus MCDM and MOP methods were able to provide an effective solution for evaluating metropolitan parks.

Several authors[12] applied MODNP by formulating a problem to solve budget optimization in the stock market, they proposed a new approach as a case study based on data collected from the Bomba Stock Exchange (BSE).

New study [13] applied DNP on PT.X company by formulating LP problem to DNP, it solved the problem by simplex method, where the DNP technique achieved the optimal number of productions.

They[14] proposed general method for solving MODNP, by assuming the problem has two types of objectives (Max and Min), they obtained that the proposed method gives the DM freedom to select the objectives functions which should be prioritized. another workers[15] suggested new approach named 'one-step method' for solving general DNP using min-max GP technique, the solution obtained from the one-step method is more efficient than the classical DNP with crisp parameters.

In spite of Zeleny approach gives an optimal system design when he applied DNP at certainty conditions only, it did not work with uncertainty conditions[6]. Many systems analysis methods were developed for solving DNP under uncertainty, such as fuzzy, interval, and stochastic programming. For example,

The author[16] applied the fuzzy goal programming approach to a multi-criterion de novo linear programming problem (α -MDNPLPP) by defining appropriate membership functions and aspiration levels, she found that the main advantage of this approach gives to the DM more freedom to determine the α-level and thus evaluate the effective solution to reduce his incomplete knowledge about the field.

The same above author[17] introduced a new approach to solve MODNP by assuming possibilistic objective functions coefficients. The solution of the problem is achieved by using an efficient and necessary condition.

The researchers[18] proposed two concepts of fuzzy and interval type-II fuzzy resources. The main targets of their study are developed for resource allocation and target setting using DNP.

The author[19] used the fuzzy goal with fuzzy parameters model and then integrating (positive and negative) ideal solutions, also introduced a new

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fuzzy DNP technique, the recommended method, which combined fuzzy resource unit pricing and fuzzy constraint amount was used to construct the fuzzy budget.

The researcher[20] is mainly concerned with optimization, both static and dynamic. Under ambiguous (Fuzzy) information, the optimization problem is formulated as maximizing (or minimizing) some utility function. He applied fuzzy DNP model on sustainable regional development. The interval DNP method for planning water resources systems was used under uncertainty conditions by[21]. The interval-fuzzy DNP for planning water resources systems was used by[22]. Monte-Carlo[23]-based interval fuzzy DNP method developed for land-use planning under uncertainty. Luhandjula's compensatory $\mu\theta$ - operator used to solve the general MODNP problem under a fuzzy environment in one step[23,24]. The fuzzy budget is constructed by [25] using fuzzy unit pricing of resources and fuzzy resource amounts of restrictions

In our study, rough interval coefficient (RIC) used to develop the Zeleny approach by applying uncertainty conditions. RIC has a main advantage that makes it applicable when data are not available or vague [26,27].

The main question of our work is "How to allocate resources under uncertainty conditions with an un-determent budget?"

In this paper, a proposed model is presented to solve the uncertainty problem using multi-objective linear programming with rough interval coefficients (RIC) combine with de novo programming, the proposed model would be Rough Interval Multi-Objective De Novo Programming (RIMODNP), three methods are used to solve RIMODNP (First: WSM before formulation, assume right- hand side known, Second: Zeleny's approach, Third: Optimal path-ratios, two methods are assumed right-hand side of constraints unknown), rest of this research is organized as follows: Section 2 presents "Methodology" which includes multi-objective linear programming, WSM, Rough interval linear programming model, DNP and MORIDNP and the steps of the proposed method. Section 3 considers "applied example and results and discussions ". And finally, the conclusion is given in Section 4.

Materials and Methods

Methodology

Multi-objective Linear Programming Model

Multi-objective linear programming (MOLP) is a model of optimizing a given system by multiple objectives. It is usually impossible to optimize all objectives simultaneously in a given system. A trade-off means that one cannot increase the level of satisfaction for an objective without decreasing it for another one. Trade-offs is property of an inadequately designed system and thus it can be eliminated through designing a better one. MOLP problem can be described as follows:

Consider the standard model of MOLP

$$max\ f_k = \sum_{j=1}^{n} C_{kj}\, X_j, \qquad k = 1,2,\dots,l,$$

subject to :

$$\sum_{j=1}^{n} a_{ij}\, X_{j_i} \le b_i, \quad i = 1,2,\dots,m,$$

1

$$X_j \ge 0, j = 1,2,\dots,n$$

Where:

The parameters $b_i\ (i = 1,2,\dots,m)$ represent the given available resources as constants. The efficient solution concept results from the solution of the MOLP model see for more details[27-29].

The Weighting Sum Method (WSM)

WSM[3,5,14] is used to solve multi-objective functions, The basic idea of WSM is that it is uses non-negative weights $w_1, w_2, \dots . w_k$ multiplied by the corresponding objective and then a composite objective is calculated using summation of the weighted objectives. Then, the objective is modified for different weight combinations over and over again.

Rough Interval Linear Programming Model:

The rough interval linear programming Model (RILP) is extension of the linear programming problem with rough interval coefficients, to predict when a data value is not properly known, but can be estimated with upper interval and lower interval bounds, a rough interval linear programming problem can be formulated as follows:

$$Max\ or\ Min\ \ f = \sum_{j=1}^{n}([\underline{c}_j^L, \underline{c}_j^U], [\overline{c}_j^L, \overline{c}_j^U])x_j$$

$subject\ to$: $\sum_{j=1}^{n}([\underline{a}_{ij}^{L}\ \underline{a}_{ij}^{U}],[\overline{a}_{ij}^{L},\overline{a}_{ij}^{U}])x_j \le ([\underline{b}_i^L,\underline{b}_i^U],[\overline{b}_i^L,\overline{b}_i^U])$ 2
$x_j \ge 0,\ j=1,2,\dots,n, \quad i=1,2,\dots,m$
Where:
$([\underline{c}_j^L,\underline{c}_j^U],[\overline{c}_j^L,\overline{c}_j^U])$,($[\underline{a}_{ij}^L,\underline{a}_{ij}^U],[\overline{a}_{ij}^L,\overline{a}_{ij}^U])$, and$([\underline{b}_i^L,\underline{b}_i^U],[\overline{b}_i^L,\overline{b}_i^U])$are rough interval coefficients of objective function and constraints and also, let x = $(x_1,x_2,\dots,x_n)^t$ represent the vector of all decision variables see for more details[21,28,29].

Properties of Rough Interval (RIC)

In order to validate the proposed model, three properties must be met[30]:

$$\left.\begin{array}{l}\left[\underline{f}_{ij}^L,\underline{f}_{ij}^U\right]\subseteq[\overline{f}_{ij}^L,\overline{f}_{ij}^U]\Rightarrow\overline{f}_{ij}^L\le\underline{f}_{ij}^U\le\underline{f}_{ij}^L\le\overline{f}_{ij}^U\\ \left[\underline{C}_{ij}^L,\underline{C}_{ij}^U\right]\subseteq[\overline{C}_{ij}^L,\overline{C}_{ij}^U]\Rightarrow\overline{C}_{ij}^L\le\underline{C}_{ij}^U\le\underline{C}_{ij}^L\le\overline{C}_{ij}^U\\ \left[\underline{a}_{ij}^L,\underline{a}_{ij}^U\right]\subseteq[\overline{a}_{ij}^L,\overline{a}_{ij}^U]\Rightarrow\overline{a}_{ij}^L\le\underline{a}_{ij}^U\le\underline{a}_{ij}^L\le\overline{a}_{ij}^U\end{array}\right\}$$
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De Novo Programming Model

DNP is used for reshaping feasible sets in linear systems, it is utilized as an approach of optimum system design. Given resource pricing and a budget, the MODNP problem is reformulated. To get the DNP formulation from the problem 1, it is necessary to convert b_i from constants to variables, and then determine their values as follows:

$max\ f_k = \sum_{j=1}^{n} C_{kj}\, X_j,\qquad k=1,2,\dots,l,$
subject to :
$\sum_{j=1}^{n} a_{ij}\, X_{j_i} \le b_i, i=1,2,\dots,m,$
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$\sum_{j=1}^{m} p_i\, b_i \le B,$
$X_j \ge 0, j=1,2,\dots,n$
Where:
$X_j,.\ b_i$ are decision variables for products and available resources respectively, p_i, B are the given of both the unit price of resource *i* and total available budget respectively.
For single or multiple objective problems, $\boldsymbol{f_k}$ is for maximizing profit.
From 2 follows: $PAx \le Pb \le B$
Defining n-vector of unit cost $V = Pb$ it can be rewriting problem 4 as the follows:
$Max\quad f_k = CX$
$s.t.\quad VX \le B \qquad , \qquad X \ge 0$
5
Solving single objective problems
$Max\quad f^i = C^i X$ i= 1, 2, 3,…, k
$s.t.\quad VX \le B$
$X \ge 0$
6
f^* is k- vector of objective values for the ideal system with respect to B.
the meta-optimum problem can be formulated as follows:
$Min\quad Z = VX$
$s.t.\quad CX \ge f^*$
$X \ge 0$
7
Solving problem 5 provides the solution:
$X^*\quad ,\ B^* = VX^*\quad ,b^* = AX^*$, for more details see[9,10].

Optimum-Path Ratio Method for Solving DNP

The optimum-path ratio[31] for achieving the best performance for a given budget $\boldsymbol{B}$ is defined as: $\boldsymbol{r_1} = \frac{\boldsymbol{B}}{\boldsymbol{B^*}}$ the given budget level $\le B^*$. Optimal system design for $\boldsymbol{B}$: $X = r_1 X^*, b = r_1 b^*, Z = r_1 f^*$, the optimum-path ratio represents an effective and fast tool for the efficient optimal redesign of large-scale linear systems. There are possible define six types of optimum-path ratios as shown in Table 1:

Table 1. Six types of optimum-path ratios.

Ratio 1	Ratio 2	Ratio 3	Ratio 4	Ratio 5	Ratio 6
$\boldsymbol{r_1} = \frac{\boldsymbol{B}}{\boldsymbol{B^*}}$	$r_2 = \frac{B}{B^{**}}$	$r_3 = \frac{\sum_i \alpha_i B_i^j}{B^{**}}$	$r_4 = \frac{B}{B^{**}}$	$r_5 = \frac{\sum_i \alpha_i B_i^j}{B^*}$	$r_6 = \frac{\sum_i \alpha_i B_i^j}{B^{**}}$

Where: $(X^*, B^* = V X,^* b^* = AX^*)$ represent the results of a meta-optimum and the value B^*identifies the minimum budget to achieve f^*
So, $(X^{**}, B^{**} = V X^{**}, b^{**} = AX^{**})$ represent the results of a synthetic optimal solution: where the value B^{**} identifies the synthetic-optimum performance f^{**} related to given combined budget level $\sum_i \alpha_i B_i^j$, (α_i represent the weight of benefit each $\boldsymbol{B_i^j}$ to produce X_i^j in terms of the j[th] criterion)[31-33].

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Optimal System Design

It's a set of designs that can be found through optimum-path ratios as in Table 1, the following optimum system designs can be determined:

(i) $x^1 = r^1 x^{**}, \quad b^1 = r^1 b^{**} \quad and \;\; f^1 = r^1 f^{**}$ 8

(ii) $x^2 = r^2 x^{**}, \quad b^2 = r^2 b^{**} \quad and \;\; f^2 = r^2 f^{**}$ 9

(iii) $x^3 = r^3 x^{**}, \quad b^3 = r^3 b^{**} \quad and \;\; f^3 = r^3 f^{**}$ 10

(iv) $x^4 = r^4 x^{*}, \quad b^4 = r^4 b^{*} \quad and \;\; f^4 = r^4 f^{*}$ 11

(v) $x^5 = r^5 x^{*}, \quad b^5 = r^5 b^{*} \quad and \;\; f^5 = r^5 f^{*}$ 12

(vi) $x^6 = r^6 x^{nd}, \quad b^6 = r^6 b^{nd} \quad and \;\; f^6 = r^6 f^{nd}$ 13

The optimum system design above (x^i, b^i, f^i), i=1,….,6, Where:

b^i: Optimum portfolio of resources to be acquired at the current market prices, p, allows one to produce x^i and realize the multi-criteria performance f^i see[10-12].

The Proposed Model of Rough Interval Multi-Objective De Novo Programming (RIMODNP)

The general mathematical model 14 rough interval multi-objective de novo programming problem (RIMODNP) is as follows:

$$Min\ or\ Max\ f^K(X) = \sum_{i=1}^{m}\sum_{j=1}^{n}([\underline{C}_{ij}^{KL}, \underline{C}_{ij}^{KU}], [\overline{C}_{ij}^{KL}, \overline{C}_{ij}^{KU}])x_{ij}$$

$where\ k = 1,2..,K$

$subject\ to$

$$\sum_{j=1}^{n}([\underline{a}_{ij}^{L}, \underline{a}_{ij}^{U}], [\overline{a}_{ij}^{L}, \overline{a}_{ij}^{U}])x_j \le b_i$$

$$\sum_{j=1}^{n}([\underline{P}_i^{L}, \underline{P}_i^{U}], [\overline{P}_i^{L}, \overline{P}_i^{U}])b_i \le ([\underline{B}^{L}, \underline{B}^{U}], [\overline{B}^{L}, \overline{B}^{U}]) \quad 14$$

$x_j \ge 0, \quad i = 1,2,\dots,m, \; j = 1,2,\dots,n, and \;\; k = 1,2,\dots,l\dots.$

Where:

$([\underline{C}_{ij}^{KL}, \underline{C}_{ij}^{KU}], [\overline{C}_{ij}^{KL}, \overline{C}_{ij}^{KU}])$ is a vector of rough interval coefficients for multi objective function $([\underline{a}_{ij}^{L}, \underline{a}_{ij}^{U}], [\overline{a}_{ij}^{L}, \overline{a}_{ij}^{U}])$: is a matrix of rough interval coefficients for constraints of multi objective function, $([\underline{P}_i^{L}, \underline{P}_i^{U}], [\overline{P}_i^{L}, \overline{P}_i^{U}])$: is a vector of rough interval coefficients of unit price of resources i and $\left([\underline{B}^{L}, \underline{B}^{U}], [\overline{B}^{L}, \overline{B}^{U}]\right)$: is a rough interval of total available budget.

where $(i = 1,2,\dots,m; \;\; j = 1,2,\dots,n)$, $x = (x_1, x_2, \dots, x_n)^t$ denote the vector of all decision variables.

$f^{RI(k)} = \left([\underline{f}^{kL}, \underline{f}^{kU}] : [\overline{f}^{kL}, \overline{f}^{kU}]\right)$ respectively and $k = 1,2,\dots,$ K is the number of objectives.

(i) The rough interval $\left([\underline{f}^{kL}, \underline{f}^{kU}] : [\overline{f}^{kL}, \overline{f}^{kU}]\right)$ is called the surely (possibly) optimal range of problem 3, if optimal range is subset of $\left([\underline{f}^{kL}, \underline{f}^{kU}] : [\overline{f}^{kL}, \overline{f}^{kU}]\right)$.

(ii) Let $[\underline{f}^{kL}, \underline{f}^{kU}] : [\overline{f}^{kL}, \overline{f}^{kU}]$ be surely optimal (possibly) optimal range of the problem 14. Then the rough interval $\left([\underline{f}^{kL}, \underline{f}^{kU}], [\overline{f}^{kL}, \overline{f}^{kU}]\right)$ is called the rough optimal range of problem 14.

(iii) The optimal solution of each corresponding MODNP problem 14 which its optimal value belongs to $[\underline{f}^{kL}, \underline{f}^{kU}] : [\overline{f}^{kL}, \overline{f}^{kU}]$ is called a completely (rather) satisfactory solution of the problem 14.

$$\left.\begin{aligned}[\underline{P}_i^{L}, \underline{P}_i^{U}] \subseteq [\overline{P}_i^{L}, \overline{P}_i^{U}] \Rightarrow \overline{P}_i^{L} \le \underline{P}_i^{U} \le \underline{P}_i^{L} \le \overline{P}_i^{U} \\ [\underline{B}^{L}, \underline{B}^{U}] \subseteq [\overline{B}^{L}, \overline{B}^{U}] \Rightarrow \overline{B}^{L} \le \underline{B}^{U} \le \underline{B}^{L} \le \overline{B}^{U}\end{aligned}\right\}$$

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Converting the Proposed Model "Rough Interval Multi-Objective De Novo Programming" into Four Sub-models:

The rough interval multi-objective de novo programming problem can be transformed into a linear multi-objective program using Tong-Shochang[28] Method, this method is used for solving the problem by converting the major problem into two classical sub-problems (Lower interval sub-problem and Upper problem) and then convert lower interval into two (1st bound of lower rough interval and 2nd bound of lower rough interval) also convert

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upper interval into two (1st bound of upper rough interval and 2nd bound of upper rough interval) as shown in 16:

$Min\ or\ Max\ f^{K}(X)\ =$
$\sum_{i=1}^{m}\sum_{j=1}^{n}([\underline{C}_{ij}^{KL}, \underline{C}_{ij}^{KU}], [\overline{C}_{ij}^{KL}, \overline{C}_{ij}^{KU}])x_{ij}$
Subject to
$\sum_{j=1}^{n}([\underline{P}_{i}^{L}, \underline{P}_{i}^{U}], [\overline{P}_{i}^{L}, \overline{P}_{i}^{U}])b_i \le ([\underline{B}^{L}, \underline{B}^{U}], [\overline{B}^{L}, \overline{B}^{U}])$
$\sum_{j=1}^{n}([\underline{a}_{ij}^{L}, \underline{a}_{ij}^{U}], [\overline{a}_{ij}^{L}, \overline{a}_{ij}^{U}])x_j \le b_i$
$where\ k = 1,2, \dots, K, x_j \ge 0$

$Min\ or\ Max\ \underline{f}^{kL}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\underline{C}^{kL}x_{ij}$
s.to $\sum_{j=1}^{n}\underline{a}_{ij}^{L}x_{ij} \le bi$
$\sum_{j=1}^{n}\underline{P}_{i}^{L}b_i \le \underline{B}^{U}$, $x_j \ge 0$

$Min\ or\ Max\ \underline{f}^{kU}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\underline{C}^{kU}x_{ij}$
s.to $\sum_{j=1}^{n}\underline{a}_{ij}^{U}x_{ij} \le bi$
$\sum_{j=1}^{n}\underline{P}_{i}^{U}b_i \le \underline{B}^{L}$, $x_j \ge 0$

$Min\ or\ Max\ \overline{f}^{kL}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\overline{C}^{kL}x_{ij}$
s.to $\sum_{j=1}^{n}\overline{a}_{ij}^{L}x_{ij} \le b_i$,
$\sum_{j=1}^{n}\overline{P}_{i}^{L}b_i \le \overline{B}^{U}$, $x_j \ge 0$

$Min\ or\ Max\ \overline{f}^{kU}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\overline{C}^{kU}x_{ij}$
s.to $\sum_{j=1}^{n}\overline{a}_{ij}^{U}x_{ij} \le b_i$,
$\sum_{j=1}^{n}\overline{P}_{i}^{U}b_i \le \overline{B}^{L}$,
$x_j \ge 0 \quad , i = 1,2 \dots, m , j =$
$1,2, \dots, n\ and\ and\ \ k = 1,2, \dots, l$
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Where:
$\underline{f}^{kL}(x)$: the multi-objective of the 1st bound of lower rough interval.
$\underline{f}^{kU}(x)$: the multi-objective of the 2nd bound of lower rough interval.
$\overline{f}^{kL}(x)$: the multi-objective of the 1st bound of upper rough interval.
$\overline{f}^{kU}(x)$: the multi-objective of the 2nd bound of upper rough interval.

Results and Discussion

In order to check our proposed model, numerical example can be applied as follows:

The Steps of Proposed Model are as Follows:

Step1: Converting RIMOP to MOP with four sub-models with fixed right-hand side (resources).

Step2: Solving MOP model using WSM with value of $wi = between\ (0,1)$ reaching to the optimal values.

Step 3: Reformulating RIMOP obtain to RIMODNP with unknown right-hand side (resources).

Step 4: Converting RIMODNP into four sub-models using Tong-Shochang method.

Step 5: Solving each multi-objective function individually under the set of constrains using POM- QM for windows v5 software.

Step 6: Reformulating model 2 with multi-objectives to allow the change the value of b_i, as in model 16 with unknown variables x which represents unknown values of capacities and requirements respecting budget B that will be used.

Step 7: Computing structure design for model 16 separately for the individual objective functions.

Step 8: Checking the results as to whether it is feasible or not based on Shi's theorem, if the results gave infeasible solution go to Step 9.

Step 9: Checking the results (bounds of rough interval for objectives f and resources b of optimal system design) for proposed model 16 if the results according to the properties go to step 10, otherwise go to step 6.

Step 10: Calculating optimum-path ratios to find the optimum system design.

Step 11: Choosing a design from among designs by the decision maker.

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Applied Example:

The following example Rough Interval Multi-Objective Linear Programming problem 17 (RIMOP):

$Max\ f_1 = ([2.5,3],[1.5,3.5])x_1 + ([2.5,3],[2,3.5])x_2$ (profit)

$Max\ f_2 = ([3,2.5],[2.5,4])x_1 + ([3.5,4],[3,5])x_2$ (quality)

$s.t.\ ([3,3.5],[2.5,5])x_1 + ([2.5,3],[2,4])x_2 \leq 60$ (raw materal1) 17

$([2.5,3],[2,4])\ x_1\ +\ ([3,3],[1.5,\ 4])\ x_2 \leq 40$ (raw materal2)

$([1.5,2],[1,2.5])\ x_1\ +\ ([3.5,4],[3,5])\ x_2 \leq 30$ (raw materal3) $, x_1, x_2 \geq 0$

Step1: Solving RIMOP before reformulation using WSM: RIMOP is converted into four sub-models of MOP as shown in Table 2 and then each sub-model is solved individually.

Table 2. Convert RIMOP to MOP into four sub-models.

1st problem	2nd problem	3th problem	4th problem
$\boldsymbol{Max\ \underline{f}_1^L = 2.5x_1 + 2.5x_2}$	$Max\ \underline{f}_1^U = 3x_1 + 3x_2$	$Max\ \bar{f}_1^L = 1.5x_1 + 2x_2$	$Max\ \bar{f}_1^U = 3.5x_1 + 3.5x_2$
$\boldsymbol{Max\ \underline{f}_2^L = 3x_1 + 3.5x_2}$	$Max\ \underline{f}_2^U = 2.5x_1 + 4x_2$	$Max\ \bar{f}_2^L = 2.5x_1 + 3x_2$	$Max\ \bar{f}_2^U = 4x_1 + 5x_2$
$\boldsymbol{s.t.}$	$s.t.$	$s.t.$	$s.t.$
$\boldsymbol{5x_1 + 4x_2 \leq 60}$	$2.5x_1 + 2x_2 \leq 60$	$3.5x_1 + 3x_2 \leq 60$	$3x_1 + 2.5x_2 \leq 60$
$\boldsymbol{4x_1} + 4\boldsymbol{x_2} \leq \boldsymbol{40}$	$2x_1 + 1.5x_2 \leq 40$	$3x_1 + 3x_2 \leq 40$	$2.5x_1 + 3x_2 \leq 40$
$2.5\ \boldsymbol{x_1} + 5\ \boldsymbol{x_2} \leq \boldsymbol{30}$	$x_1 + 3\ x_2 \leq 30$	$2\ x_1 + 4\ x_2 \leq 30$	$1.5\ x_1 + 3.5\ x_2 \leq 30$
$\boldsymbol{x_1, x_2 \geq 0}$	$x_1, x_2 \geq 0$	$x_1, x_2 \geq 0$	$x_1, x_2 \geq 0$

Step 2: Table 3 represents the results obtained using the WSM under the ratio between $w_i = (0,1)$. The method is as follows: Determine the proportion for each objective functions and then multiply the objective function by the proportion that has been determined, then collect the objective functions, getting a composite objective function, finally, the model is solved by PRO-QM to plot functions, as shown in Table 3, and Figs. 1-4 for each sub-problem.

Table 3. Results obtained from solving WSM provide optimal solutions for 1st problem.

	Weight $\boldsymbol{w = (w_1, w_2)}$	Composite's objective	Optimal solution $(\boldsymbol{x_1, x_2})$	Optimal objective value $(\boldsymbol{f_1, f_2})$	Optimal Composite's objective
1st problem	(0.0,1.0)	$3x_1 + 3.5x_2$	(0,6)	(15,21)	30.00
	(0.1, 0.9)	$2.65x_1 + 3.15x_2$	(2,8)	(25,31)	30.50
	(0.2,0.8)	$2.9x_1 + 3.4x_2$	(8,2)	(25,31)	30.00
	(0.3,0.7)	$2.85x_1 + 3.2x_2$	(8,2)	(25,31)	29.20
	(0.4,0.6)	$2.8x_1 + 2.9x_2$	(8,2)	(25,31)	28.20
	(0.5,0.5)	$2.75x_1 + 3x_2$	(8,2)	(25,31)	28.00
	(0.6,0.4)	$2.5x_1 + 2.9x_2$	(8,2)	(25,31)	25.80
	(0.7,0.3)	$2.65x_1 + 2.8x_2$	(8,2)	(25,31)	26.80
	(0.8,0.2)	$2.6x_1 + 2.7x_2$	(8,2)	(25,31)	26.20
	(0.9, 0.1)	$2.55x_1 + 2.6x_2$	(8,2)	(25,31)	27.16
	(1.0,0.0)	$2.5x_1 + 2.5x_2$	(10,0)	(25,30)	25.00

Below is a graphical depiction of feasible decision spaces, feasible objective space and optimal solution of WSM for each sub-problem.

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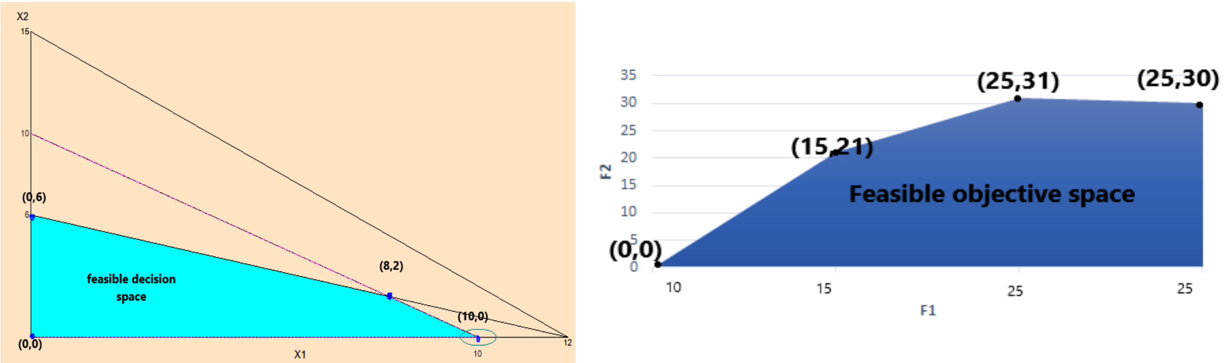

A

B

Figure 1. Feasible of WSM for 1st bound lower rough interval (1st problem): A) decision space, B) objective space.

From the results obtained in Table 3, optimizing the first objective with weight (0,1) the result of optimal solution is ($\boldsymbol{x_1}$ =0, $\boldsymbol{x_2}$ =6) with optimal objective values ($f_1 = 15, f_2 = 21$), the second objective results in the optimal solution ($\boldsymbol{x_1}$ =8, $\boldsymbol{x_2}$ =2), the unique optimal solution ($f_1 = 25, f_2 = 31$), and the third objective with weight (1,0) the results of optimal solution ($\boldsymbol{x_1}$ =10, $\boldsymbol{x_2}$ =0), the objective values ($f_1 = 25, f_2 = 30$), it is clear from the results the weight of second objective dominates the weights of first objective and third objective, so the second objective with point ($\boldsymbol{x_1}$ =8, $\boldsymbol{x_2}$ = 2) and ($f_1 = 25, f_2 = 31$) is optimal for **1st** problem. As shown in Fig 1 B. **1st** problem the point (25,31) in feasible objective space is dominates all points.

According to results are obtained from Table 4. using upper for lower rough interval 2nd problem**,** it is noticed that the objective results in the optimal solution ($\boldsymbol{x_1}$ =16.67, $\boldsymbol{x_2}$ =4.44) the unique optimal solution ($f_1 = 63.33, f_2 = 59.44$), as shown in Fig 2 B. of 2nd problem.

Table 4. Results obtained from solving WSM provide optimal solutions for 2nd problem.

	Weight $\boldsymbol{w} = (\boldsymbol{w_1}, \boldsymbol{w_2})$	Composite's objective	Optimal solution $(\boldsymbol{x_1}, \boldsymbol{x_2})$	Optimal objective value $(\boldsymbol{f_1}, \boldsymbol{f_2})$	Optimal Composite's objective
2nd problem	(0.0,1.0)	$2.5x_1 + 4x_2$	(16.67,4.44)	(63.33,59.44)	59.44
	(0.1, 0.9)	$2.55x_1 + 3.9x_2$	(16.67,4.44)	(63.33,59.44)	59.83
	(0.2,0.8)	$2.6x_1 + 3.8x_2$	(16.67,4.44)	(63.33,59.44)	60.22
	(0.3,0.7)	$2.65x_1 + 3.7x_2$	(16.67,4.44)	(63.33,59.44)	60.61
	(0.4,0.6)	$2.7x_1 + 3.6x_2$	(16.67,4.44)	(63.33,59.44)	61
	(0.5,0.5)	$2.75x_1 + 3.5x_2$	(16.67,4.44)	(63.33,59.44)	61.39
	(0.6,0.4)	$2.8x_1 + 3.4x_2$	(16.67,4.44)	(63.33,59.44)	61.78
	(0.7,0.3)	$2.85x_1 + 3.3x_2$	(16.67,4.44)	(63.33,59.44)	62.17
	(0.8,0.2)	$2.9x_1 + 3.2x_2$	(16.67,4.44)	(63.33,59.44)	62.56
	(0.9, 0.1)	$2.95x_1 + 3.1x_2$	(16.67,4.44)	(63.33,59.44)	62.94
	(1.0,0.0)	$3x_1 + 3x_2$	(16.67,4.44)	(63.33,59.44)	63.33

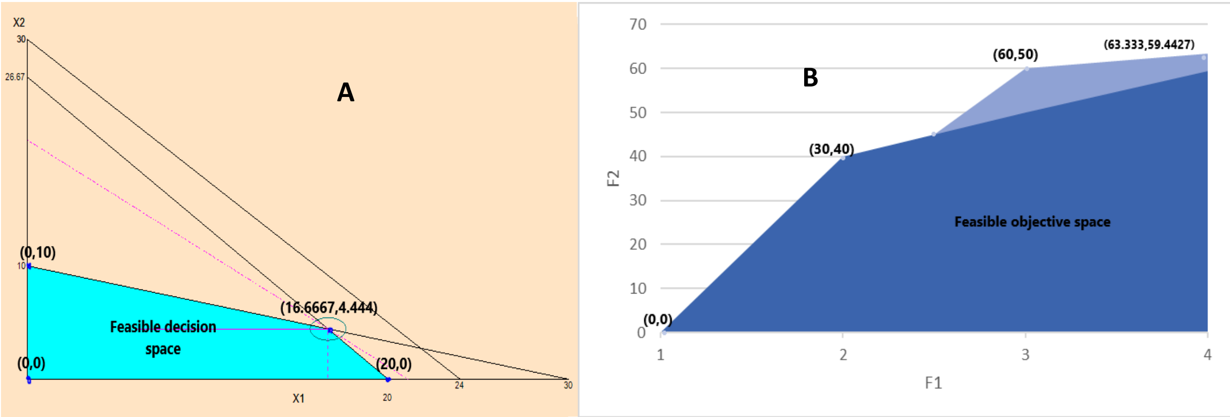

Figure 2. Feasible of WSM for 2nd bound lower rough interval (2nd problem): A) decision space, B) objective space.

The results obtained from 3th problem, is the best results were at the weight is (0,1) the objective results in the optimal solution (x_1 =11.67, x_2 =1.67) the unique optimal solution ($f_1 = 20.83, f_2 = 34.17$), Fig. 3 B. explains feasible objective space and the point ($f_1 = 20.83, f_2 = 34.17$) is dominated for all points, as shown in Table 5.

Table 5. Results obtained from solving WSM provide optimal solutions for 3th problem.

	Weight $w = (w_1, w_2)$	Composite's objective	Optimal solution (x_1, x_2)	Optimal objective value (f_1, f_2)	Optimal Composite's objective
3th problem	(0.0,1.0)	$2.5x_1 + 3x_2$	(11.67,1.67)	(20.83,34.17)	34.17
	(0.1, 0.9)	$2.4x_1 + 2.9x_2$	(11.67,1.67)	(20.83,34.17)	32.85
	(0.2,0.8)	$2.3x_1 + 2.9x_2$	(11.67,1.67)	(20.83,34.17)	31.67
	(0.3,0.7)	$2.2x_1 + 2.7x_2$	(11.67,1.67)	(20.83,34.17)	30.17
	(0.4,0.6)	$2.1x_1 + 2.6x_2$	(11.67,1.67)	(20.83,34.17)	28.83
	(0.5,0.5)	$2x_1 + 2.5x_2$	(11.67,1.67)	(20.83,34.17)	27.50
	(0.6,0.4)	$1.9x_1 + 2.4x_2$	(11.67,1.67)	(20.83,34.17)	26.17
	(0.7,0.3)	$1.8x_1 + 2.3x_2$	(11.67,1.67)	(20.83,34.17)	24.83
	(0.8,0.2)	$1.7x_1 + 2.2x_2$	(11.67,1.67)	(20.83,34.17)	23.50
	(0.9, 0.1)	$1.6x_1 + 2.1x_2$	(11.67,1.67)	(20.83,34.17)	22.18
	(1.0,0.0)	$1.5x_1 + 2x_2$	(11.67,1.67)	(20.83,34.17)	20.83

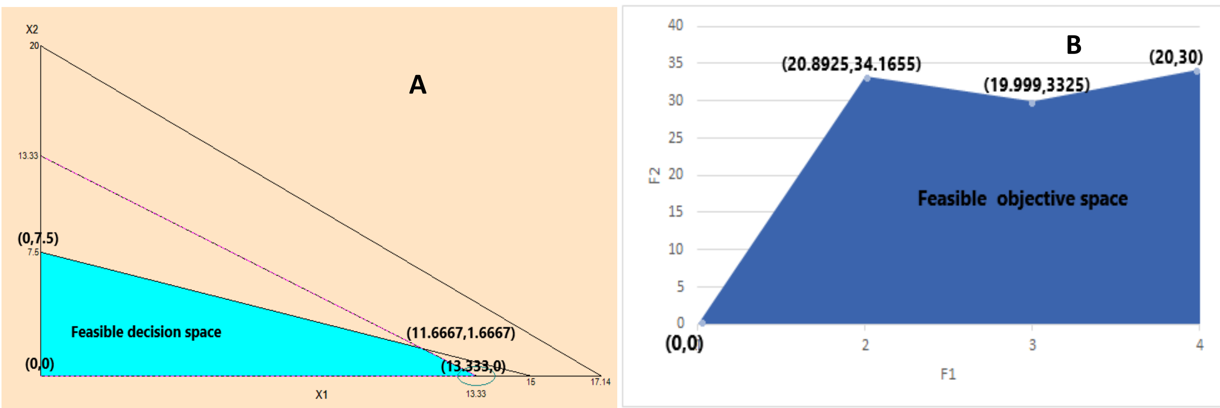

Figure 3. Feasible of WSM for 1st bound upper rough interval (3thproblem): A) decision space, B) objective space.

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Optimizing the first objective for the 4th problem with w = [0,1] the optimal solution is ($\boldsymbol{x_1}$ =11.77, $\boldsymbol{x_2}$ =3.53) with optimal objective value ($f_1 = 53.53, f_2 = 64.71$), and optimizing second objective with w = [1,0] the results of optimal solution ($\boldsymbol{x_1}$ =16., $\boldsymbol{x_2}$ =0) the unique optimal solution ($f_1 = 56, f_2 = 64$), it is clear second objective dominates the weight of the first objective, so the point ($\boldsymbol{x_1}$ =16., $\boldsymbol{x_2}$ =0) with ($f_1 = 56, f_2 = 64$) is optimal. So as shown in Fig 4 B., all results of 4th problem are explained in Table 6.

Table 6. Results obtained from solving WSM provide optimal solutions for 4th problem.

	Weight $w = (w_1, w_2)$	Composite's objective	Optimal solution (x_1, x_2)	Optimal objective value (f_1, f_2)	Optimal Composite's objective
4th problem	(0.0,1.0)	$4x_1 + 5x_2$	(11.77,3.53)	(53.53,64.71)	64.71
	(0.1, 0.9)	$3.95x_1 + 4.85x_2$	(11.77,3.53)	(53.53,64.71)	63.61
	(0.2,0.8)	$3.9x_1 + 4.7x_2$	(11.77,3.53)	(56,64)	62.47
	(0.3,0.7)	$3.85x_1 + 4.55x_2$	(16,0)	(56,64)	61.60
	(0.4,0.6)	$3.8x_1 + 4.4x_2$	(16,0)	(56,64)	60.80
	(0.5,0.5)	$3.75x_1 + 4.25x_2$	(16,0)	(56,64)	60.00
	(0.6,0.4)	$3.7x_1 + 4.1x_2$	(16,0)	(56,64)	59.20
	(0.7,0.3)	$3.65 + 3.95x_2$	(16,0)	(56,64)	58.40
	(0.8,0.2)	$3.6x_1 + 3.8x_2$	(16,0)	(56,64)	57.60
	(0.9, 0.1)	$3.55x_1 + 3.65x_2$	(16, 0)	(56, 64)	56.80
	(1.0,0.0)	$3.5x_1 + 3.5x_2$	(16,0)	(56,64)	56.00

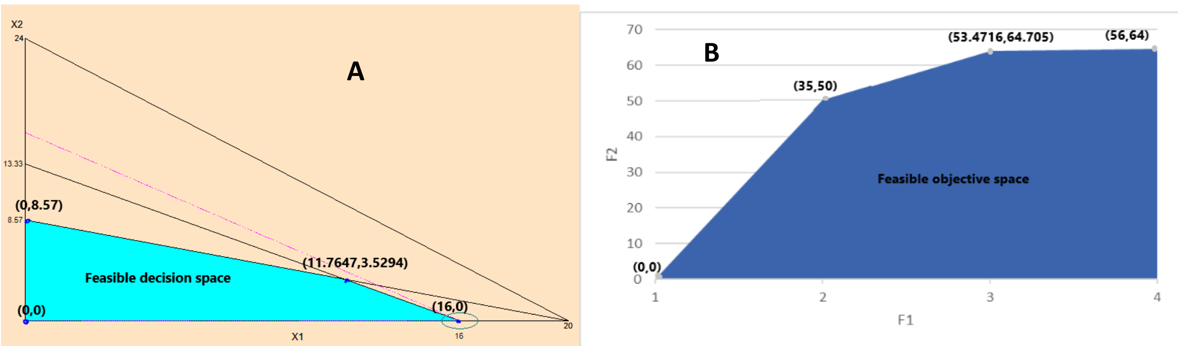

Figure 4. Feasible of WSM for 2nd bound upper rough interval (4th problem): A) decision space, B) objective space.

As a result, it's difficult to use WSM when the DM is unable to determine the weights for each problem as shown in Tables 3,4,5,6; also, this WSM method is limited because it deals with one type of objective function (Max or Min). Thus, when using this method, it is necessary to make all objective functions in one type.

Solving Example Problem 17 Using Proposed Model RIMODNP:

To apply RIMODNP for problem 17, to provide the problem with the data: Input Unit prices of resources are P_1 = ([0.5,0.65], [0.4.0.75]), P_2 = ([0.3,0.4],[0.35,0.5]) , P_3=([0.4,0.5],[0.45,0.6]), and the initial budget B=([40,55],[50,65]).

The RIMODNP problem 17 is formulated as the following:

$Max\ f_1 = ([2.5,3],[1.5,3.5])x_1 + ([2.5,3],[2,3.5])x_2$ (profit)

$Max\ f_2 = ([3,2.5],[2.5,4])x_1 + ([3.5,4],[3,5])x_2$ (quality)

$s.t.\ ([3,3.5],[2.5,5])x_1 + ([2.5,3],[2,4])x_2 \leq b1$ (raw materal1)

([2.5,3],[2,4]) x_1 + ([3,3],[1.5, 4]) $x_2 \leq b2$ (raw materal2)

([1.5,2],[1,2.5]) x_1 + ([3.5,4],[3,5]) $x_2 \leq b3$ (raw materal3),$x_1, x_2 \geq 0$ 17

([0.5,0.65],[0.4.0.75]) b_1+([0.3,0.4],[0.35,0.5]) b_2+ ([0.4,0.5],[0.45,0.6]) b_3 ≤([40,55],[50,65])

$x_1, x_2 \geq 0$

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x_1 : represent product 1, x_2 : represent product 2.

Step 3: To solve problem 17, the problem can be converted into two sub-problems (interval multi-objective de novo programming (IMODNP) as shown in Table 7:

Table 7. Convert RIMODNPP into two sub-problems Lower and Upper interval.

IMODNP/ Lower	IMODNP/ Upper
$\boldsymbol{Max \underline{f}_1^L = [2.5,3]x_1 + [2.5,3]x_2}$ $\boldsymbol{Max \underline{f}_2^L = [3,2.5]x_1 + [3.5,4]x_2}$ $\boldsymbol{s.t.}$ $\boldsymbol{[3,3.5]x_1 + [2.5,3]x_2 \le b_1}$ $[2.5,3]\ \boldsymbol{x_1} + [3,3]\boldsymbol{x_2} \le \boldsymbol{b_2}$ $[1.5,2]\boldsymbol{x_1} + [3.5,4]\ \boldsymbol{x_2} \le \boldsymbol{b_3}$ $[0.5,0.65]\ \boldsymbol{b_1} + [0.3,0.4]\ \boldsymbol{b_2} + [0.4,0.5]\ \boldsymbol{b_3} \le [40,55]$ $\boldsymbol{x_1, x_2 \ge 0}$ 18	$Max\ \bar{f}_1^U = [1.5,3.5]x_1 + [2,3.5]x_2$ $Max\ \bar{f}_2^U = [2.5,4]x_1 + [3,5]x_2$ $s.t.$ $[2.5,5]x_1 + [2,4]x_2 \le b_1$ $[2,4]x_1 + [1.5, 4]x_2 \le b_2$ $[1,2.5]\ x_1 + [3,5]\ x_2 \le b_3$ $[0.4.0.75]\ b_1 + [0.35,0.5]\ b_2 + [0.45,0.6]\ b_3 \le [50,65]$ $x_1, x_2 \ge 0$ 19

Step 4: The IMODNP problem 18 is converted to MODNP problems 20 and 21. Also, the IMODNP problem 19 is converted to MODNP problems 22 and 23, as the below Table 8, Table 9, respectively.

Table 8. Convert IMODNP into two sub-problems IMODNP for the lower interval.

1st bound of lower rough interval	2nd bound of Lower rough interval
$\boldsymbol{Max\ \underline{f}_1^L = 2.5x_1 + 2.5x_2}$ $\boldsymbol{Max\ \underline{f}_2^L = 3x_1 + 3.5x_2}$ $\boldsymbol{s.t.}$ $\boldsymbol{5x_1 + 4x_2 \le b_1}$ $\boldsymbol{4x_1} + 4\boldsymbol{x_2} \le \boldsymbol{b_2}$ $2.5\ \boldsymbol{x_1} + 5\ \boldsymbol{x_2} \le \boldsymbol{b_3}$ $0.75\ \boldsymbol{b_1} + 0.5\ \boldsymbol{b_2} + 0.6\ \boldsymbol{b_3} \le 40$ $, \boldsymbol{x_1, x_2 \ge 0}$ 20	$Max\ \underline{f}_1^U = 3x_1 + 3x_2$ $Max\ \underline{f}_2^U = 2.5x_1 + 4x_2$ $s.t.$ $2.5x_1 + 2x_2 \le b_1$ $2x_1 + 1.5x_2 \le b_2$ $x_1 + 3\ x_2 \le b_3$ $0.4\ b_1 + 0.35\ b_2 + 0.45\ b_3 \le 55$ $, x_1, x_2 \ge 0$ 21

Table 9. Convert IMODNP into two sub-problems LMODNP for the upper interval.

1st bound of Upper rough interval	2nd bound of Upper rough interval
$\boldsymbol{Max\ \bar{f}_1^L = 1.5x_1 + 2x_2}$ $\boldsymbol{Max\ \bar{f}_2^L = 2.5x_1 + 3x_2}$ $\boldsymbol{s.t.}$ $\boldsymbol{3.5x_1 + 3x_2 \le b_1}$ $\boldsymbol{3x_1} + 3\boldsymbol{x_2} \le \boldsymbol{b_2}$ $2\ \boldsymbol{x_1} + 4\ \boldsymbol{x_2} \le \boldsymbol{b_3}$ $0.65\ \boldsymbol{b_1} + 0.4\ \boldsymbol{b_2} + 0.5\ \boldsymbol{b_3} \le 50$ $, \boldsymbol{x_1, x_2 \ge 0}$ 22	$Max\ \bar{f}_1^U = 3.5x_1 + 3.5x_2$ $Max\ \bar{f}_2^U = 4x_1 + 5x_2$ $s.t.$ $3x_1 + 2.5x_2 \le b_1$ $2.5x_1 + 3x_2 \le b_2$ $1.5\ x_1 + 3.5\ x_2 \le b_3$ $0.5\ b_1 + 0.3\ b_2 + 0.4\ b_3 \le 65$ $, x_1, x_2 \ge 0$ 23

Results obtained by Zeleny's approach (Structure Design) for problem (RIMODNP):

Step 5: Calculating optimal system design for 1st problem 20 represent (1st bound of lower rough interval) using problem 2, solve problem 20:

$$Max\ \underline{f}_1^L = 2.5x_1 + 2.5x_2$$
$$Max\ \underline{f}_2^L = 3x_1 + 3.5x_2$$
$$s.t.$$
$$5x_1 + 4x_2 \le b_1$$
$$4x_1 + 4x_2 \le b_2$$
$$2.5\ x_1 + 5\ x_2 \le b_3$$
$$0.75\ b_1 + 0.5\ b_2 + 0.6\ b_3 \le 40$$
$$x_1, x_2 \ge 0$$

Take a budget constraint and replace b_1 , b_2 , b_3 with constraints of problem as the following:

$0.75(5x_1 + 4x_2\ ) + 0.5(4x_1 +\ 4x_2) + 0.6(2.5\ x_1 +\ 5\ x_2) \le 40$ easily the budget constraint was gotten as follows:

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$7.5\, x_1 + 8\, x_2 \leq 40$ 24

To find $\underline{f}_1^L$ with respect to given budget equal 40, let $\underline{x}_1^{2L}$ =0, where $\underline{x}_2^{1L}$ = 5 and the value of $\underline{f}_1^L = 12.5$, to obtain of the value of resources, the values are substituted of ($\underline{x}_1^{2L}$ =0, $\underline{x}_2^{1L}$ = 5) in constraints of problem 20:

$\underline{b}_1^L = 5 * 0 + 4 * 5 = 20$

$\underline{b}_2^L = 4 * 0 + 4 * 5 = 20$

$\underline{b}_3^L = 2.5 * 0 + 5 * 5 = 25$. so, the value of $\underline{B}_2^{1L}$ using budget constraint 24 as follows: $\underline{B}_2^{1L}$ = 7.25*0 + 8*5 = 40,

as well as for $\underline{x}_2^{1L}$ = 5.517 when $\underline{x}_1^{2L}$ =0, so $\underline{f}_2^L$ = 16.552, and then substitute the values in constraints of problem 20:

$\underline{b}_1^L = 5 * 5.517 + 4 * 0 = 27.585$

$\underline{b}_2^L = 4 * 5.517 + 4 * 0 = 22.069$

$\underline{b}_3^L = 2.5 * 5.517 + 5 * 0 = 13.793$.

So, the value of B_1^2 obtained from budget constraint 24 as follows:

$\underline{B}_1^{2L} = 7.25 * 5.517 + 8 * 0 = 39.998$). The rest of results is as shown in the Table 10.

Table 10. Results obtain Zeleny's approach (structure design) for problems 20,21,22,23.

	Optimal System Design			
1st Bound of Lower Rough Interval Problem 20	$\underline{x}_2^{1L}$	5	$\underline{x}_1^{2L}$	5.517
	$\underline{b}_1$	20	$\underline{b}_1^L$	27.585
	$\underline{b}_2$	20	$\underline{b}_2^L$	22.069
	$\underline{b}_3$	25	$\underline{b}_3^L$	13.793
	$\underline{f}_2^{1L}$	12.5	$\underline{f}_1^2$	16.552
	$\underline{B}_2^{1L}$	40	$\underline{B}_1^2$	39.998
	$\underline{B}^L$ 40			
	Optimal System Design			
2nd Bound of Lower Rough Interval Problem. 21	$\underline{x}_2^{1U}$	20.561	$\underline{x}_1^{2U}$	25.582
	$\underline{b}_1$	41.122	$\underline{b}_1^U$	63.954
	$\underline{b}_2$	30.841	$\underline{b}_2^U$	51.163
	$\underline{b}_3$	61.682	$\underline{b}_3^U$	25.582
	$\underline{f}_2^{1U}$	82.243	$\underline{f}_1^{2U}$	76.744
	$\underline{B}_2^{1U}$	55.00014	$\underline{B}_1^{2U}$	55.000014
	$\underline{B}^U$ 55			
	Optimal System Design			
1st Bound of Upper Rough Interval Problem 22	$\overline{x}_2^{1L}$	9.7087	$\overline{x}_1^{2L}$	11.173
	$\overline{b}_1^L$	29.126	$\overline{b}_1^L$	39.106
	$\overline{b}_2^L$	29.126	$\overline{b}_2^L$	33.520
	$\overline{b}_3^L$	38.835	$\overline{b}_3^L$	22.346
	$\overline{f}_2^{1L}$	19.142	$\overline{f}_1^{2L}$	27.933
	$\overline{B}_2^{1L}$	49.9998	$\overline{B}_1^{2L}$	50.0001
	$\overline{B}^L$ 50			
	Optimal System Design			
2nd Bound of Upper Rough Interval Problem 23	$\overline{x}_2^{1U}$	18.3099	$\overline{x}_1^{2U}$	22.807
	$\overline{b}_1^U$	45.775	$\overline{b}_1^U$	68.421
	$\overline{b}_2^U$	54.929	$\overline{b}_2^U$	57.018
	$\overline{b}_3^U$	64.085	$\overline{b}_3^U$	34.211
	$\overline{f}_2^{1U}$	64.085	$\overline{f}_1^{2U}$	91.228
	$\overline{B}_2^{1U}$	65	$\overline{B}_1^{2U}$	64.99995
	$\overline{B}^U$ 65			

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From Table 10, it is clear from the results obtained that it gives one optimal system design for each bound. In spite of get an optimal result, little alternative was found to provide to the DM. these results are in agreement with the results obtained by Zeleny approach[16].

Step 6: After getting the results from solving problem 20, find the 1st bound of lower rough interval ($\underline{f_1^L} = 12.5, \underline{f_2^L} = 16.552$) with budget 40, Meta-optimum solution can be easily found depending on problem 5 to solve problem 20 as follows:

$Min\ \underline{B}^L = 7.5\ \underline{x}_1 + 8\ \underline{x}_2$

s.t $2.5\ \underline{x}_1 + 2.5\ \underline{x}_2 \geq 12.5$

$3\ \underline{x}_1 + 3.5\ \underline{x}_2 \geq 16.5516, \qquad \underline{x}_1, \underline{x}_2 \geq 0$

25

using computer software POM- QM Windows V5 to solve the problem 10b, the results are as follows: $\underline{x}^{*L} = (6.6206, 0)$, $\underline{f}^{*L} = (16.552,\ 12.5)$, $\underline{b}^{*L} = (33.103, 26.482, 16.552)$, $\underline{B}^{*L} = 47.99935$, where, the value limits the minimum budget to realize $\underline{f}^{*L}$ through solutions $\underline{x}^{*L}$ and $\underline{b}^{*L}$. The given budget level $\underline{B}^L = 40 \leq \underline{B}^{*L} = 47.99935$.

The optimum-path ratio for implementing the best achievement for given budget $\underline{B}^L$ is defined as in Table 1, the optimum-path ratio $\underline{r}^L = \frac{\alpha_{i}B_i}{\underline{B}^L}$ with $0 \leq \alpha_i \leq 1$, $\sum \alpha i = 1$, where represented $\alpha_1 = \alpha_2 = 0.5$ respectively. the $\underline{r}^L$ = $(\alpha_1 B_2^1 + \alpha_2 B_1^2 = 39.99985)/47.99935))$ = 59.99%. so that can be found the values of Table 1.

Step 7: To find Synthetic optimum solution ($\underline{x}^{**L}$ is solved for jth single criterion DNP and obtain solution $\{X_i^{\mathbf{1}}, X_i^{\mathbf{2}}, X_i^{\mathbf{3}}, \ldots.\}$ form optimum solution system design, get the values as follows:

$\underline{x}^{**L} = (5.517, 5)$, take the values and substitute in constraints of problem 20 get the values of resources according to the formula $\underline{b}^{**L} = A\underline{x}^{**L} = (47.586, 42.069, 38.793)$, so that substitute in objectives of problem 14 obtain to $\underline{f}^{**L} = C\underline{x}^{**L} = (26.293, 34.052)$, and then, take the budget constraint 24 and substitute the values $\underline{x}^{**L}$ by the value of budget is $\underline{B}^{**L} = V\underline{x}^{**L} = 79.9997$, applying to the rest of problems 21,22,23.

Step 8: According to the Theorem's Shi, that means $\underline{x}_i^{kL}$ is feasible solutions for problem 14, where the results in the above Table 7. refer to $\underline{B}^L \geq \underline{B}_i^{kL} = 40, V\underline{x}_i^{kL} \leq \underline{B}^L$, this implies $\underline{B}_i^{kL} \leq \underline{B}^L = 40$ for problem 20, so $\underline{B}^{*L} = 47.99935 \geq \underline{B}^L = 40$, that refer to the Meta-optimum solution $\underline{x}^{*L}$ is feasible for problem 20. Finally, $\underline{B}^{**L} = 79.9997 \geq \underline{B}^{*L} = 47.99935$ that's mean both solutions $\underline{x}^{*L} = (6.621, 0)$and $\underline{x}^{**L}$ =(5.517, 5) are feasible for problem 11 and so on for the rest of the problems.

Step 9: Test bounds of rough interval for objectives *f* and resources *b* for problem 20,21,22,23, the results as shown in Table 9.

$\underline{f_1^L} = 12.5$, $\qquad \underline{f_2^L} = 16.552$

$\underline{f_1^U} = 82.243$, $\qquad \underline{f_2^U} = 76.744$

$\overline{f}_1^L = 19.142$, $\qquad \overline{f}_2^L = 27.933$

$\overline{f}_1^{1U} = 64.085$, $\qquad \overline{f}_2^U = 91.228$

$(\underline{f_1^L}, \underline{f_1^U}) = (12.5,\ 82.243)$, it is surely optimal range.

$(\overline{f}_1^L, \overline{f}_1^{1U}) = (\ 19.142,\ 64.085)$, it is possibly optimal range.

$[(\underline{f_1^L}, \underline{f_1^U}), (\overline{f}_1^L, \overline{f}_1^{1U})] = [(\ (12.5,\ 82.243), (\ 19.142,\ 64.085)]$ is the rough optimal range.

$(\underline{f_2^L}, \underline{f_2^U}) = (16.155,\ 76.744)$ it is surely optimal range.

$(\overline{f}_2^L, \overline{f}_2^U) = (27.933,\ 91.228)$ it is possibly optimal range.

$[\underline{f_2^L}, \underline{f_2^U}), (\overline{f}_2^L, \overline{f}_2^U)] = [\ (16.155,\ 76.744), (27.933,\ 91.228)]$ is the rough optimal range.

($\underline{b_1^L}$ =20, $\underline{b_1^U}$ =41.1216), it is surely optimal range.

($\overline{b}_1^L$ =29.126, $\overline{b}_1^U$ =45.775,) , it is possibly optimal range.

[($\underline{b_1^L}$ =20, $\underline{b_1^U}$ =41.122), ($\overline{b}_1^L$ =29.1261, $\overline{b}_1^U$ =45.775)], is the rough optimal range.

($\underline{b_2^L}$ =20, $\underline{b_2^U}$ =30.8412), it is surely optimal range.

($\overline{b}_2^L$ =29.126, $\overline{b}_2^U$ =54.929), it is possibly optimal range.

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[($\underline{b}_2^L$ =20, $\underline{b}_2^U$ =30.841), ($\overline{b}_2^L$ =29.126, $\overline{b}_2^U$ =54.929)], is the rough optimal range.

($\underline{b}_3^L$ =25, $\underline{b}_3^U$ =61.682), it is surely optimal range.

($\overline{b}_3^L$ =38.835, $\overline{b}_3^U$ =64.085), it is possibly optimal range.

[($\underline{b}_3^L$ =25, $\underline{b}_3^U$ =61.682), ($\overline{b}_3^L$ =38.835, $\overline{b}_3^U$ =64.085)], is the rough optimal range.

It is clear that the results of problems are an optimal solution because of the achieved properties of rough interval of proposed model, that means the proposed model to RIMODNP is able to solve problems under uncertainty conditions.

Step 10: The six types optimal path-ratios are calculated to find the optimum system design using the following formulas of Table 1. Table 11 explains the results of three ratios for each problem as shown below:

Table 11. Results of Optimum-path ratios for problems 20,21,22,23.

Bounds of rough interval for objectives problem	Ratios		
1st Bound of Lower Rough Interval-Problem 20	$\underline{r}^{1L} = \frac{\underline{B}^{*L}}{\underline{B}^{**L}} = 59.99\%$	$\underline{r}^{2L} = \frac{\underline{B}^{L}}{\underline{B}^{**L}} = 50.00019\%$	$\underline{r}^{3L} = \frac{\sum_i \alpha_i \underline{B}_i^{jL}}{\underline{B}^{**L}}$ =50%
2nd Bound of Lower Rough Interval-Problem 21	$\overline{r}^{1L} = \frac{\overline{B}^{*L}}{\overline{B}^{**L}} = 57.54\%$	$\overline{r}^{2L} = \frac{\overline{B}^{L}}{\overline{B}^{**L}} = 50.001\%$	$\overline{r}^{3L} = \frac{\sum_i \alpha_i \overline{B}_i^{jL}}{\overline{B}^{**L}} = 50.0006\%$
1st Bound of Upper Rough Interval-Problem 22	$\underline{r}^{1U} = \frac{\underline{B}^{*U}}{\underline{B}^{**U}} = 61.48\%$	$\underline{r}^{2U} \frac{\underline{B}^{U}}{\underline{B}^{**U}} = 49.99\%$	$\underline{r}^{3U} = \frac{\sum_i \alpha_i \underline{B}_i^{jU}}{\underline{B}^{**U}} = 50\%$
2nd Bound of Upper Rough Interval-Problem 23	$\overline{r}^{1U} = \frac{\overline{B}^{*U}}{\overline{B}^{**U}} = 71.14\%$	$\overline{r}^{2U} = \frac{\overline{B}^{U}}{\overline{B}^{**U}} = 50\%$	$\overline{r}^{3U} = \frac{\sum_i \alpha_i \overline{B}_i^{jU}}{\overline{B}^{**U}} = 49.99\%$

The optimum system design is calculated for problems 20,21,22,23 using equations 8,9,10, the results summarized in Table 12.

Table 12. The results of optimum system design for problems 20,21,22,23 for ratio 1.

designs	Optimal values & Optimal solution of problem 20	Optimal values & Optimal solution of problem 21	Optimal values & Optimal solution of problem 22	Optimal values & Optimal solution of problem 23
Design1.for Ratio 1	$\underline{x}^{1L} = (3.309, 2.995)$ $\underline{b}^{1L}$ = (28.547, 25.237, 23.272) $\underline{f}^{**1L}$ = (15.773, 20.428) $\underline{r}^{1L}$ = 59.99% with using $\underline{B}^{*L}$ =47.99935, out of $\underline{B}^{**L}$=79.9997	$\underline{x}^{1U} = (15.727, 12.641)$ $\underline{b}^{1U}$ = (64.600, 50.416, 53.650) $\underline{f}^{1U}$ = (85.105, 89.882) $\underline{r}^{1U}$= 61.48%with using $\underline{B}^{*U}$=67.628, out of $\underline{B}^{**U}$=110.00015	$\overline{x}^{1L} = (6.429, 5.586)$ $\overline{b}^{1L}$ = (39.261, 36.046, 35.204) $\overline{f}^{1L}$ = (20.8164, 32.8318) $\overline{r}^{1L}$= 57.54% with using $\overline{B}^{*L}$=57.542, out of $\overline{B}^{**L}$=99.99875	$\overline{x}^{1U} = (16.225, 13.026)$ $\overline{b}^{*U}$ = (81.239, 79.639, 69.927) $\overline{f}_2^{1U}$ = (102.377, 130.028) $\overline{r}^{1U}$= 71.14% with using $\overline{B}^{*U}$=92.483, out of $\overline{B}^{**U}$=130

Design under ratio1 1st Bound of Lower Rough Interval-Problem 20, After obtaining the values of ratios as shown in the Table 12, the first ratio is taken $\underline{r}^{1L} = \frac{\underline{B}^{*L}}{\underline{B}^{**L}}$ = 59.99% represent the optimal path ratio for achieving the synthetic optimal performance $\underline{f}^{**1L}$ = (26.293, 34.052) related to a given meta-optimal budget level $\underline{B}^{*L}$= 47.99935, optimal system design under Ratio1 is as follows: optimal solutions for (product1 and product2) are $(3.310, 2.999)$ respectively, $\underline{b}^{1L}$ resources of raw materials are (28.547, 25.237, 23.272) respectively, also the optimal values of maximization (profit and quality) are (15.773, 20.428), all results of 1st Bound of

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Lower Rough Interval-Problem 20 under budget $\underline{B}^{L} = 40$.

And so on for problem 21 under same ratio the design is as follows:

For the second bound of lower rough interval the first ratio is taken $\underline{r}^{1U}$= 61.48%, optimal system design under Ratio1 equal 57.54% is as follows: optimal solutions for (product1 and product2) are $(15.727, 12.641)$ respectively, $\underline{b}^{1U}$ resources of raw materials are (64.600, 50.416, 53.649) respectively, also the optimal values of maximization (profit and quality) are (85.105, 89.882), all results of 2nd Bound of Lower Rough Interval-Problem 20 under budget $\underline{B}^{U} = 55$.

So is the problem 22 Optimal system design under Ratio1 equal $\overline{r}^{1L}$= 57.54% is as follows: optimal solutions for (product1 and product2) are $(6.429, 5.586)$ respectively, $\overline{b}^{1L}$ resources of raw materials are (39.2609, 36.046, 35.204) respectively, also the optimal values of maximization (profit and quality) are (20.816, 32.832), all results of 1st Bound of Upper Rough Interval-Problem 22 under budget $\overline{B}^{L} = 50$.

Design for Ratio1 2nd Bound of Upper Rough Interval-Problem 23, Optimal system design under Ratio1 equal $\overline{r}^{1U}$= 71.14% is as follows: optimal solutions for (product1 and product2) are $(16.225, 13.026)$ respectively, $\overline{b}^{1U}$ resources of raw materials are (81.239, 79.639, 69.927) respectively, also the optimal values of maximization (profit and quality) are (102.377, 130.028), all results of 2nd Bound of Upper Rough Interval-Problem 23 under budget

$\overline{B}^{U} = 65$.

The results of ratios (2,3) can be summarized as in the Table 13.

Table 13. The results of optimum system design for problems 20,21,22,23 for ratios (2,3).

Desi	Optimal values & Optimal solution of problem 20	Optimal values & Optimal solution of problem 21	Optimal values & Optimal solution of problem 22	Optimal values & Optimal solution of problem 23
Design2. For Ratio 2	$x^{2L} = (2.759, 2.5)$ $\underline{b}^{2L}$ = (23.793, 21.034, 19.397) f^{**2L} = (13.147, 17.026) $\underline{r}^{2L}$ = 50.00019%with using $\underline{B}^{L}$ =40, out of $\underline{B}^{**L}$=79.9997	$\underline{x}^{2U} = (12.788, 10.278)$ $\underline{b}^{2U}$ = (52.527, 40.994, 43.623) $\underline{f}^{2U}$ = (69.199, 73.084) $\underline{r}^{2U}$= 49.99%with using $\underline{B}^{U}$=55, out of $\underline{B}^{**U}$=110.00015	$\overline{x}^{2L} = (5.587, 4.855)$ $\overline{b}^{2L}$ = (34.117, 31.324, 30.591) $\overline{f}^{2L}$ = (18.0889, 28.5301) $\overline{r}^{2L}$= 50.001% with using $\overline{B}^{L}$=50, out of $\overline{B}^{**L}$=99.99875	$\overline{x}^{2U} = (11.4035, 9.155)$ $\overline{b}^{2U}$ = (57.098, 55.974, 49.148) $\overline{f}^{2U}$ = (71.955, 91.389) $\overline{r}^{2U}$ = 50% with using $\overline{B}^{U}$=65, out of $\overline{B}^{**U}$=130
Design3. For Ratio 3	$\underline{x}^{3L} = (2.759, 2.5)$ $\underline{b}^{3L}$ = (23.793, 21.034, 19.397) $\underline{f}^{**3L}$ = (13.147, 17.026) $\underline{r}^{3L}$ = 50% with using $\alpha_1\underline{B}_1^{1L}+\alpha_2\underline{B}_2^{2L}$=39.99985 , out of $\underline{B}^{**L}$=79.9997	$\underline{x}^{3U} = (12.791, 10.281)$ $\underline{b}^{3U}$ = (52.5375, 41.002, 43.6319) $\underline{f}^{3U}$ = (69.213, 73.098) $\underline{r}^{3U}$= 50%with using $\alpha_1\underline{B}_1^{1U}+\alpha_2\underline{B}_2^{2U}$=55.000075 , out of $\underline{B}^{**U}$=110.00015	$\overline{x}^{3L} = (5.587, 4.854)$ $\overline{b}^{3L}$ = (34.117, 31.323, 30.591) $\overline{f}^{3L}$ = (18.089, 28.529) $\overline{r}^{3L}$= 50.0006% with using $\alpha_1\overline{B}_1^{1L}+\alpha_2\overline{B}_2^{2L}$=49.99993 5 out of $\overline{B}^{**L}$=99.99875	$\overline{x}^{3U} = (11.403, 9.154)$ $\overline{b}^{3U}$ = (57.0968, 55.9725, 49.147) $\overline{f}^{3U}$ = (71.953, 91.387) $\overline{r}^{3U}$= 49.99% with using $\alpha_1\overline{B}_1^{1U}+\alpha_2\overline{B}_2^{2U}$=64.99997 5 out of B^{**U}=130

From Table 13 , it is found that all solutions under each ratio gives an optimal system design, meaning that this method is the most efficient method when compared with other methods, because it gives more flexibility for DM by choosing suitable alternative, it gives twelve alternatives while the other methods give four alternatives for each one. These results are confirmed by[23].

After obtaining the results from each method can be described separately, as shown in Table 14.

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Table 14. Described of methods for solving proposed model.

Method	Right hand of constraints $\boldsymbol{b_i}$	Model usability	Results	The number of optimal system designs
WSM	b_i: resources right hand side of constraints are Known and fixed.	Before formulation proposed model RIMOP.	Select one solution from the set of feasible solutions.	Four optimal solutions, but not re-designable system.
Zeleny's approach	b_i: resources hand side of constraints are unknown and variable.	after formulation proposed model RIMODNP.	All solutions are optimal.	Four optimal solutions, but re-designable system.
Optimal path-ratios	b_i: resources hand side of constraints are unknown and variable.	after formulation proposed model RIMODNP.	All solutions are optimal.	Twelve optimal designs for the three ratios used, and the system can be redesigned.

From the results, we concluded that WSM is useless to use, especially when the conditions are uncertainty, because it is not possible to control re-allocation resources or improve the current system. And also, it's noted that the optimal path-ratios method is more efficient, than other methods, because it gives more alternatives to the DM, and it is also possible to re-improve system design without returning to create new system design.

Conclusion

In this paper, an ideal resource allocation system under conditions of uncertainty is designed using the proposed model to reconfigure the possible combination to obtain the optimal combination of resources that produce a system with no or minimal waste. The optimal system design of the proposed model is obtained by solving in three ways: WSM, Zeleny approach and optimal path -ratios method.

The first method (WSM) gives one result among a set of feasible solutions (ideal system design) under each level of problem can't improve RIMOP, the second method (Zeleny's approach) gives one optimal system design under each level which can be improved by redesigning, while the third method (optimal path-ratios) gives three optimum system designs for each sub-model (levels), from this method twelve designs are obtained for RIMODNP problem. According to the results obtained from our propose model, we found that the third, method (optimal path-ratios) is more efficient compared with the other methods, it gives more alternative solutions (12 solutions) which represents the optimal system design, thus, the results of the proposed model conform to the conditions and theorems, in spite of the high efficiency of our model it faces difficulties when applied on large scales because it needs special computer program designs.

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Author's Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Cairo University, Giza, Egypt.

Authors' Contributions Statement

I. A.H. did the conception, and design of paper,the acquisition, and analysis of data H.Z., N. R. and H. S. did analysis, interpretation, revision and proofreading.

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تصميم نظام أمثل باستخدام الفاصل التقريبي لبرمجة دي نوفو متعددة الأهداف

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[2] قسم بحوث العمليات، كلية الدراسات العليا للبحوث الإحصائية، جامعة القاهرة، الجيزة، مصر.

الخلاصة

تُعد برمجة دي نوفو متعددة الأهداف، أداة فعالة تتعامل مع تصميم نظام أمثل، وذلك من خلال تحديد المستوى الأمثل لتخصيص الموارد، وتحسين قيمة دوال الهدف وفقاً لسعر الموارد (هذا فيما لو كانت الظروف طبيعية ولا يوجد تقلب في الاسعار). في هذه البحث أُقتِرحَ نهجاً جديداً لحل مشكلة عدم اليقين لمشكلة برمجة دي نوفو، وذلك باستخدام أنموذج مركب يتضمن نوعين من البرمجة: الأول هو برمجة متعددة الأهداف ذات الفاصل التقريبي (RIMOP)، والثاني: برمجة دي نوفو(DNP)، علماً بأن معاملات متغيرات القرار لدالة الهدف والقيود عبارة عن فاصل تقريبي RIC. وقد استخدامنا ثلاث طرائق لحل الانموذج المقترح وإيجاد تصميم نظام أمثل، الطريقة الاولى هي طريقة المجموع الموزون WSM والتي تُستخدم قبل اعادة صياغة النموذج المقترح RIMOP (وكانت الموارد معروفة)، وقد وجد ان نتائج طريقة WSM تعطي حلاً واحداً (حل وسط قريب من المثالية) من بين الحلول المقبولة تحت كل حد من حدود المشكلة، وبالتالي يمكن تقديم أربع حلول (بدائل) لصانع القرار(DM) . اما منهجية زيلني وطريقة نسب المسار الأمثل فقد استخدمت بعد صياغة الانموذج المقترح (RIMODNP) (وكانت الموارد للطرف الأيمن للقيود ليست معروفة). وجد ان منهجية زيلني تعطي تصميم نظام أمثل واحداً لكل حد من حدود المشكلة، في حين ان طريقة المسار الأمثل طبقت بعد ان تم التأكد من حدود النموذج المقترح وفقاً النظرية شاي فقد استخدمنا ثلاثة أنواع من النسب تحت كل مشكلة فرعية، فقد وجد ان هذِهِ الطريقة تعطي ثلاث تصاميم انظمة مثلى تحت كل حد للمشكلة، وقد اتضح من خلال النتائج ان طريقة نسب المسار الامثل هي أكثر كفاءة من غيرها بعد تطبيقها على النموذج المقترح، لأنها اعطت اثنا عشر بديلاً لـ(DM)، لوحظ ان الانموذج المقترح يتوافق مع شروط ونظريات RIC. يُعد الانموذج المقترح مناسباً جداً لظروف عدم اليقين، اخيراً، طبق مثال رقمي على الانموذج المقترح.

الكلمات المفتاحية: برمجة دي نوفو، البرمجة الخطية متعددة الاهداف، نسب المسار الأمثل، تصميم النظام الأمثل، برمجة الفاصل التقريبية الخطية.