2024, 21(7): 2413-2424
https://doi.org/10.21123/bsj.2023.9137
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Semi-Analytical Solutions for Time-Fractional Fisher Equations via New Iterative Method

Shivaji Ashok Tarate [*1] ***A. P. Bhadane*** [2] ***S.B. Gaikwad*** [1] ***K.A. Kshirsagar*** [1]
[1] Department of Mathematics, New Arts, Commerce and Science College, Ahmednagar, Maharashtra, India.
[2] Department of Mathematics, Loknete Vyankatrao Hiray Arts, Science and Commerce College, Nashik, Maharashtra, India.
*Corresponding Author.

Received 27/05/2023, Revised 15/10/2023, Accepted 17/10/2023, Published Online First 25/12/2023, Published 1/7/2024

Abstract

An effective method for resolving non-linear partial differential equations with fractional derivatives is the New Sumudu Transform Iterative Method (NSTIM). It excels at solving difficult mathematical puzzles and offers insightful information about the behaviour of time-fractional Fisher equations. The method, which makes use of Caputo's sense derivatives and Wolfram in Mathematica, is reliable, simple to use, and gives a visual depiction of the solution. The analytical findings demonstrate that the proposed approach is effective and simple in generating precise solutions for the time-fractional Fisher equations. The results are made more reliable and applicable by including Caputo's sense derivatives. Mathematical modelling relies on the effectiveness and simplicity of the NSTIM approach to solve time-fractional Fisher equations since it enables precise solutions without the use of a lot of processing power. The NSTIM approach is a useful tool for researchers in a variety of domains since it also offers a flexible framework that is easily adaptable to other fractional differential equations. It now becomes possible to examine the dynamics and behaviour of complex systems governed by time-fractional Fisher equations with efficiency and reliability, opening up new research avenues. The ability to solve time-fractional Fisher equations efficiently and reliably using the NSTIM approach has significant implications for various fields such as population dynamics, mathematical biology, and epidemiology. Researchers can now analyze the spread of diseases or study the population dynamics of species with higher accuracy and less computational effort. This advancement in solving fractional differential equations paves the way for deeper insights into the behavior and patterns of complex systems, ultimately advancing scientific understanding and offering new possibilities for practical applications.

Keywords: Caputo fractional derivative, Fisher equations, Fractional Calculus, Iterative method, Sumudu Transform.

Introduction

In recent decades, fractional differential equations have fascinated mathematicians, physicists and engineering researchers[1–3]. A fractional theory, which includes fractional derivatives and fractional integration, can be used to model a wide range of problems[4–7]. Various methods have been developed to solve both linear and nonlinear fractional differential equations[8–10], including the Cauchy reaction-diffusion method, the Adomian decomposition method(A.D.M.)[11,12], the homotopy method (H.A.M.)[13], the variational method of iteration (V.I.M.)[14,15], and the perturbation method of homotopy (H.P.M.)[16]. These methods have been applied to the Cauchy-diffusion of time-fractional equations, which are used to model nonlinear and linear systems in fields such as engineering, biology, ecology, chemistry, and physics[17–19].

The study of investigates solutions for nonlinear generalized proportional, fractional functional integro-differential Langevin equations using fixed point theorems and Ulam-Hyers stability. It creates a mathematical model to analyse Wolbachia dispersal among Aedes aegypti mosquitoes, analysing symmetrical characteristics[20–22]. The model is physically meaningful and assessed for equilibrium points in the presence and absence of disease. Eight equilibrium points are determined, and the basic reproduction number is calculated using the next-generation matrix method. Numerical simulations are conducted to evaluate the basic reproduction number and identify the optimal CI value for reducing disease spread. The study also examines the interaction between prey and predator populations, focusing on the additive Allee effect and intraspecific competition. The study highlights the importance of considering precautionary measures in controlling disease spread, with the rate of precautionary measures playing a crucial role in reducing the chance of infection by the Chickenpox virus[23–25].

The authors of an article obtained both numerical and analytical solutions to the time-fractional Fisher equations using the New Sumudu transform iterative method (NSTIM). The benefit of this new method is that it makes the calculations easy and gives the most accurate estimate of the exact answer[26–28]. There are many problems in fractional derivatives[29,30], hydrodynamics[31], chemical diffusion[32], and option pricing[33]. Partial differential equations[34] can be used to model computational fluid dynamics[35–38] and control theory[39–42]. Nonlinear P.D.E.s and processes for finding numerical solutions to nonlinear problems have gotten much attention recently (P.D.E.s)[43–45]. The main theme of this research is focused on the solution and analysis of a nonlinear time fractional Fisher's equation with specific boundary conditions.

The analytical method focuses on finding an exact solution to PDEs, but solving the time fractional Fisher equation is challenging due to the fractional derivative. The iterative method, using techniques like the New Sumudu transform iterative method, discretizes the equation in space and time, and updates it iteratively until it converges to the desired accuracy. The analytical method approximates the fractional derivative term, while the iterative method transforms the equation into linear or nonlinear algebraic equations that can be solved iteratively.

The software Mathematica provides powerful tools for creating visualizations and graphical representations of nonlinear time fractional Fisher's equations. It supports 2D and 3D plotting, graphing, and interactive visualizations, which aids in better understanding solutions of Fisher's equations.

The new Sumudu transform iterative method offers several advantages, including faster convergence, improved accuracy and precision, wide applicability, robustness, memory and computational efficiency, parallelization, adaptability to problem structure, trade-offs, comparative analysis, and experimental results.

Fisher's equation is a mathematical model that describes the spread of a mutant gene through a population. It is a partial differential equation with constant coefficients.

$$y_\omega(\xi,\omega) = y_{\xi,\xi}(\xi,\omega) + y(\xi,\omega)(1 - y(\xi,\omega)). \qquad 1$$

This model shows the population density by $y(\xi,\omega)$, and the logistic form is indicated by $y(y-1)$. In chemical kinetics and population dynamics, this equation solves problems like the nonlinear growth of a population in a habitat of 1-dimensional and the number of neutrons in a nuclear reaction. Also, one of the same equations is used in models growth of a logistic population[46–48], the spread of a flame, neurophysiology, chemical reactions that happen on their own and branching Brownian motion. In this article, Fisher's equation of time-fractional can be written as follows:

$$D_\omega^\beta y(\xi,\omega) = y_{\xi,\xi}(\xi,\omega) + \lambda y(\xi,\omega)(1 - y(\xi,\omega)), \qquad 2$$

$$0 < \beta \leq 1,$$

where $D_\omega^\beta y(\xi,\omega)$ denotes the caputo fractional derivative (C.F.D.) of order β and λ is a parameter (accurate)[49–51].

Background:

2024, 21(7): 2413-2424
https://doi.org/10.21123/bsj.2023.9137
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Definition 1: The $\beta \in \mathbb{C}, Re(\beta) > 0$ R-L fractional integral $I_{p+}^{\beta} f$ of order is defined by [50],

$$\left({}_pD_q^{-\beta} f\right)(q) = \left(I_{p+}^{\beta} f\right)(q)$$
$$= \frac{1}{\Gamma(\beta)} \int_p^q \frac{f(c)}{(q-c)^{1-\beta}} dc, (q > p, Re(\beta) > 0). \quad 3$$

Definition 2: The R-L fractional derivatives $\left({}_pD_q^{\beta} z\right)(x)$ of order $\beta \in \mathbb{C}, Re(\beta) > 0$ is defined by [50],

$$\left({}_pD_\xi^{\beta} z\right)(\xi) = \frac{d^{J}}{d\xi^{J}} \left(J_{p+}^{n-\beta} z\right)(\xi)$$
$$= \frac{1}{\Gamma(J-\beta)} \frac{d^{J}}{d\xi^{J}} \int_p^{\xi} (\xi - c)^{n-\beta-1} z(c) dc,$$
$$c > p, \ J = Re(\beta) + 1. \quad 4$$

Definition 3: Function of Mittag-Leffler and generalation[50]

$$E_\delta(z) = \sum_{m=0}^{\infty} \frac{z^m}{\Gamma(\delta^m + 1)} (\delta \in C, Re(\delta) > 0), \quad 5$$

$E_{\delta,\omega}$ is Mittag-Leffler function in two parameters.

$$E_{\delta,\omega}(z) = \sum_{m=0}^{\infty} \frac{y^m}{\Gamma(\delta^m + \omega)} \quad \omega, \delta \in C, Re(\delta) > 0,$$
$$Re(\omega) > 0. \quad 6$$

Definition 4: A caputo fractional derivative of function $y(\xi, t)$ is defined as [49],

$$D_\xi^{\beta} y(\xi, t) = \frac{1}{\Gamma(J-\beta)} \int_0^{\xi} (\xi - c)^{(J-\beta-1)} \frac{\partial^{J} y(c,t)}{\partial c^{J}} dc,$$
$$J - 1 < \beta \leq J, J \in N \quad 7$$

$d^J \equiv \frac{d^J}{dx^J}$ and j_x^{β} denote the R-L fractional integral operator of order $\beta > 0$ defined as $d^J \equiv \frac{d^J}{dx^J}$ and j_x^{β} respectively.

$$J_\xi^{\beta} y(\xi, t) = \frac{1}{\Gamma\beta} \int_0^{\xi} (\xi - c)^{(\beta-1)} y(c, t) dc, c > 0, z - 1 < \beta \leq z, z \in N. \quad 8$$

Definition 5: The Sumudu transform of a function $f(p), p > 0$ is defined as [51]

$$S[f(p)] = \int_0^{\infty} e^{-p} f(vp) dp, v \in (-P_1, P_2) \quad 9$$
$$\text{and } f(p) \in W,$$

where W =

$$\left\{ f(p), | \exists M, P_1, P_2 > 0, |f(p)| \leq Me^{\frac{|p|}{P_j}}, \text{ if } p \in (-1)^j \times [0, \infty) \right\}. \quad 10$$

Definition 6: The Sumudu transform of the Caputo fractional derivative is defined as[51],

$$S\left[D_\Upsilon^{n\beta} y(\Upsilon, \omega)\right] = v^{-n\beta} S[y(\Upsilon, \omega)] - \sum_{i=0}^{J-1} v^{-n\beta+i} y^{(i)}(0, \omega), \ J - 1 < n\beta < J. \quad 11$$

The New Sumudu transform Iterative Method (NSTIM):

To illustrate this New Iterative Transform of Sumudu Method [51–53] take into account a fractional partial differential equation with the initial conditions, which is both non-homogeneous and nonlinear:

$$D_\omega^{n\beta} z(\Upsilon, \omega) + Lz(\Upsilon, \omega) + R\big(z(\Upsilon, \omega)\big) = g(\Upsilon, \omega),$$
$$n - 1 < n\beta \leq n, z(\Upsilon, 0) = h(\Upsilon) \quad 12$$

where $D_\omega^{n\beta}$ is the fractional Caputo derivative operator, $D_\omega^{n\beta} = \frac{\partial^{n\beta}}{\partial \omega^{n\beta}}$, L-operator(linear), R-operator(non-linear), $g(\Upsilon, \omega)$ is continuous function.

Employing the Sumudu transform to Eq 12 obtain,

$$S\left[D_\omega^{n\beta} z(\Upsilon, \omega)\right] + S[L(z(\Upsilon, \omega)) + R(z(\Upsilon, \omega))] = S[g(\Upsilon, \omega], \quad 13$$

employing the property of sumudu transformation, obtain,

$$S[z(\Upsilon, \omega)] - v^{n\beta} \sum_{k=0}^{J-1} v^{-n\beta+k} z^{(k)}(\Upsilon, 0) + v^{n\beta} S\big[L\big(z(\Upsilon, \omega)\big) + R\big(z(\Upsilon, \omega)\big) - g(\Upsilon, \omega)\big] = 0. \quad 14$$

employing the Sumudu transform of inverse to Eq 14,

$$z(\Upsilon, \omega) = S^{-1}\big[v^{n\beta} \sum_{k=0}^{J-1} v^{-n\beta+k} z^{(k)}(\Upsilon, 0) - v^{n\beta} S\big[L\big(z(\Upsilon, \omega)\big) + R\big(z(\Upsilon, \omega)\big) - g(\Upsilon, \omega)\big]\big]. \quad 15$$

Next, assume that,

2024, 21(7): 2413-2424
https://doi.org/10.21123/bsj.2023.9137
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

$$f(\Upsilon,\omega) = S^{-1}\left[v^{n\beta}\sum_{k=0}^{J-1} v^{-n\beta+k} z^{(k)}(\Upsilon,0) + v^{n\beta}[g(\Upsilon,\omega)]\right]; \quad 16$$

$$N(z(\Upsilon,\omega)) = -S^{-1}\left[v^{n\beta}S[R(z(\Upsilon,\omega))]\right]; \quad 17$$

$$K(z(\Upsilon,\omega)) = -S^{-1}\left[v^{n\beta}S[L(z(\Upsilon,\omega))]\right]. \quad 18$$

Thus, Eq 15 will be reduced in the following form,

$$z(\Upsilon,\omega) = f(\Upsilon,\omega) + K(z(\Upsilon,\omega)) + N(z(\Upsilon,\omega)). \quad 19$$

The solution of the equation is given in the series form,

$$z(\Upsilon,\omega) = \left(\sum_{m=0}^{\infty} z_m(\Upsilon,\omega)\right),$$

Obtaining

$$K\left(\sum_{m=0}^{\infty} z_m(\Upsilon,\omega)\right) = \sum_{m=0}^{\infty} K(z_m(\Upsilon,\omega)). \quad 20$$

Operator N (nonlinear) is decomposed as

$$N\left(\sum_{m=0}^{\infty} z_m\right) = N(z_0) + \left\{N\left(\sum_{j=0}^{m} z_j\right) - N\left(\sum_{j=0}^{m-1} z_j\right)\right\}. \quad 21$$

Therefore, Eq 12 can be represented in the following form, Defining the recursive relation

$$z_0(\Upsilon,\omega) = f(\Upsilon,\omega),$$
$$z_1(\Upsilon,\omega) = K(z_0(\Upsilon,\omega)) + N(z_0(\Upsilon,\omega)),$$
$$z_{r+1}(\Upsilon,\omega) = K(z_m(\Upsilon,\omega)) + \left\{N\left(\sum_{j=0}^{r} z_j(\Upsilon,\omega)\right) - N\left(\sum_{j=0}^{r-1} z_j(\Upsilon,\omega)\right)\right\}, \quad \text{for all } r \geq 1 \quad 22$$

Thus, $(z_1 + z_2 + \ldots\ldots + z_{m+1}) = K(z_0 + \ldots. + z_m) + N(z_0 + \ldots. + z_m)$ 23

namely,

$$z(\Upsilon,\omega) = \sum_{m=0}^{\infty} z_m(\Upsilon,\omega) = f + K\left(\sum_{m=0}^{\infty} z_m(\Upsilon,\omega)\right) + N\left(\sum_{m=0}^{\infty} z_m(\Upsilon,\omega)\right) \quad 24$$

The m-term approximate solution of. Eq 12 is given by

$$z_m(\Upsilon,\omega) = z_0(\Upsilon,\omega) + z_1(\Upsilon,\omega) + \cdots + z_{m-1}(\Upsilon,\omega). \quad 25$$

Convergence and Error Analysis:

Theorem 1: Let $z_p(\Upsilon,\omega)$ and $z_n(\Upsilon,\omega)$ be the members of Banach space H and the exact solution of Eq 1 be $z(\Upsilon,\omega)$. The Series solution $\sum_{p=0}^{\infty} z_p(\Upsilon,\omega)$ converges to $z(\Upsilon,\omega)$, if $z_p(\Upsilon,\omega) \leq \lambda z_{p-1}(\Upsilon,\omega)$ for $\lambda \in (0,1)$, that is for any $z > 0, \exists E$ such that $||z_{p+n}(\Upsilon,\omega)|| \leq z, \forall p, n > E$.

Proof: Let $u_p(\Upsilon,\omega) = z_0(\Upsilon,\omega) + z_1(\Upsilon,\omega) + z_2(\Upsilon,\omega) + \cdots + z_p(\Upsilon,\omega)$ be the sequence of p^{th} partial sum of series $\sum_{p=0}^{\infty} z_p(\Upsilon,\omega)$.

Now, consider

$$\begin{aligned} ||u_{p+1}(\Upsilon,\omega) - u_p(\Upsilon,\omega)|| &= ||z_{p+1}(\Upsilon,\omega)|| \\ &\leq \lambda||z_p(\Upsilon,\omega)|| \\ &\leq \lambda^2||z_{p-1}(\Upsilon,\omega)|| \\ &\leq \lambda^3||z_{p-2}(\Upsilon,\omega)|| \\ &\vdots \\ &\leq \lambda^{p+1}||z_0(\Upsilon,\omega)||. \end{aligned} \quad 26$$

for $\forall n, p \in E$

Consider,

$||u_p(\Upsilon,\omega) - u_n(\Upsilon,\omega)||$

2024, 21(7): 2413-2424
https://doi.org/10.21123/bsj.2023.9137
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

$$= ||z_{p+n}(\Upsilon,\omega)||$$
$$= ||(u_p(\Upsilon,\omega) - u_{p-1}(\Upsilon,\omega))$$
$$+(u_{p-1}(\Upsilon,\omega) - u_{p-2}(\Upsilon,\omega))$$
$$+(u_{p-2}(\Upsilon,\omega) - u_{p-3}(\Upsilon,\omega))$$
$$+\cdots+(u_{n+1}(\Upsilon,\omega) - u_n(\Upsilon,\omega))||$$
$$\le ||(u_p(\Upsilon,\omega) - u_{p-1}(\Upsilon,\omega))||$$
$$+||(u_{p-1}(\Upsilon,\omega) - u_{p-2}(\Upsilon,\omega))||$$
$$+||(u_{p-2}(\Upsilon,\omega) - u_{p-3}(\Upsilon,\omega))|| \quad 27$$
$$+\cdots+||(u_{n+1}(\Upsilon,\omega) - u_n(\Upsilon,\omega))||$$
$$\le \lambda^p ||z_0(\Upsilon,\omega)||$$
$$+\lambda^{p-1}||z_0(\Upsilon,\omega)||$$
$$+\lambda^{p-2}||z_0(\Upsilon,\omega)||$$
$$+\cdots+\lambda^{p-1}||z_0(\Upsilon,\omega)||$$
$$= ||z_0(\Upsilon,\omega)||(\lambda^p + \lambda^{p-1} + \cdots + \lambda^{p+1})$$
$$= ||z_0(\Upsilon,\omega)||(\frac{1-\lambda^{p-n}}{1-\lambda})\lambda^{n+1}$$

Since $0 < \lambda < 1$ and $z_0(\Upsilon,\omega)$ is bounded, so assume that,

$$z = ||z_0(\Upsilon,\omega)||(\frac{1-\lambda^{p-n}}{1-\lambda})\lambda^{n+1},$$

The anticipated outcome is achieved.. Also $\sum_{p=0}^{\infty} z_p(\Upsilon,\omega)$ is a cauchy sequence in H, which implies that there exists $z_0(\Upsilon,\omega) \in H$ such that $\lim_{p\to\infty} z_p(\Upsilon,\omega) = z(\Upsilon,\omega)$. Hence, the proof has been completed.

Theorem 2: Let $\sum_{p=0}^{q} z_p(\Upsilon,\omega)$ be the finite and approximate solution of $z(\Upsilon,\omega)$. If $||z_{p+1}(\Upsilon,\omega)|| \le \lambda||z_0(\Upsilon,\omega)||$ for $\lambda \in (0,1)$, then the maximum absolute error is

$$||z(\Upsilon,\omega) - \sum_{p=0}^{q} z_p(\Upsilon,\omega)|| \le \frac{\lambda^{q+1}}{1-\lambda}||z_0(\Upsilon,\omega)||.$$

Proof: $||z(\Upsilon,\omega) - \sum_{p=0}^{q} z_p(\Upsilon,\omega)||$

$$= ||\sum_{p=0}^{\infty} z_p(\Upsilon,\omega)||$$
$$\le \sum_{p=q+1}^{\infty} ||z_p(\Upsilon,\omega)||$$
$$\le \sum_{p=q+1}^{\infty} \lambda^q ||z_0(\Upsilon,\omega)|| \quad 28$$
$$\lambda^{q+1}(1+\lambda+\lambda^2+\cdots)||z_0(\Upsilon,\omega)||$$
$$\le \frac{\lambda^{q+1}}{1-\lambda}||z_0(\Upsilon,\omega)||$$

Thus, the proof has been completed.

Results and Discussion

Numerical Examples:

Example 1: Suppose the following nonlinear time fractional Fisher's equation[54]

$$D_{\omega}^{\beta} y(\Upsilon,\omega) = \frac{\partial^2 y(\Upsilon,\beta)}{\partial \Upsilon^2} + 6y(1-y),\ 0 < \beta \le 1 \quad 29$$

The initial condition

$$y(\Upsilon,0) = \frac{1}{(1+e^{\Upsilon})^2}, \quad 30$$

employing Sumudu transform on Eq 29 and using the initial condition of Eq 30 to obtain,

$$S[y(\Upsilon,\omega)] = \frac{1}{(1+e^{\Upsilon})^2} + \frac{1}{u^{-\beta}} S[\frac{\partial^2 y}{\partial \Upsilon^2} + 6y(1-y)], \quad 31$$

employing the Sumudu transform of the inverse formula, the following equation is obtained,

$$y(\Upsilon,\omega) = \frac{1}{(1+e^{\Upsilon})^2} + S^{-1}[\frac{1}{u^{-\beta}} S[\frac{\partial^2 y}{\partial \Upsilon^2} + 6y(1-y)]],$$
$namely,$
$$y(\Upsilon,\omega) = \frac{1}{(1+e^{\Upsilon})^2} + S^{-1}[\frac{1}{u^{-\beta}} S[\frac{\partial^2 y}{\partial \Upsilon^2} + 6y(1-y)]]. \quad 32$$

According to NSTIM, result obtain;

$$y_0 = \frac{1}{(1+e^{\Upsilon})^2},$$
$$K[y(\Upsilon,\omega)] = S^{-1}[\frac{1}{u^{-\beta}} S[\frac{\partial^2 y}{\partial \Upsilon^2} + 6y(1-y)]]. \quad 33$$

By iteration, the following results are obtained

$$y_0(\Upsilon,\omega) = \frac{1}{(1+e^{\Upsilon})^2},$$

$$y_1(\Upsilon,\omega) = S^{-1}[\frac{1}{u^{-\beta}}S[\frac{\partial^2 y_0}{\partial \Upsilon^2} + 6y_0(1-y_0)]], \quad 34$$

$$= 10\frac{e^{\xi}}{(1+e^{\Upsilon})^3}\frac{(\omega^{\beta})}{\Gamma(\beta+1)}.$$

$$y_2(\Upsilon,\omega) = S^{-1}[\frac{1}{u^{-\beta}}S[\frac{\partial^2(y_0+y_1)}{\partial \Upsilon^2}]] - S^{-1}[\frac{1}{u^{\beta}}S[\frac{\partial^2 y_0}{\partial \Upsilon^2}]] \quad 35$$

$$= 50\frac{e^{\Upsilon}(-1+2e^{\Upsilon})}{(1+e^{\Upsilon})^4}\frac{(\omega^{2\beta})}{\Gamma(2\beta+1)}$$

$$y_3(\Upsilon,\omega) = 50\frac{e^{\Upsilon}(-1+2e^{\Upsilon})}{(1+e^{\Upsilon})^4}\frac{(\omega^{2\beta})}{\Gamma(2\beta+1)} \quad 36$$

Therefore, the Operating analytical solution of the problem in the series form can be obtained as,

$$y(\Upsilon,\omega) = y_0(\Upsilon,\omega) + y_1(\Upsilon,\omega) + \cdots$$

$$y(\Upsilon,\omega) = \frac{1}{(1+e^{\Upsilon})^2} + 10\frac{e^{\Upsilon}}{(1+e^{\Upsilon})^3}\frac{(\omega^{\alpha})}{\Gamma(\beta+1)} + 50\frac{e^{\Upsilon}(-1+2e^{\Upsilon})}{(1+e^{\Upsilon})^4}\frac{(\omega^{2\beta})}{\Gamma(2\beta+1)} +$$

$$250e^{\Upsilon}\left[5 - 6e^{\Upsilon} - 15e^{\Upsilon} + 20e^{3}\Upsilon - 12e^{\Upsilon}\frac{\Gamma(2\beta+1)}{\Gamma(\beta+1)^2}\right]\frac{\omega^{3}\beta}{(1+e^{\Upsilon})^6\Gamma(3\beta+1)} + \cdots \quad 37$$

Where - $E_{\beta}(\omega^{\beta})$ is mittage leffer function defined by Eq 7.

putting $\beta = 1$, Eq 29 becomes the following equation,

$$y(\Upsilon,\omega) = \frac{\partial^2 y}{\partial \Upsilon^2} - 6y - y$$

With accurate solution

$$y(\Upsilon,\omega) = \frac{1}{(1+e^{\Upsilon-5\omega})^2}. \quad 38$$

(See Fig. 1 and Table 1)

Remark 1: The linear time fractional Fisher equations are shown above. The estimated results of fractional Fisher equations of the time linear at values of β=0.2,0.4,0.6,0.8 and the accurate solution for β=1 are shown below. Fig 1 (a), in 3-dimension view and in Fig 1 (b), in 2-dimension forms, respectively. The answer is so simple to discover that it is constantly dependent on the values of time-fractional derivatives.

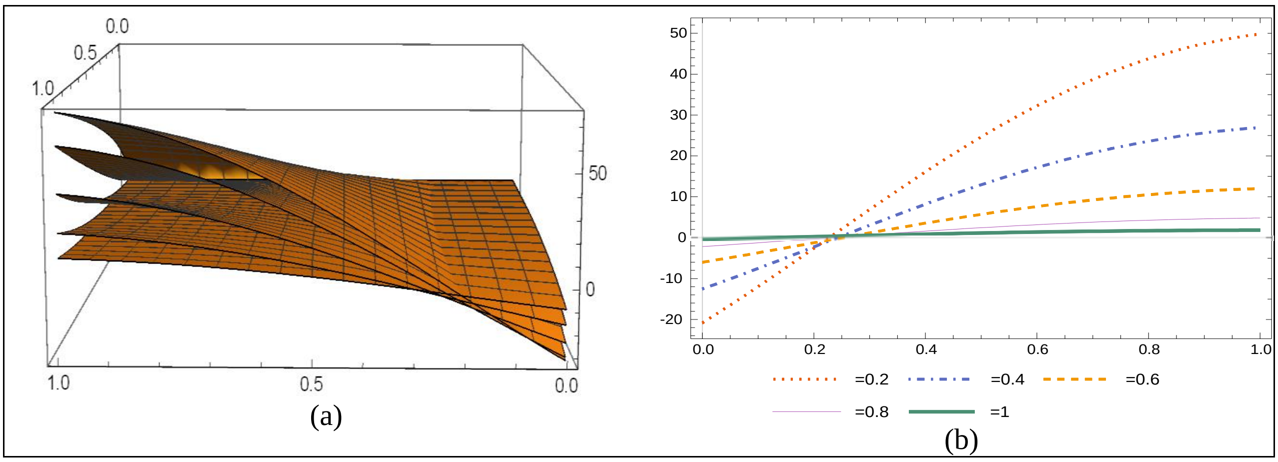


Figure 1. The estimated results of fractional Fisher equations of the time linear at values of β=0.2,0.4,0.6,0.8 and 1.

The numerical solution obtained using the NSTIM of 5th order approximation of Example 1 is compared with the accurate solution for $\beta = 1$ in Table 1, which shows the efficiency and effectiveness of the method.

Table 1. Error Analysis of Example 1.

		$\boldsymbol{\beta} = 1$		
Υ	ω	y(NSTIM)	y(accurate)	$\|yNSTIM - yaccurate\|$
0.2	0.3	0.0539935	0.0539944	9×10^{-07}
0.4	0.5	0.26375	0.263795	5×10^{-05}
0.6	0.7	0.724381	0.724951	6×10^{-04}
0.8	0.9	1.57046	1.57415	4×10^{-03}

Example 2: Suppose the nonlinear time fractional following Fisher's eq[54]

$$D_{\omega}^{\beta} y(\Upsilon,\omega) = \frac{\partial^2 y(\Upsilon,\omega)}{\partial \Upsilon^2} + y(1-y), 0<\beta\le 1 \quad 39$$

The initial condition

$$y(\Upsilon,0) = \alpha, \quad 40$$

employing Sumudu transform on the Eq 39 and using the initial condition of Eq 40 obtain,

$$S[y(\Upsilon,\omega)] = \alpha + \frac{1}{u^{-\beta}} S[\frac{\partial^2 y}{\partial \Upsilon^2} + y(1-y)], \quad 41$$

employing the Sumudu transform of the inverse formula,

$$y(\Upsilon,\omega) = \alpha + S^{-1}[\frac{1}{u^{-\beta}} S[\frac{\partial^2 y}{\partial \Upsilon^2} + y(1-y)]],$$
$$namely,$$
$$y(\Upsilon,\omega) = \alpha + S^{-1}[\frac{1}{u^{-\beta}} S[\frac{\partial^2 y}{\partial \Upsilon^2} + y(1-y)]]. \quad 42$$

According to the NSTIM, result obtain

$$y_0 = \alpha,$$
$$K[y(\Upsilon,\omega)] = S^{-1}[\frac{1}{u^{-\beta}} S[\frac{\partial^2 y}{\partial \Upsilon^2} + y(1-y)]]. \quad 43$$

By iteration, the following results are obtained

$$y_0(\Upsilon,\omega) = \alpha,$$
$$y_1(\Upsilon,\omega) = S^{-1}[\frac{1}{u^{-\beta}} S[\frac{\partial^2 y_0}{\partial \Upsilon^2} + y_0(1-y_0)]],$$
$$= \alpha(1-\alpha)\frac{(\omega^{\beta})}{\Gamma(\beta+1)}. \quad 44$$

$$y_2(\Upsilon,\omega) = S^{-1}[\frac{1}{u^{-\beta}} S[\frac{\partial^2 (y_0+y_1)}{\partial \Upsilon^2}]]$$
$$-S^{-1}\left[\frac{1}{u^{-\beta}} S\left[\frac{\partial^2 y_0}{\partial \Upsilon^2}\right]\right], \quad 45$$
$$= \alpha(1-\alpha)(1-2\alpha)\frac{(\omega^{2\beta})}{\Gamma(2\beta+1)}.$$

$$y_3(\Upsilon,\omega) = (\alpha - 5\alpha^2 + 8\alpha^3 - 4\alpha^4)\frac{(\omega^{3\beta})}{\Gamma(3\beta+1)}$$
$$-(\alpha^2 - 2\alpha^3 + \alpha^4)\frac{\Gamma(2\beta+1)}{[\Gamma(\beta+1)]^2}\frac{(\omega^{3\beta})}{\Gamma(3\beta+1)}. \quad 46$$

$$y_4(\Upsilon,\omega) = (1-2\alpha)(\alpha - 5\alpha^2 + 8\alpha^3 - 4\alpha^4)$$
$$-(\alpha^2 - 2\alpha^3 + \alpha^4)\frac{\Gamma(2\beta+1)}{[\Gamma(\beta+1)]^2}\frac{(\omega^{4\beta})}{\Gamma(4\beta+1)}$$
$$-2(\alpha-\alpha^2)(\alpha - 3\alpha^3 + 2\beta^3)\frac{\Gamma(3\alpha+1)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}\frac{(\omega^{4\beta})}{\Gamma(4\beta+1)}. \quad 47$$

Therefore, the analytical estimated results of the problem in the series form can be found as,

$$y(\Upsilon,\omega) = y_0(\Upsilon,\omega) + y_1(\Upsilon,\omega) + \cdots \quad 48$$

$$y(\Upsilon,\omega) = \begin{bmatrix} \alpha + \alpha(1-\alpha)\frac{(\omega^{\beta})}{\Gamma(\beta+1)} + \alpha(1-\alpha)(1-2\alpha)\frac{(\omega^{2\beta})}{\Gamma(2\beta+1)} \\ +(\alpha - 5\alpha^2 + 8\alpha^3 - 4\alpha^4)\frac{(\omega^{3\beta})}{\Gamma(3\beta+1)} \\ -(\alpha^2 - 2\alpha^3 + \alpha^4)\frac{\Gamma(2\beta+1)}{[\Gamma(\beta+1)]^2}\frac{(\omega^{3\beta})}{\Gamma(3\beta+1)} \\ +(1-2\alpha)(\alpha - 5\alpha^2 + 8\alpha^3 - 4\alpha^4) \\ -7(\alpha^2 - 2\alpha^3 + \alpha^4)\frac{\Gamma(2\beta+1)}{[\Gamma(\beta+1)]^2}\frac{(\omega^{4\beta})}{\Gamma(4\beta+1)} \\ -2(\alpha-\alpha^2)(\alpha - 3\alpha^3 + 2\alpha^3)\frac{\Gamma(3\beta+1)}{\Gamma(\beta+1)\Gamma(2\beta+1)}\frac{(\omega^{4\beta})}{\Gamma(4\beta+1)} + \cdots \end{bmatrix} \quad 49$$

Where - $E_{\beta}(\omega^{\beta})$ is mittage leffer function defined by Eq 7.

putting $\beta = 1$, Eq 39 becomes the following equation,

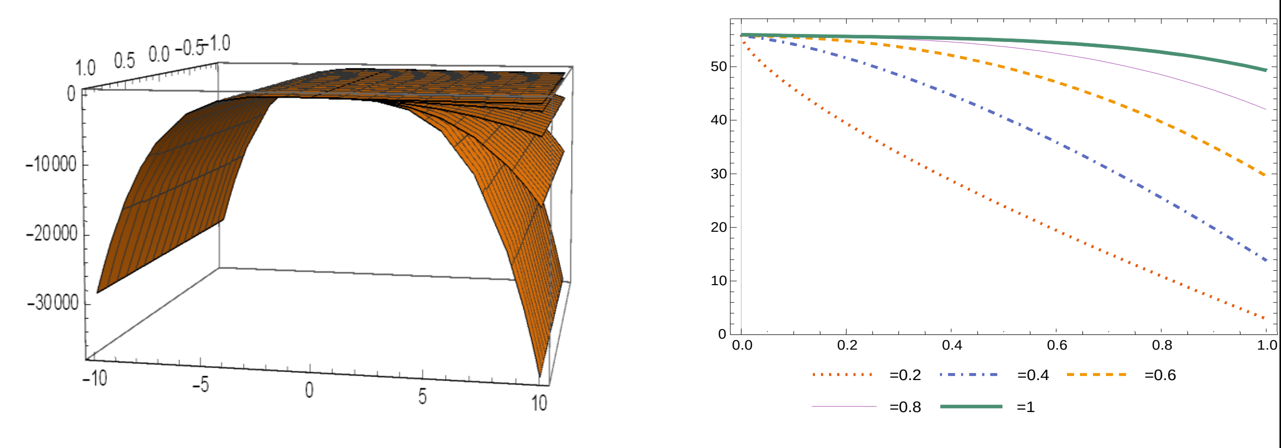
$$y(\Upsilon,\omega) = \frac{\partial^2 y}{\partial \Upsilon^2} + y(1-y),$$

with accurate solution 50

$$y(\Upsilon,\omega) = \frac{\alpha e^{\Upsilon}}{(1-\alpha+\alpha e^{\omega})}$$

(See Fig. 2 and Table 2)

Remark 2: The linear time fractional Fisher equations are shown above. The estimated results of the linear time fractional Fisher equations at values of β=0.2,0.4,0.6,0.8 and the accurate solution for β=1 are shown below in Fig. 2 (a), in 3-dimension view and in Fig. 2 (b), in 2-dimension forms respectively. The solution is so simple to discover that it is constantly dependent on the values of time-fractional derivatives.



(a) (b)

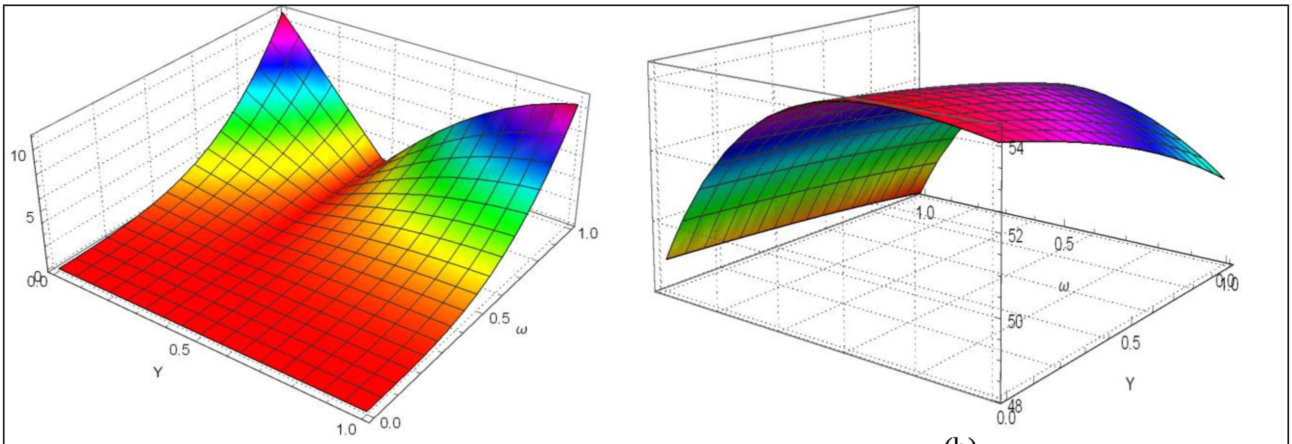
Figure 2. The estimated results of the linear time fractional Fisher equations at values of $\boldsymbol{\beta}$=0.2,0.4,0.6,0.8 and 1

The numerical solution obtained using the NSTIM of 5th order approximation of Example 2 is compared with the accurate solution for $\beta = 1$ in Table 2, which shows the efficiency and effectiveness of the method

Table 2. Error Analysis of Example 2

		$\boldsymbol{\beta} = 1$		
Y	ω	y(NSTIM)	y(accurate)	$\|y_{NSTIM} - y_{accurate}\|$
0.2	0.3	0.109149	0.109032	1×10^{-04}
0.4	0.5	0.146032	0.143259	3×10^{-03}
0.6	0.7	0.159643	0.139239	2×10^{-02}
0.8	0.9	0.204733	0.118578	9×10^{-02}

Remark 3: Fig 3(a) and Fig 3(b) depict the absolute error between estimated and accurate solutions for β=1. By comparison, it is clear that by computing additional terms, the efficiency and accuracy of this method (NSTIM) can be significantly improved. The authors have used a few iterations in this post. However, the precision of the estimated solution could be substantially enhanced if they employed additional terms. As a result, the recommended method for solving the linear differential equation is both precise and efficient.



(a) (b)

Figure 3. Absolute error between estimated and accurate solutions for β=1.

Conclusion

This research used the novel Sumudu transform iterative technique to solve linear time fractional Fisher equations. The novel Sumudu transform iterative approach (NSTIM) combines NIM and Sumudu to solve linear time fractional Fisher equations. The iterative Sumudu transform approach is more structured and accurate and requires less numerical calculation, according to the numbers.

The approach reduces computational effort compared to conventional methods while maintaining good numerical precision. It can aid mathematicians and researchers working in the field of partial differential equations. The key advantage of this method is quick convergence and accuracy. NSTIM is an effective tool for discovering approximation and semi-analytical solutions.

Acknowledgment

For their help and direction, while writing this research work, all the co-authors and teachers deserve gratitude, including Professors Ashok P. Bhadane, S.B. Gaikwad, and Kishor A. Kshirsagar, among others.

Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Commerce and Science College, Ahmednagar, Maharashtra, India.

Authors' Contribution Statement

S. T. proposed the Concepts, ideas, method, and analysis. A. B. designed the manuscript. S. G. read the manuscript and revised it. K. K. Drafting of the manuscript with interpreted and plotted the graphs of the solution of examples using Mathematica 11.3. All authors read and approved the final manuscript.

References

1. Rafeiro H, Samko S. Fractional integrals and derivatives: Mapping properties. Vol. 19, Fractional Calculus and Applied Analysis. Gordon and Breach; 2016. p. 580–607. http://tocs.ulb.tu-

darmstadt.de/32759916.pdf
2. Yang XJ, Baleanu D, Srivastava HM. Local Fractional Integral Transforms and Their Applications. Local Fractional Integral Transforms and Their Applications. Elsevier; 2015. 1–249 p. https://linkinghub.elsevier.com/retrieve/pii/C20140047685
3. Debnath L. Nonlinear partial differential equations for scientists and engineers. Nonlinear Partial Differential Equations for Scientists and Engineers. Boston: Birkhäuser Boston; 2012. 1–860 p. https://link.springer.com/10.1007/978-0-8176-8265-1
4. Abukhaled M, Khuri SA. RLC electric circuit model of fractional order: a Green's function approach. Int J Comput Math. 2023 Apr 17; 1–0. https://www.tandfonline.com/doi/abs/10.1080/00207160.2023.2203787
5. Bin-Mohsin B, Awan MU, Javed MZ, Khan AG, Budak H, Mihai M V., et al. Generalized AB-Fractional Operator Inclusions of Hermite–Hadamard's Type via Fractional Integration. Symmetry (Basel). 2023 May 1; 15(5): 1012. https://www.mdpi.com/2073-8994/15/5/1012
6. Sadri K, Hosseini K, Baleanu D, Salahshour S, Hinçal E. A robust scheme for Caputo variable-order time-fractional diffusion-type equations. J Therm Anal Calorim. 2023 Jun 28; 148(12): 5747–64. https://link.springer.com/10.1007/s10973-023-12141-0
7. Logeswari K, Ravichandran C, Nisar KS. Mathematical model for spreading of COVID-19 virus with the Mittag–Leffler kernel. Numer Methods Partial Differ Equ. 2020; https://onlinelibrary.wiley.com/doi/full/10.1002/num.22652
8. Nisar KS, Akinyemi L, Inc M, Şenol M, Mirzazadeh M, Houwe A, et al. New perturbed conformable Boussinesq-like equation: Soliton and other solutions. Results Phys. 2022 Feb 1; 33: 105200. https://linkinghub.elsevier.com/retrieve/pii/S221137972200016X
9. Nisar KS, Ali KK, Inc M, Mehanna MS, Rezazadeh H, Akinyemi L. New solutions for the generalized resonant nonlinear Schrödinger equation. Results Phys. 2022 Feb 1; 33: 105153. https://linkinghub.elsevier.com/retrieve/pii/S2211379721011128
10. Ghode K, Takale K, Gaikwad S. Traveling Wave Solutions of Fractional Differential Equations Arising in Warm Plasma. Baghdad Sci J. 2023 Mar 1; 20(1(SI)): 318–25. https://doi.org/10.21123/bsj.2023.8394
11. Wazwaz AM. A reliable modification of Adomian decomposition method. Appl Math Comput. 1999 Jul; 102(1): 77–86. https://linkinghub.elsevier.com/retrieve/pii/S0096300398100243
12. Nikam VR, Gaikwad SB, Tarate SA, Kshirsagar KA. Fuzzy Laplace-Adomian Decomposition Method for Approximating Solutions of Time Fractional Klein-Gordan Equations in a Fuzzy Environment. Eur Chem Bull. 2023; 12(8): 5926–43. https://www.researchgate.net/publication/372677786_Fuzzy_Laplace
13. Ahmed RF, Al-Hayani WM, Al-Bayati AY. The Homotopy Analysis Method to Solve the Nonlinear System of Volterra Integral Equations and Applying the Genetic Algorithm to Enhance the Solutions. Eur J Pure Appl Math. 2023 Apr 30; 16(2): 864–92. https://ejpam.com/index.php/ejpam/article/view/4693
14. Rehman G, Qin S, Ain QT, Ullah Z, Zaheer M, Talib MA, et al. A study of moisture content in unsaturated porous medium by using homotopy perturbation method (HPM) and variational iteration method (VIM). GEM - Int J Geomathematics. 2022; 13(1). https://doi.org/10.1007/s13137-021-00193-z
15. Tarate, S. A., Bhadane, A. P., Gaikwad, S. B., & Kshirsagar, K. A. A Semi-Analytic Solution For Time-Fractional Heat Like And Wave Like Equations Via Novel Iterative Method.Eur. Chem. Bull. 2023,12(Specialissue8),6164-6187 https://doi.org/10.48047/ecb/2023.12.si8.5242023.27/07/2023 www.eurchembull.com
16. Dumka P, Pawar PS, Sauda A, Shukla G, Mishra DR. Application of He's homotopy and perturbation method to solve heat transfer equations: A python approach. Adv Eng Softw. 2022; 170(June): 103160. https://doi.org/10.1016/j.advengsoft.2022.103160
17. da C. Sousa JV, Kucche KD, de Oliveira EC. Stability of mild solutions of the fractional nonlinear abstract Cauchy problem. Electron Res Arch. 2021; 30(1) : 272–88. http://www.aimspress.com/article/doi/10.3934/era.2022015
18. Sonawane J, Sontakke B, Takale K. Approximate Solution of Sub diffusion Bio heat Transfer Equation. Baghdad Sci J. 2023 Mar 4; 20(1(SI)): 0394. https://doi.org/10.21123/bsj.2023.8410
19. Gaikwad V. Fractional Hartley Transform and its Inverse. Baghdad Sci J. 2023; 20(1(SI)): 339–44. https://doi.org/10.21123/bsj.2023.8396
20. Joseph D, Ramachandran R, Alzabut J, Jose SA, Khan H. A Fractional-Order Density-Dependent Mathematical Model to Find the Better Strain of Wolbachia. Symmetry (Basel). 2023 Apr 1; 15(4).
21. Anggriani N, Panigoro HS, Rahmi E, Peter OJ, Jose SA. A predator–prey model with additive Allee effect and intraspecific competition on predator involving Atangana–Baleanu–Caputo derivative. Results Phys. 2023 Jun 1; 49.
22. songkran pleumpreedaporn wsctsaj. qualitative analysis of generalized proportional fractional functional integro-differential langevin equation with variable coefficient and nonlocal integral conditions. *Mem Differ Equ Math Phys* 2021;83:99–120. https://rmi.tsu.ge/jeomj/memoirs/vol83/abs83-

8.htm
23. Jose SA, Ramachandran R, Baleanu D, Panigoro HS, Alzabut J, Balas VE. Computational dynamics of a fractional order substance addictions transfer model with Atangana-Baleanu-Caputo derivative. Math MethodsApplSci.2023; 46(5). https://api.semanticscholar.org/CorpusID:253608454
24. Jose SA, Raja R, Alzabut J, Rajchakit G, Cao J, Balas VE. Mathematical modeling on transmission and optimal control strategies of corruption dynamics. NonlinearDyn.2022;109(4). https://doi.org/10.1142/S1793557118500900
25. Jose SA, Raja R, Dianavinnarasi J, Baleanu D, Jirawattanapanit A. Mathematical modeling of chickenpox in Phuket: Efficacy of precautionary measures and bifurcation analysis. Biomed Signal Process Control. 2023; 84. https://api.semanticscholar.org/CorpusID:25719386526.
26. Zhang Y, Cattani C, Yang XJ. Local fractional homotopy perturbation method for solving non-homogeneous heat conduction equations in fractal domains. Entropy. 2015 Oct 5; 17(10): 6753–64. http://www.mdpi.com/1099-4300/17/10/6753
27. Bhalekar S, Daftardar-Gejji V. Convergence of the New Iterative Method. Int J Differ Equ. 2011; 2011: 1–10. http://www.hindawi.com/journals/ijde/2011/989065/
28. Gupta VG, Shrama B, Kiliçman A. A note on fractional sumudu transform. J Appl Math. 2010; 2010: 1–9. http://www.hindawi.com/journals/jam/2010/154189/
29. Almeida R, Malinowska AB, Torres DFM. A fractional calculus of variations for multiple integrals with application to vibrating string. J Math Phys. 2010; 51(3). http://jmp.aip.org/jmp/copyright.jsp
30. Khader MM, Sweilam NH, Mahdy AMS. Two computational algorithms for the numerical solution for system of fractional differential equations. Arab J Math Sci. 2015 Jan 1; 21(1): 39–52. https://doi.org/10.1016/j.ajmsc.2013.12.001.
31. Abbasbandy S. The application of homotopy analysis method to nonlinear equations arising in heat transfer. Phys Lett A. 2006 Dec 18; 360(1): 109–13. https://linkinghub.elsevier.com/retrieve/pii/S0375960106011984
32. He J, Yu Z, Cao J, Song W, Xu K, Fan W, et al. Rationally selecting the chemical composition of the Nd-Fe-B magnet for high-efficiency grain boundary diffusion of heavy rare earths. J Mater Chem C. 2022 Feb10;10(6):2080–8. https://pubs.rsc.org/en/content/articlehtml/2022/tc/d1tc05469d
33. Saadatmandi A, Dehghan M. A tau approach for solution of the space fractional diffusion equation. Comput Math with Appl. 2011 Aug 1; 62(3): 1135–42. https://linkinghub.elsevier.com/retrieve/pii/S0898122111003014
34. the double fuzzy elzaki transform for solving fuzzy partial differential equations. https://doi.org/10.14403/jcms.2022.35.2.177
.35. Mahdy AMS. A numerical method for solving the nonlinear equations of Emden-Fowler models. J OceanEngSci.2022; https://doi.org/10.1016/j.joes.2022.04.019
36. Mohamed MS, Elagan SK, Almalki SJ, Alharthi MR, El-Badawy MF, Najati SA, et al. Optimal Control and Solving of Cellular DNA Cancer Model. Appl Math Inf. Sci. 2022;16(1):109–19. https://doi.org/10.32604/cmc.2021.017208
37. Mahdy AMS, Higazy M, Mohamed MS. Optimal and Memristor-Based Control of A Nonlinear Fractional Tumor-Immune Model. Comput Mater Contin. 2021; 67(3). https://doi.org/10.32604/cmc.2021.015161
38. Gepreel KA, Mohamed MS, Alotaibi H, Mahdy AMS. Dynamical behaviors of nonlinear coronavirus (COVID-19) model with numerical studies. Comput Mater Contin. 2021; 67(1): 675–86. https://doi.org/10.32604/cmc.2021.012200
39. Yildiz AR. Hybrid immune-simulated annealing algorithm for optimal design and manufacturing. Int J Mater Prod Technol. 2009; 34(3): 217–26. http://www.inderscience.com/link.php?id=24655
40. Gepreel KA, Higazy M, Mahdy AMS. Optimal control, signal flow graph, and system electronic circuit realization for nonlinear Anopheles mosquito model. Int J Mod Phys C. 2020; 31(9). https://api.semanticscholar.org/CorpusID:225288440
41. Alotaibi H, Gepreel KA, Mohamed MS, Mahdy AMS. An Approximate Numerical Methods for Mathematical and Physical Studies for Covid-19 Models. Comput Syst Sci Eng. 2022; 42(3). https://doi.org/10.32604/csse.2022.020869.
42. Mahdy AMS, Mohamed MS, Amiri AYA, Gepreel KA. Optimal control and spectral collocation method for solving smoking models. Intell Autom Soft Comput.2022;31(2). https://doi.org/10.32604/iasc.2022.017801
43. Mahdy AMS, Mohamed MS, Lotfy K, Alhazmi M, El-Bary AA, Raddadi MH. Numerical solution and dynamical behaviors for solving fractional nonlinear Rubella ailment disease model. Results Phys. 2021 May1;24. https://doi.org/10.1016/j.rinp.2021.104091
44. Mahdy AMS, Higazy M. Numerical Different Methods for Solving the Nonlinear Biochemical Reaction Model. Int J Appl Comput Math. 2019 Dec 1;5(6):1–17. https://link.springer.com/article/10.1007/s40819-019-0740-x
45. Higazy M, El-Mesady A, Mahdy AMS, Ullah S, Al-Ghamdi A. Numerical, Approximate Solutions, and Optimal Control on the Deathly Lassa Hemorrhagic Fever Disease in Pregnant Women. J Funct Spaces. 2021;2021. https://api.semanticscholar.org/CorpusID:245240155

2024, 21(7): 2413-2424
https://doi.org/10.21123/bsj.2023.9137
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

46. Mahdy AMS, Amer YAE, Mohamed MS, Sobhy E. General fractional financial models of awareness with Caputo–Fabrizio derivative. Adv Mech Eng. 2020; 12(11). https://doi.org/10.1177/1687814020975525
47. Khader MM, Swetlam NH, Mahdy AMS. The chebyshev collection method for solving fractional order klein-gordon equation. Vol. 13, WSEAS Transactions on Mathematics. 2014. About 218,000 search results https://api.semanticscholar.org/CorpusID:16884907
48. Mahdy AMS, Gepreel KA, Lotfy K, El-Bary AA. A numerical method for solving the Rubella ailment disease model. Int J Mod Phys C. 2021; 32(7). https://api.semanticscholar.org/CorpusID:233656249
49. Ortigueira MD. A travel through the world of fractional calculus. Lect Notes Electr Eng. 2011; 84 LNEE:1–3. https://link.springer.com/chapter/10.1007/978-94-007-0747-4_1
50. Kochubei A, Luchko Y. Fractional Differential Equations. Vol 2 Fractional Differential Equations. De Gruyter; 2019. 1–519 p. https://www.degruyter.com/document/doi/10.1515/9783110571660/html
51. Kapoor M. Analytical Approach for Solution of Linear and Non-linear Time-Fractional Schrödinger's Equations by Employing Sumudu Transform Iterative Method. Int J Appl Comput Math. 2023 Jun 27; 9(3): 38. https://link.springer.com/10.1007/s40819-023-01508-4
52. Tarate SA, Bhadane AP, Gaikwad SB, Kshirsagar KA. Sumudu-iteration transform method for fractional telegraph equations. J Math Comput Sci. 2022;12(0):ArticleID127. http://scik.org/index.php/jmcs/article/view/7255
53. Tarate S A, Bhadane A P, Gaikwad S B, Kshirsagar K A. Solution of time-fractional equations via Sumudu-Adomian decomposition method. Comput. Methods Differ. Equ. 2023; 11(2): 345-356. http://cmde.tabriz.ac.ir. https://doi.org/10.22034/cmde.2022.51421.2139
54. Sharma SC, Bairwa RK. Iterative Laplace Transform Method for Solving Fractional Heat and Wave-Like Equations. Res J Math Stat Sci. 2015; 3(2): 4–9. https://api.semanticscholar.org/CorpusID:124713816

الحلول شبه التحليلية لمعادلات فيشر الكسرية للزمن عبر الطريقة التكرارية الجديدة

شيفاجي أشوك تارات [1]*، أ.ب. بهادان [2]،إس.بي. جايكواد [1] ، ك. كشيرساجار [1]

[1]قسم الرياضيات، كلية الفنون الجديدة والتجارة والعلوم، أحمد نجار، ماهاراشترا، الهند.
[2] قسم الرياضيات، لوكنيتي فيانكاتراو هيراي للفنون، كلية العلوم والتجارة، ناشيك، ماهاراشترا، الهند.

الخلاصة

إحدى الطرق الفعالة لحل المعادلات التفاضلية الجزئية غير الخطية ذات المشتقات الكسرية هي الطريقة التكرارية لتحويل سومودو الجديدة (NSTIM). إنه يبرع في حل الألغاز الرياضية الصعبة ويقدم معلومات ثاقبة حول سلوك معادلات فيشر ذات الكسر الزمني. الطريقة، التي تستخدم مشتقات كابوتو الحسية وWolfram في Mathematica، موثوقة وسهلة الاستخدام وتعطي تصويرًا مرئيًا للحل. أظهرت النتائج التحليلية أن الطريقة المقترحة فعالة وبسيطة في توليد حلول دقيقة لمعادلات فيشر الكسرية للزمن. أصبحت النتائج أكثر موثوقية وقابلة للتطبيق من خلال تضمين مشتقات كابوتو الحسية. تعتمد النمذجة الرياضية على فعالية وبساطة منهج NSTIM لحل معادلات فيشر ذات الكسر الزمني لأنها تتيح حلولاً دقيقة دون استخدام الكثير من قوة المعالجة. يعد نهج NSTIM أداة مفيدة للباحثين في مجموعة متنوعة من المجالات لأنه يوفر أيضًا إطارًا مرنًا يمكن تعديله بسهولة مع المعادلات التفاضلية الكسرية الأخرى. أصبح من الممكن الآن فحص ديناميكيات وسلوك الأنظمة المعقدة التي تحكمها معادلات فيشر الكسرية الزمنية بكفاءة وموثوقية، مما يفتح طرقًا بحثية جديدة. إن القدرة على حل معادلات فيشر ذات الكسور الزمنية بكفاءة وموثوقية باستخدام نهج NSTIM لها آثار مهمة على مجالات مختلفة مثل الديناميات السكانية والبيولوجيا الرياضية وعلم الأوبئة. يمكن للباحثين الآن تحليل انتشار الأمراض أو دراسة الديناميكيات السكانية للأنواع بدقة أعلى وجهد حسابي أقل. يمهد هذا التقدم في حل المعادلات التفاضلية الكسرية الطريق لرؤى أعمق حول سلوك وأنماط الأنظمة المعقدة، مما يؤدي في نهاية المطاف إلى تعزيز الفهم العلمي وتقديم إمكانيات جديدة للتطبيقات العملية.

الكلمات المفتاحية: مشتقة كابوتو الكسرية، معادلات فيشر، حساب التفاضل والتكامل الكسري، الطريقة التكرارية، تحويل سومودو.